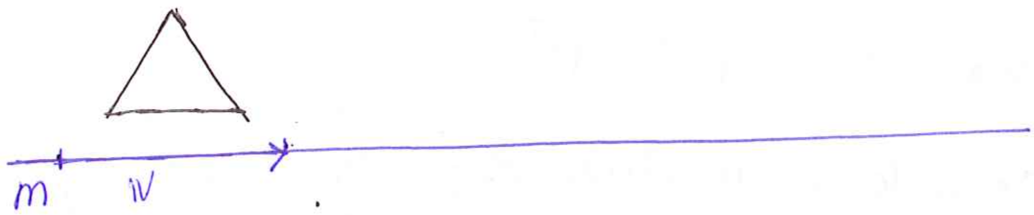
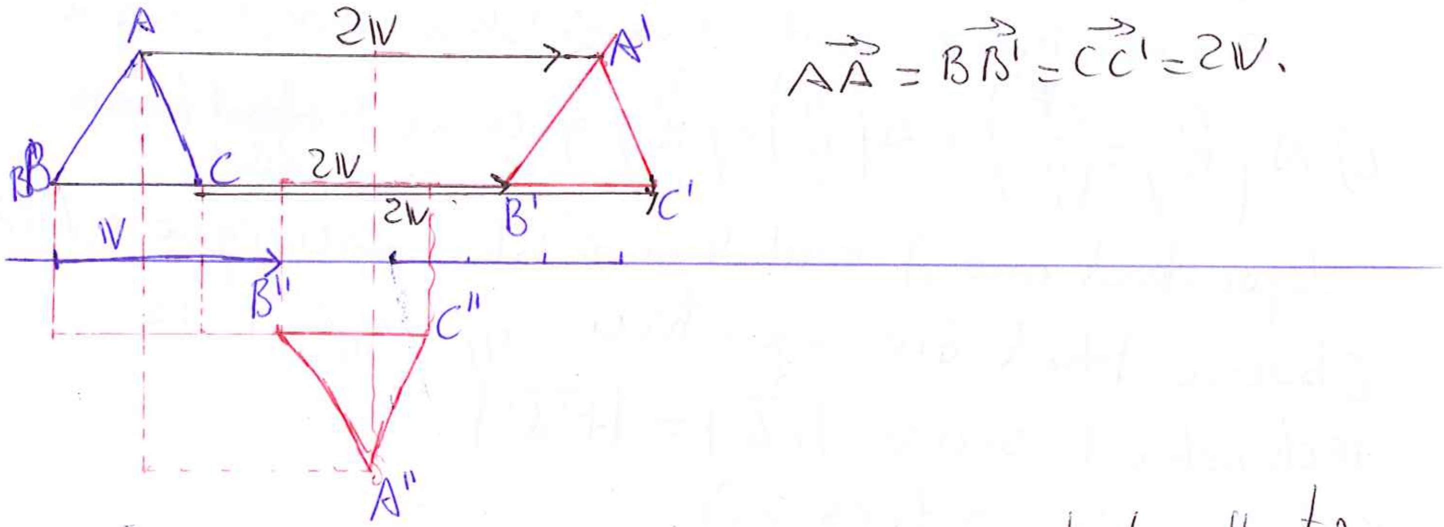
3.9.10 Look at the drawing.



The product $G_{m,v} G_{m,v} = T_{2v}$



$\vec{AA'} = \vec{BB'} = \vec{CC'} = 2v.$

3.9.13 Let P' be the image of P under a glide reflection with axis m. Show that m bisects the segment $\overline{PP'}$

To show that m bisects the segment PP', observe that m is a transversal to the triangel $\Delta PP_sP'$ (it is parallel to P_sP'). As P_s is the image of P under the reflection in m, then m bisects PP_s and so PP'

3.9.14 (a) Find a glide reflection that maps $\overline{PQ}$ to $\overline{P'Q'}$ (b) Is it unique? (c) Is ~~there~~ an unique glide-reflection sending P to P'

Method 1 consider a matrix $A_G = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & -a_{11} & a_{23} \\ 0 & 0 & 1 \end{pmatrix}$

$a_{11}^2 + a_{12}^2 = 1$

c) $A\begin{pmatrix} p_1 \\ p_2 \\ 1 \end{pmatrix} = \begin{pmatrix} p_1' \\ p_2' \\ 1 \end{pmatrix}$ gives us

