2.(62)

a) $f(x,y) = 1-|x|-y^2 \leq 1$, $f(0,0) = 1 \Rightarrow$ lok. (strängt) max

b) $f(x,y) = |x| - \cos y$. $f(0,0) = |0| - \cos 0 = -1$,

$|x| - \cos y \geq -1$, så lokalt min.

c) $f(x,y) = |x| + \cos y$ $\quad f(x,0) > f(0,0)$, men $f(0,y) < f(0,0)$

för (x,y) nära 0 så ej extrempunkt

d) $f(x,y,z) = x^2 - y^2$. $f(0,0,0) = 0$, $f(x,0,0) = x^2 > 0$, $x \neq 0$

$f(0,t,t) = -t^2 < 0$ så ej extrempunkt

$t \neq 0$

e) $f(x,y,z) = \cos xyz$ $\quad f(0,0,0) = 1 \geq \cos xyz$

så lokalt max.

f) $f(x,y,z) = (1+x^2)e^{-y^2-z^2}$

$f(0,0,0) = 1$, $f(x,0,0) = 1+x^2 > 1$, $x \neq 0$

$f(0,y,0) = e^{-y^2} < 1$ om $y \neq 0$

så ej extrempunkt

g) $f(x,y) = (x+y)^2 + xy^3$, $f(0,0) = 0$.

$f(x,0) = x^2 > 0$ om $x \neq 0$, $f(x,-x) = -x^4 < 0$, $x \neq 0$

så ej extrempunkt

h)
$$f(x,y) = \begin{cases} y^2 \arctan \frac{x}{y^2} & y \neq 0 \\ 0 & y = 0 \end{cases}$$

$f(-t,t) = t^2 \arctan(-1/t) < 0$ om $t > 0$

$f(t,t) = t^2 \arctan(1/t) > 0$ om $t > 0$

så ej lok. extrempunkt

i)

$f(x,y) = (x-y)^2 + xy^3 = x^2 + y^2 - 2xy + xy^3$

$= x^2 + y^2 + xy(y^2-2) = \left(x + \frac{y(y^2-2)}{2}\right)^2 - \frac{y^2(y^2-2)^2}{4} + y^2$

$$= \underbrace{\left(x + \frac{y(y^2-2)}{2}\right)^2}_{\geq 0} + y^2 \underbrace{\left(\frac{4 - y^4 + 4y^2 - 4}{4}\right)}_{\geq 0 \text{ om } y \text{ nära } 0} \geq 0$$

Lokalt min.

2.64)

a) $(x^2+y^2-1)e^y = (x^2+y^2-1)\left(1+y+\frac{y^2}{2}+O(y^3)\right) =$

$= -1-y+x^2+y^2-\frac{y^2}{2}+\cancel{O(y^3)} = -1-y+x^2+\frac{y^2}{2}+O(|(x,y)|^3)$

$O(|(x,y)|^3)$

b) $2\sqrt{1+x^2+y} - \cos(x-z) - y =$

$= \left/ \begin{array}{l} \sqrt{1+t} = 1+\frac{t}{2}-\frac{t^2}{8}+O(t^3) \\ \cos s = 1-\frac{s^2}{2}+O(s^4) \end{array} \right/ =$

$= 2\left(1+\frac{(x^2+y)}{2}-\frac{(x^2+y)^2}{8}+O((x^2+y)^3)\right) -$

$- 1 - \frac{(x-z)^2}{2} + O((x-z)^4) - y =$

$= \ldots = 1 + \frac{3x^2}{2} - xz - \frac{y^2}{4} + \frac{z^2}{2} + O\left(|(x,y,z)|^3\right).$

2.65) $f(x,y) = \ln(2x^2+xy+y^2)$, $f(2,-1) = \ln 7$

$f'_x = \frac{4x+y}{2x^2+xy+y^2}$, $f'_x(2,-1) = \frac{8-1}{8-2+1} = 1$

$f'_y = \frac{x+2y}{2x^2+xy+y^2}$, $f'_y(2,-1) = \frac{2-2}{7} = 0$

$f''_{xx} = \frac{4(2x^2+xy+y^2)-(4x+y)(4x+y)}{(2x^2+xy+y^2)^2}$, $f''_{xx}(2,-1) = \ldots = -\frac{3}{7}$

$f''_{xy} = \frac{(2x^2+xy+y^2)-(4x+y)(x+2y)}{(2x^2+xy+y^2)^2}$, $f''_{xy}(2,-1) = \ldots = \frac{1}{7}$

$f''_{yy} = \frac{2(2x^2+xy+y^2)-(x+2y)(x+2y)}{(2x^2+xy+y^2)^2}$, $f''_{yy}(2,-1) = \ldots = \frac{2}{7}.$

$f(2+h,-1+k) = f(2,-1) + f'_x(2,-1)h + f'_y(2,-1)k + \frac{f''_{xx}(2,-1)}{2}h^2 +$

