Exercise 1i) Determine the affine transfomration that takes P1(1,1,1) to P1'(−1,2,1), P2(3,3,1),to P2'(3,4,1) and P3(−1,3,1) to P3'(−3,6,1).
ii) Determine the affine transfomration that takes l[1,−2,5] to l'[1,−1,0], m[2,1,0] to l'[1,1,−2], and n[1,3,−15] to n'[10,1−3].
iii) Give the coordinates of the lines supporting the sides of the triangle P1P2P3 above
iv) Determine the corners of the triangle with sides on the lines l, m and n above
v) what is the relation between the transformations in point i) and ii) ?

1i) Consider A thematrix of the input−points and B the matrix of the value−points. The matrix of the transformation, SOl1, is BA^{-1}.

ii)−v) Using the determinant function we see that the sides of the triangle P1P2P3 are in fact the lines l' m' and n'. And in the same way we see that the corners in the triangle with sides l, m and n are the points P1', P2' and P3'. So the second transformation is the inverse of the first transformation. Then ist matrix SOL22 and the first six coordinates in solalt is the inverse of SOL1 as we see in the calculations. The three methods used are
First A2 is the matrix of the points P1'| P2'| P3'. B2 is the matrix of P1|P2|P3 so SOL22 = $B2A2^{-1}$.

If we used the equation $u'A_{f2} = su$ (Bpar and bb) or $tu' = uA_{f2}^{-1}$ (Bs and b) we get a system 9 x9 where the first six variables are the parameters in A_{f2} and A_{f2}^{-1} respectively, and the three last paameters are s1, s2, s3, and t1, t2, t3 respectively.

We see that SOL22 = InvSOL1 and the first six parameters in sol are the parameters in SOL1. So The two transformations are the inverse of each other!!

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> with(LinearAlgebra) :
  A := Matrix([[1, 3,-1], [1, 3, 3], [1, 1, 1]]);
  B := Matrix([[-1, 3,-3], [2, 4, 6], [1, 1, 1]]);
  Ainv := MatrixInverse(A) :
  SOL1 := MatrixMatrixMultiply(B, Ainv); InvSOL1 := MatrixInverse(SOL1);



  A2 := Matrix([[-1, 3,-3], [2, 4, 6], [1, 1, 1]]);
  B2 := Matrix([[1, 3,-1], [1, 3, 3], [1, 1, 1]]);
  SOL22 := MatrixMatrixMultiply(B2, MatrixInverse(A2));
  Bpar := Matrix([[1,-1, 0, 0, 0, 0,-1, 0, 0], [0, 0, 1,-1, 0, 0, 2, 0, 0], [0, 0, 0, 0, 1,
      -1,-5, 0, 0], [1, 1, 0, 0, 0, 0, 0,-2, 0], [0, 0, 1, 1, 0, 0, 0,-1, 0], [0, 0, 0, 0, 1, 1,
      0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0,-1], [0, 0, 0, 1, 0, 0, 0, 0,-3], [0, 0, 0, 0, 0, 1, 0, 0,
      15]]);
  bb := Vector([0, 0, 0, 0, 0, 2, 0, 0, 3]);
  solalt := LinearSolve(Bpar, bb);
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Bs := Matrix([[1,-2, 0, 0, 0, 0,-1, 0, 0], [0, 0, 1,-2, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1,-2,
    0, 0, 0], [2, 1, 0, 0, 0, 0, 0,-1, 0], [0, 0, 2, 1, 0, 0, 0,-1, 0], [0, 0, 0, 0, 2, 1, 0, 2,
    0], [1, 3, 0, 0, 0, 0, 0, 0, 0], [0, 0, 1, 3, 0, 0, 0, 0,-1], [0, 0, 0, 0, 1, 3, 0, 0, 3]]);
b := Vector([0, 0,-5, 0, 0, 0, 0, 0, 15]);
sol := LinearSolve(Bs, b);
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$$A := \begin{bmatrix} 1 & 3 & -1 \\ 1 & 3 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$B := \begin{bmatrix} -1 & 3 & -3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{bmatrix}$$

$$SOL1 := \begin{bmatrix} \frac{3}{2} & \frac{1}{2} & -3 \\ -\frac{1}{2} & \frac{3}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$InvSOL1 := \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} & 2 \\ \frac{1}{5} & \frac{3}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A2 := \begin{bmatrix} -1 & 3 & -3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{bmatrix}$$

$$B2 := \begin{bmatrix} 1 & 3 & -1 \\ 1 & 3 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$SOL22 := \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} & 2 \\ \frac{1}{5} & \frac{3}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Bpar := \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -5 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 15 \end{bmatrix}$$

$$bb := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$

$$solalt := \begin{bmatrix} \frac{3}{5} \\ \frac{1}{5} \\ -\frac{1}{5} \\ \frac{3}{5} \\ 2 \\ 0 \\ \frac{2}{5} \\ \frac{2}{5} \\ \frac{1}{5} \end{bmatrix}$$

$$Bs := \begin{bmatrix} 1 & -2 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 2 & 0 \\ 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 & 3 \end{bmatrix}$$

$$b := \begin{bmatrix} 0 \\ 0 \\ -5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 15 \end{bmatrix}$$

$$sol := \begin{bmatrix} \frac{3}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ \frac{3}{2} \\ -3 \\ 1 \\ \frac{5}{2} \\ \frac{5}{2} \\ 5 \end{bmatrix} \qquad (1)$$

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