3 equations and 4 variables $\Rightarrow$ not unique

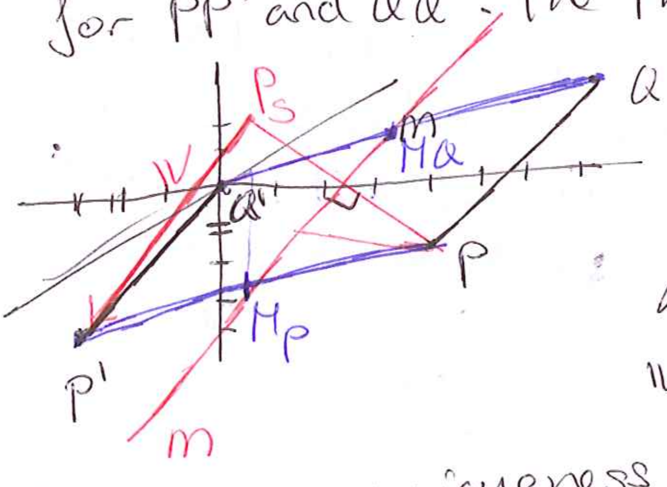
b) $A\begin{pmatrix} p_1 \\ p_2 \\ 1 \end{pmatrix} = \begin{pmatrix} p_1' \\ p_2' \\ 1 \end{pmatrix}$; $A\begin{pmatrix} q_1 \\ q_2 \\ 1 \end{pmatrix} = \begin{pmatrix} q_1' \\ q_2' \\ 1 \end{pmatrix}$ gives us 4 ~~[illegible]~~ equat. dependent ones) and 4 variables $\Rightarrow$ unique solution

Observe that the equation $a_{11}^2 + a_{12}^2 = 1$ is redundant since $|\overline{PQ}| = |\overline{P'Q'}|$

Solving b) solves a).

Method 2

a) and b) Consider $M_P\begin{pmatrix} 1/2 \\ -3 \\ 1 \end{pmatrix}$ and $M_Q\begin{pmatrix} +7/2 \\ +1/2 \\ 1 \end{pmatrix}$ the midpoints for $\overline{PP'}$ and $\overline{QQ'}$. The the axis of G is the line $M_P M_Q = m$



The translation vector is obtained by considerer P_S: the reflection of P in m

now, the translation vector $w = \overrightarrow{P_S P'}$

For the non-uniqueness of G in c) consider that the only condition we have is that M_P belongs to the axis; we may choose infinite many axis through M_P

Svar till uppgifter om Euklidiska Geometri

3.5.1 $u[-2,5,7]$ a) $[2,-5,-7]$, $[4,-10,-14]$, $[-4,10,14]$

b) $2x_1 - 5x_2 = 7$

c) $X(1,-1,1)$, $Y(7/2, 0, 1)$

3.5.2 (dual av 3.5.1) $X(4,-7)$ a) $\begin{bmatrix}-4\\-7\\1\end{bmatrix}, \begin{bmatrix}-4\\7\\-1\end{bmatrix}, \begin{bmatrix}8\\-16\\2\end{bmatrix}$

b) $4u_1 - 7u_2 + u_3 = 0$

c) $u_1[2,1,-1]$, $u_2[1,1,3]$

3.5.5 $\begin{vmatrix}10 & -7 & x_1\\ 2 & 3 & x_2\\ 1 & 1 & 1\end{vmatrix} = 0 = \begin{vmatrix}17 & -7-x_1 & x_1\\ -1 & 3-x_2 & x_2\\ 0 & 0 & 1\end{vmatrix}$ $17(3-x_2)-(7-x_1)$ $\underset{0}{\shortparallel}$ $1x_1 - 17x_2 + 44 = 0$

3.5.10 $u[u_1,u_2,u_3]$ och $v[v_1,v_2,v_3]$ är parallella omm $\begin{cases}u_1x_1 + u_2x_2 = -u_3\\ v_1x_1 + v_2x_2 = -v_3\end{cases}$ har ingen lösning $\Leftrightarrow$ det finns $h \neq 0$ så att

~~$\begin{vmatrix}u_1 & u_2\\ v_1 & v_2\end{vmatrix} = 0$ och~~ $v_1x_1 + v_2x_2 + h(u_1x_1 + u_2x_2) = 0$ $\forall x_1, x_2$

men $(-u_3)h + (-v_3) \neq 0$.

d.v.s. det finns $h \neq 0$ s.a. $v_1 = -hu_1$, $v_2 = -hv_2$ och $v_3 \neq -hu_3$

3.5.13 (i) Ekvation till punkt $X(0,0,1)$: $u_3 = 0$.

