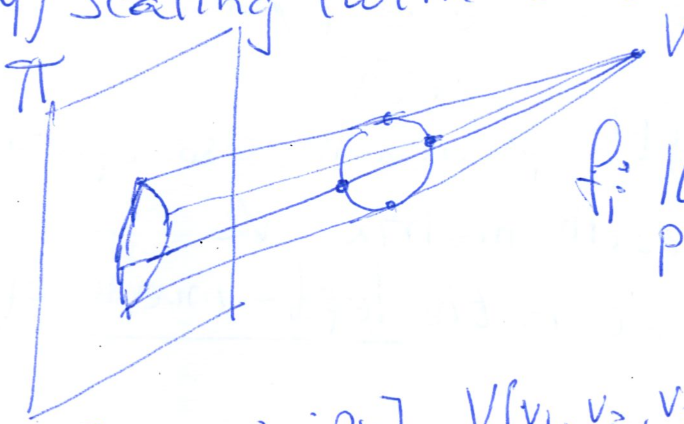
Viewing Pipeline

Process to visualizing 3D-objects in 2D-devices.

Steps

1) Projection
2) Coordinate systems for the plane
3) Viewplane mapping: Coordinates of an object in the viewing plane (with choosen coord. system)
4) Scaling (with a 2D-affine transformation)



1) Perspective projection

$$f_1 : \mathbb{E}^3 \longrightarrow \pi$$
$$P \longmapsto f(P) = VP \cap \pi$$

$\mathbb{n} = [n_1 : n_2 : n_3 : n_4]$, $V(v_1, v_2, v_3, 1)$, $\mathbb{n} \cdot V \neq 0$ $(V \notin \pi)$

f has matrix $\boxed{M = \underbrace{V\mathbb{n}}_{(4\times1)\times(1\times4)} - \underbrace{(\mathbb{n}\cdot V)}_{(1\times4)\times(4\times1)} I_4}$

<u>Proof</u> Let $P \in \mathbb{E}^3$ i) If $\mathbb{n}\cdot P = 0$ $(P \in \pi)$

$M(P) = V\mathbb{n}(P) - (\mathbb{n}\cdot V) I_4 (P) = 0 - \lambda P = P$ (projectively)

ii) If $\mathbb{n}\cdot P \neq 0$. Any point in PV has homegeneous coordinates $\lambda P + \mu V$. We look for the point $(\lambda : \mu)$ such that $\mathbb{n}\cdot(\lambda P + \mu V) = 0$, $\lambda = \dfrac{-\mu(\mathbb{n}\cdot V)}{(\mathbb{n}\cdot P)}$

$$f(P) = \frac{-\mu(\mathbb{n}\cdot V)}{(\mathbb{n}\cdot P)} P + \mu V = \frac{-\mu}{(\mathbb{n}\cdot P)}\left(V\mathbb{n}(P) - \mathbb{n}\cdot V(P)\right)$$

$$f(P) = \left(V\mathbb{n} - (\mathbb{n}\cdot V) I_4\right)(P)$$

(ii) Viewplane Coordinate Mapping

Π coordinate system consists of the origin $\mathbb{O}(q_1, q_2, q_3, 1)$ and two (unitary) vectors $\mathbb{r} = (r_1, r_2, r_3, 0)$ and $\mathbb{s} = (s_1, s_2, s_3, 0)$ for the X- and Y- directions

$\mathcal{U}(t_1, t_2, t_3, 1) = (q_1 + r_1 + s_1, q_2 + r_2 + s_2, q_3 + r_3 + s_3, 1) = \mathbb{O} + \mathbb{r} + \mathbb{s}$

(iii) A point $f(P) = P'$ has 3D-coord $P'(x; y; z; \omega)$ and coordinates $P''(X; Y; W)$ in the system $(\mathbb{O}, \mathbb{r}, \mathbb{s})$

We know that
$$\begin{pmatrix} q_1 & r_1 & s_1 & t_1 \\ q_2 & r_2 & s_2 & t_2 \\ q_3 & r_3 & s_3 & t_3 \\ 1 & 0 & 0 & 1 \end{pmatrix} = \underbrace{\begin{pmatrix} r_1 & s_1 & q_1 \\ r_2 & s_2 & q_2 \\ r_3 & s_3 & q_3 \\ 0 & 0 & 1 \end{pmatrix}}_{4 \times 3} \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

with other words $k(P'') = P'$ (the pseudo-inverse of our map)

We want $f_2(P') = P''$ with matrix VC

Then $VC(k(P'')) = P''$, VC is the <u>left-inverse of k</u>

$$\boxed{VC = (k^t k)^{-1} k^t}$$

Observe that VC is independent of f, depends on the choice of $\mathbb{O}$, $\mathbb{r}$, $\mathbb{s}$

(iv) Fitting the piece of the plane to the screen: We need two translations (for the origins to fit) and two strains (to fit scale)

$$DC = \begin{pmatrix} 1 & 0 & U_{min} \\ 0 & 1 & V_{min} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{U_{max} - U_{min}}{X_{max} - X_{min}} & 0 & 0 \\ 0 & \frac{V_{max} - V_{min}}{Y_{max} - Y_{min}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -X_{min} \\ 0 & 1 & -Y_{min} \\ 0 & 0 & 1 \end{pmatrix}$$

Viewing pipeline given by $DC \cdot VC \cdot M$