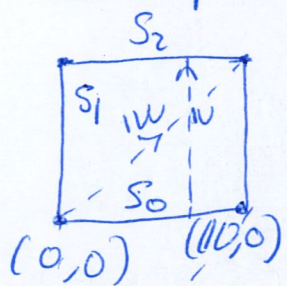


(2) Instancing

Instancing: Drawing an object by making each part of the object as a copy (instance) of a base object, called the graphic primitive. Each copy (instance) of the graphic primitive is obtained by applying an affine transformation to the graphic primitive.

Example 1: Make a square from a segment $(10, 0)$



$$S_1 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} S_0$$

rotation with angle 90° at $(0, 0)$

$$S_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix} S_0$$

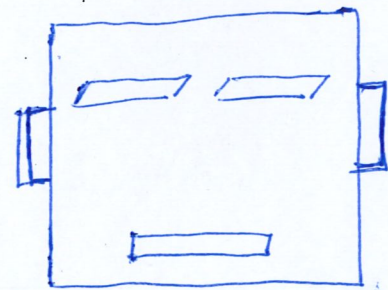
translation with vector $w = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$ of S_0

$$S_0 = \begin{pmatrix} 0 & 10 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}$$

$$S_3 = \begin{pmatrix} 0 & 1 & 10 \\ -1 & 0 & 10 \\ 0 & 0 & 1 \end{pmatrix} S_0$$

first rotation with angle -90° at $(0, 0)$ and then translation with vector $w = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$

Example 2: Self-portrait using instances of the square with corners:



Mouth with matrix $A_M = \begin{pmatrix} 1/3 & 0 & 10/3 \\ 0 & 1/50 & 10/6 \\ 0 & 0 & 1 \end{pmatrix}$

Left eye $A_{LE} = \begin{pmatrix} 1/3 & 1/50 & 1/2 \\ 0 & 1/50 & 20/3 \\ 0 & 0 & 1 \end{pmatrix}$

Right eye $A_{RE} = \begin{pmatrix} 1/3 & 1/50 & 29/3 \\ 0 & 1/50 & 20/3 \\ 0 & 0 & 1 \end{pmatrix}$

Left ear $A_{LE} = \begin{pmatrix} 1/10 & 0 & -1.00 \\ 0 & 1/4 & 4.50 \\ 0 & 0 & 1 \end{pmatrix}$

Right ear $A_{RE} = \begin{pmatrix} 1/10 & 0 & 10.0 \\ 0 & 1/4 & 4.5 \\ 0 & 0 & 1 \end{pmatrix}$

Transformations

Transformation: Drawing of an object by means of a part of the object as a copy (translation).
 The base object is called the graphic primitive.
 (Each copy (instance) of the graphic primitive is obtained by applying an affine transformation to the original primitive.)

Example: Rotate a square (side = 2) counter-clockwise by $\theta = 45^\circ$ about the origin with angle θ .



$$T_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

Translation: translation with vector $v = (a, b)$.
 $T_v = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$

Example: Elliptical with center at origin with axes of length 2.

$$T_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



Left eye $A_{LE} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
 Right eye $A_{RE} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
 Mouth with matrix $A_M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
 Nose with matrix $A_N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Instantiating Fractals

We can see a fractal as an example of instantiation, where the functions generating the IFS (iterating function system) are affine contractions

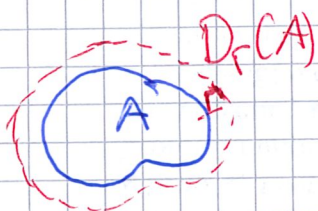
A (diff) function $f: \mathbb{E}^2 \rightarrow \mathbb{E}^2$ is a contraction if $\forall x, y \in \mathbb{E}^2, \exists \lambda \in (0, 1)$
 $d(f(x), f(y)) \leq \lambda d(x, y)$

We can define contractions for any metric space M
 $f: M \rightarrow M \quad \forall x, y \in M \quad \exists \lambda \in (0, 1)$
 $d(f(x), f(y)) \leq \lambda d(x, y)$

As our metric space we will consider $\mathcal{H}(\mathbb{E}^2)$

$\mathcal{H}(\mathbb{E}^2) = \{A; A \text{ compact in } \mathbb{E}^2\}$ (Hyperspace of \mathbb{E}^2)

We consider sets $D_r(A) = \{y; d(x, y) < r \text{ for some } x \in A\}$



We can define a metric on $\mathcal{H}(\mathbb{E}^2)$:

$d_{\mathcal{H}}(A, B) := \inf \{r > 0; A \subseteq D_r(B), B \subseteq D_r(A)\}$

Result Let $f_i: \mathbb{E}^2 \rightarrow \mathbb{E}^2$ be (similarities) of ratio $\lambda < 1$

Then the function $F: \mathcal{H}(\mathbb{E}^2) \rightarrow \mathcal{H}(\mathbb{E}^2)$

defined as $F(A) = \cup f_i(A)$, A compact is a contraction. Moreover F has a fixed point \mathcal{F} (a compact set in \mathbb{E}^2) \mathcal{F} is the fractal

Example f_1 is the dilation and translation
with matrix $\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\lambda_1 = 1/2$

f_2 is the similarity with matrix $\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.
 $\lambda_2 = 1/2$

and f_3 is the similarity with matrix $\begin{pmatrix} 1/2 & 0 & 1 \\ 0 & 1/2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.
 $\lambda_3 = 1/2$

The fractal below corresponds to 1000 iterations
of $\{f_1, f_2, f_3\}$ all three with probability $1/3$