(ii) Ekvation för en linje // med $[u_1,u_2,u_3]$ $\Rightarrow u_1x_1 + u_2x_2 = 0$

$ku_1x_1 + hu_2x_2 + u_3 = 0$
$\underset{0}{\shortparallel}$

dvs $v[u_1,u_2,0]$

3.5.14 a) $l_1[-2,1,-7]$, $\tan(\angle l_1, l_2) = \dfrac{-8-3}{-6+4} = \dfrac{11}{2}$

$l_2[3,4,17]$

b) $l_1[1,0,0]$ ortogonala

$l_2[0,1,0]$

c) $l_1[1,-1,0]$ $\tan(\angle l_1, l_2) = \dfrac{1}{1}$; $\angle l_1 l_2 = 45°$

$l_2[1,0,0]$

3.6.1 $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ med $A = \begin{pmatrix}1&5&0\\0&1&0\\0&0&1\end{pmatrix}$

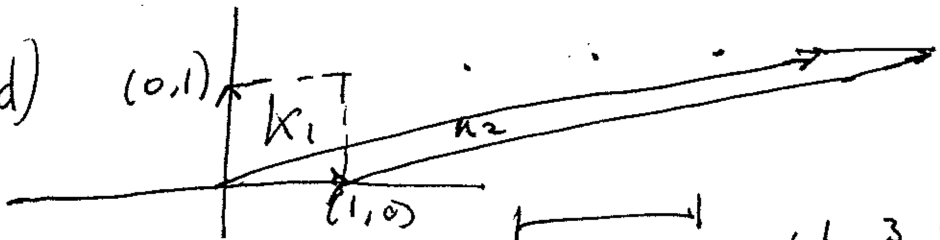
(a) $\ell[1,-2,3]: x_1 - 2x_2 = -3 \qquad x_1 = 2x_2 - 3$

$A\begin{pmatrix}2x_2-3\\x_2\\1\end{pmatrix} = \begin{pmatrix}7x_2-3\\x_2\\1\end{pmatrix}$ c) linje $[1,-7,3]$

(b) invarianta punkter i ℓ: $7x_2 - 3 = 2x_2 - 3 \to \boxed{x_2 = 0}$

$X(-3, 0, 1)$

(d)



T avbildar k_1 på k_2.

3.6.3 $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ $A = \begin{pmatrix}1&3&-7\\2&5&4\\0&0&1\end{pmatrix}$

a) $A\begin{bmatrix}1\\2\\1\end{bmatrix} = \begin{bmatrix}0\\16\\1\end{bmatrix}$, $A\begin{bmatrix}6\\4\\1\end{bmatrix} = \begin{bmatrix}11\\36\\1\end{bmatrix}$

b) $PQ[2,-5,8]$; $P'Q'[20,-11,176]$

c) A^{-1}: $\left(\begin{array}{ccc|ccc}1&3&-7&1&0&0\\2&5&4&0&1&0\\0&0&1&0&0&1\end{array}\right) \sim \left(\begin{array}{ccc|ccc}1&3&0&1&0&7\\0&1&0&2&-1&+18\\0&0&1&0&0&1\end{array}\right) \Rightarrow \begin{pmatrix}-5&3&-47\\2&-1&18\\0&0&1\end{pmatrix}$

d) $[2,-5,8]\begin{pmatrix}-2&3&-47\\2&-1&18\\0&0&1\end{pmatrix} = k[u_1', u_2', u_3']$

3.6.7 Multiplicera rad nr. 3 tillvänster med alla 3 kolonner till höger

3.7.1 $\begin{pmatrix}a&b&a_{13}\\-b&a&a_{23}\\0&0&1\end{pmatrix}\begin{pmatrix}0&2\\0&0\\1&1\end{pmatrix} = \begin{pmatrix}1&3\\1&1\\1&1\end{pmatrix}$
$\begin{cases}a_{13} = 1\\a_{23} = 1\\2a + a_{13} = 3 \to a = 1\\-2b + a_{23} = 1 \to b = 0\end{cases}$

Translation med vektor $v = \begin{pmatrix}1\\1\end{pmatrix}$

$T(1,-1,1) \longrightarrow X(2, 0, 1)$

$\begin{pmatrix}a&b&a_{13}\\b&-a&a_{23}\\0&0&1\end{pmatrix}\begin{pmatrix}0&2\\0&0\\1&1\end{pmatrix} = \begin{pmatrix}1&3\\1&1\\1&1\end{pmatrix}$ $a_{13} = 1$, $a_{23} = 1$; $a = 1$, $b = 0$

