

with(LinearAlgebra) :

A := Matrix([[1., 0., 1.], [0., 2., 0.], [-3., 0., 5.]]);

$$A := \begin{bmatrix} 1. & 0. & 1. \\ 0. & 2. & 0. \\ -3. & 0. & 5. \end{bmatrix} \quad (1)$$

B := A<sup>(-1)</sup>;

Eigenvalues(A);

$$\begin{bmatrix} 2. + 0. I \\ 4. + 0. I \\ 2. + 0. I \end{bmatrix} \quad (2)$$

Eigenvectors(A);

$$\begin{bmatrix} 2. + 0. I \\ 4. + 0. I \\ 2. + 0. I \end{bmatrix}, \begin{bmatrix} -0.707106781186547 + 0. I & -0.316227766016838 + 0. I & 0. I \\ 0. I & 0. I & 1. + 0. I \\ -0.707106781186547 + 0. I & -0.948683298050514 + 0. I & 0. I \end{bmatrix} \quad (3)$$

Tr := Transpose(B);

Eigenvalues(Tr);

Eigenvectors(Tr);

$$Tr := \begin{bmatrix} 0.625000000000000 & -0. & 0.375000000000000 \\ 0. & 0.500000000000000 & 0. \\ -0.125000000000000 & 0. & 0.125000000000000 \end{bmatrix}$$

$$\begin{bmatrix} 0.500000000000000 + 0. I \\ 0.250000000000000 + 0. I \\ 0.500000000000000 + 0. I \end{bmatrix}$$

$$\begin{bmatrix} 0.500000000000000 + 0. I \\ 0.250000000000000 + 0. I \\ 0.500000000000000 + 0. I \end{bmatrix},$$

(4)

$$\begin{bmatrix} 0.948683298050514 + 0. I & -0.707106781186547 + 0. I & 0. I \\ 0. I & 0. I & 1. + 0. I \\ -0.316227766016838 + 0. I & 0.707106781186547 + 0. I & 0. I \end{bmatrix}$$

1. Observe that the eigenvalue with multiplicity 2 for  $A$  is 2 that appears as first and third eigenvalue. The eigenvalue 4 is simple
2. The eigenvectors associated to the eigenvalue 2 for  $A$  are the points  $P(-1: 0: -1)$  and  $Y(0: 1: 0)$  that generated the line  $m[1: 0: -1]$ : a line of fixed points,  **$m$  is the axis**.
3. The point  $C(1: 0: 3)$  (second eigenvector to  $A$ ), is the eigenvector to 4.  **$C$  is the centre**.
4. Observe that  **$m$**  appears as the **eigenvector to the eigenvalue 1/4 to  $Tr$**  (of course if a line is invariant pointwise it is invariant as a set).
5. The lines  **$l[3: 0: -1]$**  (eigenvector no. 1 to  $Tr$ ) and  **$n[0,1,0]$**  (eigenvector no. 3 to  $Tr$ ) are the **eigenvector to the eigenvalue 1/2 to  $Tr$** . **They pass the centre  $C(1: 0: 3)$** .