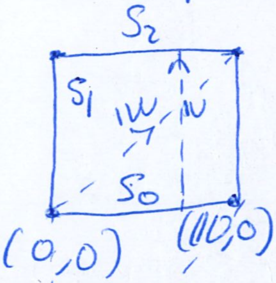
Instancing

(2)

Instancing: Drawing an object by making each part of the object as a copy (<u>instance</u>) of a base objects called the graphic <u>primitive</u>

: Each copy (instance) of the graphic primitive is obtained by applying an affine transformation to the graphic primitive

<u>Example 1</u>: Make a square from a segment (10,0)



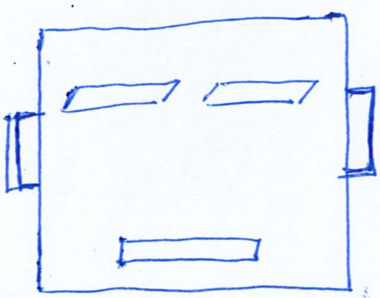
$$S_0 = \begin{pmatrix} 0 & 10 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}$$

$$S_1 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} S_0$$ rotation with angle $\pi/2$ at (0,0)

$$S_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix} S_0$$ translation with vector $w = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$ of S_0

$$S_3 = \begin{pmatrix} 0 & 1 & 10 \\ -1 & 0 & 10 \\ 0 & 0 & 1 \end{pmatrix} S_0$$ first rotation with angle $-\pi/2$ at (0,0) and then translation with vector $w = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$

<u>Example 2</u>: Selfportrait using instances of the square with corners: $Sq = \begin{pmatrix} 0 & 10 & 10 & 0 \\ 0 & 0 & 10 & 10 \\ 1 & 1 & 1 & 1 \end{pmatrix}$



Mouth with matrix $A_M = \begin{pmatrix} 1/3 & 0 & 10/3 \\ 0 & 1/50 & 10/6 \\ 0 & 0 & 1 \end{pmatrix}$

Left eye $A_{LE} = \begin{pmatrix} 1/3 & 1/50 & 1/2 \\ 0 & 1/50 & 20/3 \\ 0 & 0 & 1 \end{pmatrix}$

Right eye $A_{RE} = \begin{pmatrix} 1/3 & 1/50 & 29/3 \\ 0 & 1/50 & 20/3 \\ 0 & 0 & 1 \end{pmatrix}$

Left ear $A_{LÖ} = \begin{pmatrix} 1/10 & 0 & -1.00 \\ 0 & 1/4 & 4.50 \\ 0 & 0 & 1 \end{pmatrix}$, right ear $A_{RÖ} = \begin{pmatrix} 1/10 & 0 & 10.0 \\ 0 & 1/4 & 4.5 \\ 0 & 0 & 1 \end{pmatrix}$

Instancing Fractals

We can see a fractal as an example of instancing, where the functions generating the IFS (iterating function system) are affine contractions

A (diff) function $f: \mathbb{E}^2 \longrightarrow \mathbb{E}^2$ is a contraction if $\forall x, y \in \mathbb{E}^2$, $\exists \lambda \in (0,1)$

$$d(f(x), f(y)) \leq \lambda\, d(x,y)$$

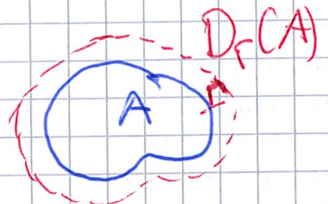
We can define contractions for any metric space $\mathbb{M}$ $\quad f: \mathbb{M} \longrightarrow \mathbb{M}$ $\quad \forall x, y \in \mathbb{M}$ $\exists \lambda \in (0,1)$

$$d(fx, fy) \leq \lambda\, d(x,y)$$

As our metric space we will consider $\mathbb{H}(\mathbb{E}^2)$

$$\mathbb{H}(\mathbb{E}^2) = \{A ;\ A \text{ compact in } \mathbb{E}^2\}$$ Hyperspace of $\mathbb{E}^2$

We consider sets $D_r(A) = \{y ;\ d(x,y) < r \text{ for some } x \in A\}$



We can define a metric on $\mathbb{H}(\mathbb{E}^2)$:

$$d_{\mathbb{H}}(A, B) := \inf\{r > 0 ;\ A \subseteq D_r(B),\ B \subseteq D_r(A)\}$$

Result Let $f_i : \mathbb{E}^2 \longrightarrow \mathbb{E}^2$ be similarities of ratio $\lambda < 1$

Then the function $F: \mathbb{H}(\mathbb{E}^2) \longrightarrow \mathbb{H}(\mathbb{E}^2)$ defined as $F(A) = \cup\, f_i(A)$, A compact is a contraction. Moreover F has a fixed point $\mathcal{F}$ (a compact set in $\mathbb{E}^2$) $\mathcal{F}$ is the fractal

<u>Example</u> f_1 is the dilation and translation with matrix $\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\lambda_1 = \frac{1}{2}$

f_2 is the similarity with matrix $\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. $\lambda_2 = 1/2$

and f_3 is the similarity with matrix $\begin{pmatrix} 1/2 & 0 & 1 \\ 0 & 1/2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ $\lambda_3 = 1/2$

The fractal below corresponds to 1000 iterations of $\{f_1, f_2, f_3\}$ all three with probability $1/3$