Glidspegling med axel $v = \begin{pmatrix}1\\1\end{pmatrix}$

3.8.1 a) $T\begin{pmatrix}1\\-2\\1\end{pmatrix} = \begin{pmatrix}3\\4\\1\end{pmatrix}$ $\quad A_T = \begin{pmatrix}1&0&2\\0&1&6\\0&0&1\end{pmatrix}$

$$[2, 3, -1]\begin{pmatrix}1&0&2\\0&1&6\\0&0&1\end{pmatrix} = [2, 3, 21]$$

b) $l_1[2, 3, -1]$ och $l_2[2, 3, 21]$ är parallella

$2 = 1 \cdot 2$
$3 = 1 \cdot 3$
$21 \neq 1(-1)$

3.8.2) $[1, -2, 5]\begin{pmatrix}1&0&a\\0&1&b\\0&0&1\end{pmatrix} = [2h, -4h, 7h]$

$1 = 2h \quad h = 1/2$
$-2 = -4h$
$a + 2b + 5 = 7/2 \quad ; \quad a = 2b - 3/2$

Exempel $b = 0$ $\quad \begin{pmatrix}1&0&-3/2\\0&1&0\\0&0&1\end{pmatrix}$

3.8.5) $\begin{pmatrix}3/5&4/5&2\\-4/5&3/5&1\\0&0&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\1\end{pmatrix} = \begin{pmatrix}x_1\\x_2\\1\end{pmatrix}$ $\quad \begin{cases}2/5\,x_1 - 4/5\,x_2 = 2\\ 4/5\,x_1 + 2/5\,x_2 = 1\\ (2) - 2(1) \rightarrow 2x_2 = -3\end{cases}$

c) Center $C(-2, -3/2, 1)$

b) vinkel $[0, 0, \cancel{3/2}]$ $A = [\cancel{4}/5, +\cancel{3}/5, 5/2]$

$\tan(\theta) = \cancel{3/4}.\ \boxed{-4/3}$

3.8.7 $u[2, 0, 3]$ $\quad \tan(\theta) = \frac{2}{2} = 1, \ \theta = 45^0$

$v[1, 1, 5]$ $\quad \begin{cases}2x_1 = -3\\ x_1 + x_2 = -5\end{cases}$ $\quad x_1 = -3/2$ $\quad x_2 = -5 + 3/2 = -7/2$

$$\begin{pmatrix}1&0&-3/2\\0&1&7/2\\0&0&1\end{pmatrix}\begin{pmatrix}1/\sqrt2&-1/\sqrt2&0\\1/\sqrt2&1/\sqrt2&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&3/2\\0&1&7/2\\0&0&1\end{pmatrix}$$

3.8.8 En möjligt rotation är den med center $M \equiv$ midpunkten mellan P och P' och vinkel $180°$. $M(m_1, m_2, 1)$

$$A = \begin{pmatrix} 1 & 0 & m_1 \\ 0 & 1 & m_2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -m_1 \\ 0 & 1 & -m_2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 2m_1 \\ 0 & -1 & 2m_2 \\ 0 & 0 & 1 \end{pmatrix}$$

b) Använd A där $m_1 = \frac{2+1}{2}$ och $m_2 = \frac{0-3}{2}$

3.8.18

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 & 10 \\ 8 & 4 & 7 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 11 & 14 \\ -2 & -4 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 7 & 11 & 14 \\ -2 & -4 & 2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 & 10 \\ 8 & 4 & 7 \\ 1 & 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 7 & 11 & 14 \\ -2 & -4 & 2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1/10 & 1/5 & 2/5 \\ -7/30 & -4/15 & 38/15 \\ 2/15 & 1/15 & -12/5 \end{pmatrix}$$

kontrollera ~~beräkningar~~

3.9.2 c) $m[\frac{\sqrt{3}}{3}, -1, 0]$ $\quad A_{Rm} = \begin{pmatrix} a & b & 0 \\ b & \cancel{a} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad a^2+b^2=1$

origo invariant

$[\frac{\sqrt{3}}{3}, -1, 0]A = [\frac{k\sqrt{3}}{3}, -k, 0]$

så $\begin{cases} \frac{\sqrt{3}}{3}\cos\theta - \sin\theta = \frac{\sqrt{3}}{3}k \\ \cos\theta + \frac{\sqrt{3}}{3}\sin\theta = -k \end{cases} \rightarrow \begin{cases} k = -2\cos\theta \\ -\sin\theta = -\sqrt{3}\cos\theta; \quad \tan\theta = \sqrt{3} \\ \sin\theta = \frac{\sqrt{3}}{2} \quad \cos\theta = 1/2 \end{cases}$

$$A = \begin{pmatrix} 1/2 & \sqrt{3}/2 & 0 \\ \sqrt{3}/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b) $$A\begin{pmatrix} 3 \\ 7 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1+7\sqrt{3}}{2} \\ \frac{3\sqrt{3}-7}{2} \\ 1 \end{pmatrix}$$

c) kontrollera beräkningar