Hand-in Exercises 1, TATA49 Geometry with Applications

Maple or Matlab may be used in the calculations.

The exercises are customized by your birthdays as follows: p_1, p_2 is the month of your birthday, where January is 0,1; February 0,2;...; and December 1,2. q_1, q_2 is the date in the month, between 13 and 29, nearest to your birthday. r_1, r_2 are the two last digits in the year you were born. However, if someone was born October 20, November 22 or December 24, the day of birth for our purposes is 23. If you were born 1990, for geometry you were born 1991.

Exercise 1 *Consider the square with sides on the lines* $l[r_2, -r_1, r_1p_2 - r_2p_1]$, $m[r_1, r_2, -(r_1p_1 + r_2p_2)], n[r_2, -r_1, r_1^2 + r_2^2 + r_1p_2 - r_2p_1], r[r_1, r_2, -(r_1^2 + r_2^2 + r_1p_1 + r_2p_2)]$

1. *Verify that the lines above determine a square.*
2. *Determine the sides, vertices and inner angles of the image of the square above under the affine transformation with matrix* $\begin{pmatrix} r_2+1 & r_2+r_1 & q_1 \\ r_1+1 & r_1+1 & q_2 \\ 0 & 0 & 1 \end{pmatrix}$.

Exercise 2 *Determine the affine transformation that takes* $P(p_1+10, p_2+5)$ *to* $P'(1,1)$; $Q(q_1, q_2)$ *to* $Q'(1,2)$, *and* $R(r_1, r_2)$ *to* $R'(2,1)$. *Decompose it as a product of a similarity, a strain and a shear.*

Exercise 3 *Determine the affine transformation that takes the line* $p[1, p_1, p_2]$ *to* $y[1, -1, 4]$; *the line* $q[q_1, q_2, -1]$ *to* $d[2, 1, -4]$, *and the line* $r[r_1 - 1, r_2, 1]$ *to* $l[2, 0, -7]$. *Decompose it as a product of a similarity, a strain and a shear.*

Exercise 4 *Determine all the direct isometries that fix each of the lines with coordinates* $r[-(p_1+p_2), q_1+q_2, a], a \in \mathbb{R}$, *as lines. Observe that we have a family of lines, depending on one parameter* a.

Exercise 5 *Identify the isometries that take the point* $Q(q_1, q_2)$ *to* $Q'(r_2 + p_2, r_1 + p_1)$.

Exercise 6 *In the previous exercise identify the element of the plane (line, point, vector, curve, ...) that is common to all but one of the direct isometries. Determine the element of the plane common to all the indirect isometries in the previous exercise.*

Exercise 7 *In the figure below you see the arm of a planar robot. Give the function* f *that determines the configuration (position and orientation) of the hand depending on the parameters of the arm.*

Exercise 8 *With the robot arm as in the figure below, determine, if possible, the parameters for the angles at the junctions (links) the controller should get if the coordinates of the point are* $P_{(H)}(-1,1)$, *relative to the local coordinates*

(x_H, y_H) at the hand, and $P_0(-\sqrt{2}/2, \sqrt{2}/2)$, relative to the coordinates (x_0, y_0) for the controller. The total angle for the hand (the part of the arm with the hand) is π rad.

Exercise 9 *Consider two glider-eflections G_m and G_n with axes the lines m and n respectively. Show that $G_n G_m$ if a translation. Determine the translation vector in terms of the axes $m[m_1, m_2, m_3]$, $n[n_1, n_2, n_3]$ and translation vectors, $\mathbf{v_m} = (k_m m_2, -k_m m_1)$, $\mathbf{v_n} = (k_n n_2, -k_n n_1)$, $k_m, k_n \neq 0$, of G_m and G_n.*

Exercise 10 *Determine the similarity(ies) that takes(take) the points $P(p_1 + r_1, p_2 + r_2)$ and $Q(r_1 - q_1, r_2 - q_2)$ to the points $P'(r_1, r_2)$ and $Q'(r_2 + q_2 + p_2, r_1 + q_1 + p_1)$ respectively. What is the ratio of the similarity(ies)? Decompose the similarity(ies) as a product of an isometry and a dilation.*

Exercise 11 *A dilation takes $P(1, 2)$ to $P'(22, 8)$ and $Q(3, 4)$ to $Q'(18, 4)$. Determine its centre, ratio and matrix.*

Exercise 12 *Show that any isometry T satisfying that $T^2 = 1_d$ is a rotation of angle π rad., or a reflection*

Exercise 13 *Consider the affine transformation T with matrix* $\begin{pmatrix} a & b & 1-a \\ b & -a & -2b \\ 0 & 0 & 1 \end{pmatrix}$, *where a and $b > 0$ are real numbers such that $a^2 + b^2 = 1$. Show that the affine transformation T is a reflection and determine its axis m, i.e. the angle that m forms with the x-axis and the intersection point of m with the x-axis.*

Exercise 14 *The hyperbola with equation $x^2 y^2 = 1$ is called the equilateral hyperbola with asymptotes $y = \pm x$, center at $O(0, 0)$ and foci $F_1(\sqrt{2}, 0)$, $F_2(-\sqrt{2}, 0)$. Calculate the equation, centre and axes of the image of the equilateral hyperbola by the affine transformation with matrix* $\begin{pmatrix} r_2 + 1 & r_2 + r_1 & q_1 \\ r_1 + 1 & r_1 + 1 & q_2 \\ 0 & 0 & 1 \end{pmatrix}$.