LINKÖPING UNIVERSITY
Department of Mathematics
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Syllabus

• TATA53 • Linear Algebra Honours Course • 6 credits • Spring 2024 •

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Web page: `https://courses.mai.liu.se/GU/TATA53/`

Literature: Treil: *Linear algebra done wrong* (2021), available free from the authors website `https://sites.google.com/a/brown.edu/sergei-treil-homepage/linear-algebra-done-wrong` Supplementary lecture notes and exercises will be posted on the course web page.

Contents
Vector spaces, direct sum, LU-factorizataion, Jordan normal form, inner products, norms, QR-factorization, spectral theorems, singular values, low rank approximation, pseudo-inverse, polar factorization, Perron-Frobenius theory, tensor products, multilinear algebra. Applications: Ranking models, neural networks, dynamical systems, least square problems, principal component analysis.

Teaching
There are typically one lecture and one exercise session per week. The lectures provide an overview of the theory, and is a complement to the course literature.

Examination
The examination consists of four sets of hand-in assignments. For each of the four assignments there is a corresponding seminar where you will present your solutions. For grade 3/4/5 you will need a total score corresponding to 60%/75%/85% on the hand-in assignments, with at least 60% on each one. You also need to be ready to present your solutions at the seminars. For grade 5 you are also expected to present solutions to a few of the more challenging problems. To allow as many students as possible to make presentations there are two instances of each seminar scheduled - you are expected to attend one of each (so four in total). There may be insufficient time for everyone to present the harder problems, in that case we will arrange some supplementary opportunity to present solutions.

Set	Deadline	Main topics
1	5/2 8.00am	Vector spaces, direct sums, LU-factorization
2	4/3 8.00am	Jordan normal form, inner product spaces
3	18/4 8.00am	QR-factorization, spectral theory, Perron-Frobenius
4	13/5 10.00am	Singular values, multilinear algebra

Hand in your assignments in the compartment labelled TATA53 in house B, entrance 21, one floor up just outside the A-corridor. Alternatively you can hand them in at the beginning of the first of the two seminars (meaning 15 minutes after the deadline listed above). The assignments will typically be marked and returned in about a week.

Schedule

Preliminary schedule for **Le**ctures, **Ex**ercise sessions, and **Se**minars of the course. Always refer to TimeEdit for the current version and exact lecture times. The numbered exercises are recommended exercises from the **supplementary lecture notes** on the course website. These are intended to prepare you for the assignment problems.

Date		Topics/suggested exercises
16/1	**Le**1	Vector spaces, basis, dimension, subspaces, direct sum, quotients, linear maps
19/1	**Ex**1	**1:** 1-10, 13, (14, 15), 16, 17, (22)
23/1	**Le**2	Echelon forms, elementary matrices, rank, trace, LU- and Cholesky-factorization
26/1	**Ex**2	**2:** 1, 2, (3), 7, 11, 15, (16), 18, 20, 22, 24, 26, 27, 28, (35)
30/1	**Le**3	Complexification, eigenvectors, spectra, multiplicities, complex diagonalization
2/2	**Ex**3	**2:** 32, 33, (34); **3:** 1, 2, 3, 4, 5, 6a(b), 7, (9) 10, (12), 13
5/2	**Se**1a	Seminar 1. Presentations of exercises. Attend 1a or 1b.
6/2	**Le**4	Cayley-Hamilton, minimal polynomial, Jordan Normal form, nilpotent structures
7/2	**Se**1b	Seminar 1. Presentations of exercises. Attend 1a or 1b.
9/2	**Ex**4	**3:** 15, 16, 18, (19); **4:** 1, 2, 3, (4), (5), (8), 9, 10, 11, 12, 13
12/2	**Le**5	Generalized eigenspaces, Jordan chains, Jordan theorem, Jordanization
16/2	**Ex**5	**4:** 14, 15, 16, 17, 18, (19), 20, 21, (22-27)
20/2	**Le**6	Matrix exponential, dynamical systems, inner product spaces
23/2	**Ex**6	**4:** 28, 29, 30, 32, (33, 34), 35, 37; **5:** 1, 2, 3, 4, (5, 6), 7
27/2	**Le**7	Norms, Gram-Schmidt, QR-factorization
1/3	**Ex**7	**5:** 8, 11, 12, (16, 17, 18), 20, 23, (24), 25, 26, (27, 28), 29, 30, 31
4/3	**Se**2a	Seminar 2. Presentations of exercises. Attend 2a or 2b.
5/3	**Le**8	Self-adjoint, unitary, normal, and positive-definite operators, spectral theorems
6/3	**Se**2b	Seminar 2. Presentations of exercises. Attend 2a or 2b.
8/3	**Ex**8	**5:** 33, 35, 36, (37, 38, 39), 40, 41, 42, (43, 44), 45, (46, 47), 48, (49), 50
26/3	**Le**9	Least squares, positive matrices, Perron's theorem
28/3	**Ex**9	**5:** 51, 52, (53); **6:** 1, 2, (3, 4), 5, (6, 7), 9, 10, 11, 12, (13), 14
2/4	**Le**10	Non-negative matrices, Frobenius' theorem, ranking models, Markov chains
2/4	**Ex**10	**6:** 15, 16, 17, (18, 19), 20, (22), 23, 24, 25, 26, (27), 28
8/4	**Le**11	Singular values and vectors, singular value decomposition
9/4	**Ex**11	**7:** 1, 2, 3, (4), 5, 6, 7, (8), 9, (10, 11), 12, 14, 15, (16, 18), 19
15/4	**Le**12	Schmidt-decomposition, low rank approximation, Eckart-Young, pseudo-inverse
16/4	**Ex**12	**7:** 20, 21, (22, 23), 24, 25, 26, 27, (28), 29, 30, 31, (32, 34, 35)
18/4	**Se**3a	Seminar 3. Presentations of exercises. Attend 3a or 3b.
19/4	**Se**3b	Seminar 3. Presentations of exercises. Attend 3a or 3b.
22/4	**Le**13	Condition number, polar factorization, principal component analysis
23/4	**Ex**13	**7:** 36, (37), 38, (39), 40, 41, 42, (43), 44, 45
29/4	**Le**14	Total least squares, multilinear algebra, tensor products, duals
30/4	**Ex**14	**7:** 46, 47, (48); **8:** 1, 2, (3), 4, 5, 8, 10, (11), 12, 13, (14, 15)
6/5	**Le**15	Kronecker product, neural networks, linear algebra over $\mathbb{Z}_p$, linear codes
7/5	**Ex**15	
13/5	**Se**4a	Seminar 4. Presentations of exercises. Attend 4a or 4b.
14/5	**Se**4b	Seminar 4. Presentations of exercises. Attend 4a or 4b.