

[> with(plots) : with(LinearAlgebra) : with(SolveTools) :

Finally the following example is a collineation and its invariant elements. It has three points and three lines invariants, and it is NOT a perspectivistic collineation.

$A := \text{Matrix}([\ [-1., -1., 2.], \ [-5., -1., 3.], \ [3., 0., -1.] ])$ ;

$$A := \begin{bmatrix} -1. & -1. & 2. \\ -5. & -1. & 3. \\ 3. & 0. & -1. \end{bmatrix} \quad (1)$$

$B := \text{Transpose}(\text{MatrixInverse}(A))$ ;

$$B := \begin{bmatrix} 1.000000000000000 & 4.000000000000001 & 3.000000000000000 \\ -1.000000000000000 & -5.000000000000001 & -3.000000000000000 \\ -1.000000000000000 & -7.000000000000001 & -4.000000000000000 \end{bmatrix} \quad (2)$$

$\text{Eigenvectors}(A)$ ;

$\text{Eigenvectors}(B)$ ;

$$\begin{bmatrix} -4.66792869559189 + 0. I \\ 1.78775903903729 + 0. I \\ -0.119830343445404 + 0. I \end{bmatrix}, \quad (3)$$

$[\ [-0.415280907253006 + 0. I, \ -0.624766541251697 + 0. I, \ 0.144545798199293 + 0. I],$

$[\ -0.843903994776894 + 0. I, \ 0.397034562962171 + 0. I, \ 0.858124740559903 + 0. I],$

$[ \ 0.339658380833921 + 0. I, \ -0.672332004850156 + 0. I, \ 0.492674783058692 + 0. I ]]$

$$\begin{bmatrix} -8.34513171912598 + 0. I \\ 0.559359498771433 + 0. I \\ -0.214227779645465 + 0. I \end{bmatrix}, \quad (3)$$

$[\ [0.454010394091017 + 0. I, \ -0.898570015650219 + 0. I, \ -0.775090647914976 + 0. I],$

$[\ -0.515821456363572 + 0. I, \ 0.322327002824651 + 0. I, \ -0.211315625858943 + 0. I$

$],$

$[ \ -0.726500369726185 + 0. I, \ -0.297787223071176 + 0. I, \ 0.595466366625845 + 0. I ]]$

;

$$\begin{bmatrix} -4.66792869559189 + 0. I \\ 1.78775903903729 + 0. I \\ -0.119830343445404 + 0. I \end{bmatrix},$$

$[\ [-0.415280907253006 + 0. I, \ -0.624766541251697 + 0. I,$

$$\begin{aligned}
& 0.144545798199293 + 0. I ], \\
& [ -0.843903994776894 + 0. I, 0.397034562962171 + 0. I, 0.858124740559903 + 0. I ], \\
& [ 0.339658380833921 + 0. I, -0.672332004850156 + 0. I, 0.492674783058692 + 0. I ] ] \\
& \left[ \begin{array}{l} -8.34513171912598 + 0. I \\ 0.559359498771433 + 0. I \\ -0.214227779645465 + 0. I \end{array} \right], \\
& [ [ 0.454010394091017 + 0. I, -0.898570015650219 + 0. I, \\
& -0.775090647914976 + 0. I ], \\
& [ -0.515821456363572 + 0. I, 0.322327002824651 + 0. I, -0.211315625858943 + 0. I \\
& ], \\
& [ -0.726500369726185 + 0. I, -0.297787223071176 + 0. I, 0.595466366625845 + 0. I ] ]
\end{aligned} \tag{4}$$

$$\text{Linear}(\{x + y + z - 3, 2x - y + 2z - 3, x + y - 2z\}, \{x, y, z\}); \\
\{x = 1, y = 1, z = 1\} \tag{5}$$

$$\text{solve}(\{x + y + z = 3, 2x - y + 2z = 3, x + y - 2z = 0\}, [x, y, z]); \\
[[x = 1, y = 1, z = 1]] \tag{6}$$

$R := \text{Matrix}([[1, 1, 1], [2, -1, 2], [1, 1, -2]]);$   
 $v := \text{Vector}([3, 2, 0]);$   
 $\text{LinearSolve}(R, v);$

$$R := \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 1 & 1 & -2 \end{bmatrix}$$

$$v := \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{3} \\ \frac{4}{3} \\ 1 \end{bmatrix}$$

(7)

$K := \langle \langle 1, 2, 1 \rangle | \langle 1, -1, 1 \rangle | \langle 1, 2, -2 \rangle \rangle;$   
 $\text{LinearSolve}(K, v);$

$$K := \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 1 & 1 & -2 \end{bmatrix}$$

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$$\begin{bmatrix} \frac{2}{3} \\ \frac{4}{3} \\ 1 \end{bmatrix}$$

**(8)**