TATA53 Introduction

Linear Algebra Honours course

Jonathan Nilsson

Linköping University

Course Information

- Webpage: https://courses.mai.liu.se/GU/TATA53/
- Linear algebra honours course
- Spring 2024
- First Cycle
- 6 credits
- Mandatory for TMA, elective for other programs
- Teaching: One lecture and one lesson per week, four seminars
- Litterature: *Treil:* Linear algebra done wrong; Supplementary lecture notes and exercises.
- Four hand-in assignments, presented at seminars
- Grading scale: 3,4,5

What's new this year?

TATA53 Spring 2023

- 71 students registered
- 21 answered the course evaluation
- Students liked the course

What's new this year?

TATA53 Spring 2023

- 71 students registered
- 21 answered the course evaluation
- Students liked the course

Changes for Spring 2024

- New examiner
- Course is now mandatory for TMA
- More exercise sessions available
- Supplementary lecture notes (continuously updated)
- New set of suggested exercises, with hints and answers
- The formal syllabus is being updated
- Modified format for assignments

Learning objectives

Updated for 2024: Upon completion of the course, the student should be able to:

- Select and apply appropriate methods for problem solving and calculation in all parts of the course as described in the course contents
- Conduct theoretical reasoning to prove theorems and other results related to the course contents
- Present and justify solutions to tasks related to the course content using relevant concepts and clear reasoning

Less formally...

Idea of the course:

- Deepen theoretical understanding of linear algebra
- Show interesting applications of the theory
- Practice communicating mathematics written/orally

Less formally...

Idea of the course:

- Deepen theoretical understanding of linear algebra
- Show interesting applications of the theory
- Practice communicating mathematics written/orally

Teaching:

- Lectures: Provide overview, highlighting what's important
- Lessons: Work on problems individually or in groups
- Seminars: Presentations and discussions
- At home: Solve problems, read details not covered on lectures

Part I

Course contents overview

Complex vector spaces and inner product spaces

- Abstract theory of vector spaces
- Generalization from real to complex vector spaces
- Complex spectral theory
- Inner products in general

Complex vector spaces and inner product spaces

- Abstract theory of vector spaces
- Generalization from real to complex vector spaces
- Complex spectral theory
- Inner products in general

Applications: Further math courses, approximation of functions, coding applications

Complex vector spaces and inner product spaces

- Abstract theory of vector spaces
- Generalization from real to complex vector spaces
- Complex spectral theory
- Inner products in general

Applications: Further math courses, approximation of functions, coding applications

What is the angle between $\sin(x)$ and $\cos(x)$?

Complex vector spaces and inner product spaces

- Abstract theory of vector spaces
- Generalization from real to complex vector spaces
- Complex spectral theory
- Inner products in general

Applications: Further math courses, approximation of functions, coding applications

What is the angle between $\sin(x)$ and $\cos(x)$?

What polynomial of degree 2 lies closest to e^x when $x \in [0, 1]$?

Matrix factorizations

LU, Cholesky, Schur, QR, Polar.

Matrix factorizations

LU, Cholesky, Schur, QR, Polar.

$$\begin{pmatrix} 1 & 1 & 3 \\ 2 & 4 & 7 \\ -1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

Applications in computer algebra.

Jordan normal form

A standard form for *any* square complex matrix.

$$\begin{pmatrix} 5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

Jordan normal form

A standard form for *any* square complex matrix.

$$\begin{pmatrix} 5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

Applications: Predator-prey models and other dynamical systems, theoretical applications.

Perron-Frobenius theory

What can be said about the eigenvalues of positive matrices?

Perron-Frobenius theory

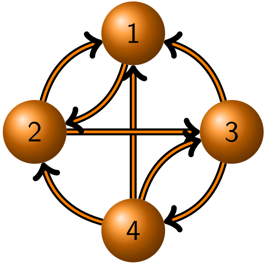
What can be said about the eigenvalues of positive matrices?

Applications: Pagerank, Markov chains

Perron-Frobenius theory

What can be said about the eigenvalues of positive matrices?

Applications: Pagerank, Markov chains
Given a network of websites, how should we rank them?



$$\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{3} \\ 1 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

Singular values

We can write $A = U\Sigma V^*$ where Σ has the *singular values* of A on the diagonal.

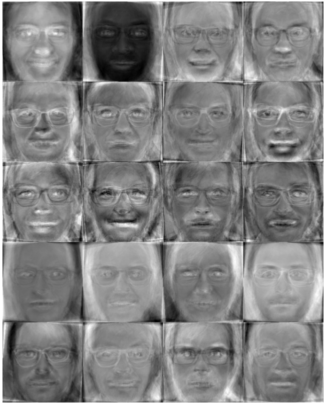
Singular values

We can write $A = U\Sigma V^*$ where Σ has the *singular values* of A on the diagonal.

Applications: Low rank approximation, principal component analysis, data compression, face recognition

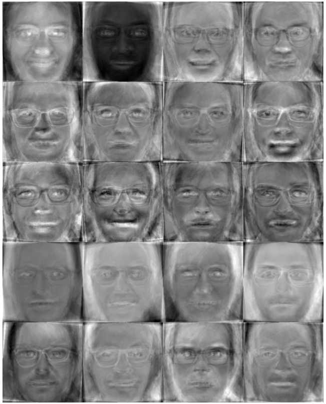
Singular values

We can write $A = U\Sigma V^*$ where Σ has the *singular values* of A on the diagonal.
Applications: Low rank approximation, principal component analysis, data compression, face recognition



Singular values

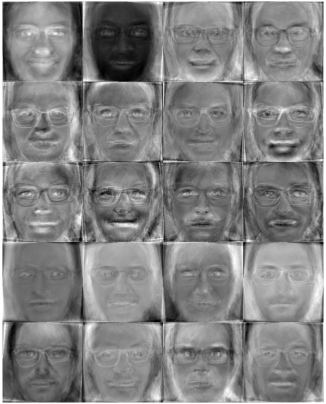
We can write $A = U\Sigma V^*$ where Σ has the *singular values* of A on the diagonal.
Applications: Low rank approximation, principal component analysis, data compression, face recognition



Singular values

We can write $A = U\Sigma V^*$ where Σ has the *singular values* of A on the diagonal.

Applications: Low rank approximation, principal component analysis, data compression, face recognition



Examination

Four assignments will be posted on the couse web page.

Set	**Deadline**	**Main topics**
1	5/2 8.00am	Vector spaces, direct sums, LU-factorization
2	4/3 8.00am	Jordan normal form, inner product spaces
3	18/4 8.00am	QR-factorization, spectral theory, Perron-Frobenius
4	13/5 10.00am	Singular values, multilinear algebra

After each deadline, there is a seminar where you present your solutions. Details will be posted with the first assignment.