$+ f''_{xy}(2,-1)hk + \frac{f''_{yy}(2,-1)}{2}k^2 + O(|(h,k)|^3)$

$= \ln 7 + h - \frac{3h^2}{14} + \frac{hk}{7} + \frac{k^2}{7} + O(|(h,k)|^3).$

2.66)

a) $h^2 - hk + k^2 = \left(h - \frac{k}{2}\right)^2 - \frac{k^2}{4} + k^2 = \left(h - \frac{k}{2}\right)^2 + \frac{3k^2}{4}$

pos. def.

b)

$h^2 + hk - k^2 = (h \ \ k)\begin{pmatrix} 1 & 1/2 \\ 1/2 & -1 \end{pmatrix}\begin{pmatrix} h \\ k \end{pmatrix}$

$\begin{vmatrix} 1-\lambda & 1/2 \\ 1/2 & -1-\lambda \end{vmatrix} = (1-\lambda)(-1-\lambda) - \frac{1}{4} = \lambda^2 - \frac{5}{4} = 0 \quad \lambda = \pm \frac{\sqrt{5}}{2}$

indefinit

c)

$hk = Q(h,k) \quad Q(h,h) = h^2 > 0, \ h \neq 0, \ Q(h,-h) = -h^2 < 0 \ h \neq 0$

indefinit

d)

$6k^2 + l^2 - 6hk - 2hl + 4kl = (l - h + 2k)^2 - h^2 + 4hk - 4k^2 + 6k^2 - 6hk$

$= (l - h + 2k)^2 - (h+k)^2 + k^2 + 2k^2 =$

$= (l - h + 2k)^2 - (h+k)^2 + 3k^2$ indefinit

e)

$4hk + 4kl - 2h^2 - 3k^2 - 4l^2 = (h \ \ k \ \ l)\begin{pmatrix} -2 & 2 & 0 \\ 2 & -3 & 2 \\ 0 & 2 & -4 \end{pmatrix}\begin{pmatrix} h \\ k \\ l \end{pmatrix}$

$\begin{vmatrix} -2-\lambda & 2 & 0 \\ 2 & -3-\lambda & 2 \\ 0 & 2 & -4-\lambda \end{vmatrix} = (-2-\lambda)\left((-3-\lambda)(-4-\lambda) - 4\right) - 2\left(2(-4-\lambda)\right)$

$= (-2-\lambda)(\lambda^2 + 7\lambda + 8) + 4\lambda + 16 =$

$= -\lambda^3 - 9\lambda^2 - 18\lambda = 0 \iff \lambda = 0$ eller $\lambda^2 + 9\lambda + 18 = 0$

$\iff \lambda = 0$ eller $\lambda = -\frac{9}{2} \pm \sqrt{\frac{81}{4} - 18} = -\frac{9}{2} \pm \sqrt{\frac{9}{4}} = -\frac{9}{2} \pm \frac{3}{2}$

$\lambda = 0, -3, -6$ Negativ semidefinit

2.66)

f) $\cancel{h^2 + 2hk + k^2 = (h+k)^2}$ pos. semidefinit.

$Q(h,k,l) = h^2 + 2hk + 2k^2 = (h+k)^2 + k^2 \geq 0$ $Q(0,0,l) = 0$ för alla l

g) $(h-k)^2 + (k-l)^2 + (l-h)^2 \geq 0$

$Q(h,h,h) = 0^2 + 0^2 + 0^2 = 0$ pos. semidefinit.

h) $h^2 + 2hk + 3k^2 - 4kl + 6l^2 =$

$= (h+k)^2 + 2k^2 - 4kl + 6l^2 = (h+k)^2 + 2(k-l)^2 + 4l^2$

pos. definit.

i) $h^2 + 2hk + k^2 - 12kl = (h+k)^2 - 12kl$

$Q(h,0,0) = h^2 > 0$ om $h \neq 0$

$Q(h,-h,h) = (h-h)^2 - 12h^2 = -12h^2 < 0$ om $h \neq 0$ indefinit

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2.68

a) $f(x,y) = -1 - y + \underbrace{x^2 - \frac{y^2}{2}}$ $+ O(\|(x,y)\|^3)$

$f'_y(0,0) = -1 \neq 0$, så ej stationär

~~indefinit => sadelpunkt~~

b) $f(x,y,z) = 1 + \underbrace{\frac{3x^2}{2} - xz - \frac{y^2}{4} + \frac{z^2}{2}} + O(\|(x,y,z)\|^3)$

Indefinit => sadelpunkt

2.69)

a)

$$f(x,y) = -1 \underbrace{- x^2 + 2xy - 2y^2}_{\text{andragradstermer}} + x^3y - x^4 = -1 - x^2 + 2xy - 2y^2 + O(|(x,y)|^3)$$

$$-x^2 + 2xy - 2y^2 = -(x-y)^2 - y^2 \quad \text{neg. definit}$$

$\Rightarrow$ lokalt max.

b)

$$f(x,y) = x^2 + x^3 - 2xy + y^2 = (x-y)^2 + x^3$$

$f(x,x) = x^3$ byter tecken i 0, så varken eller.

c) $f(x,y) = x^2 + x^4 - 2xy + y^2 = (x-y)^2 + x^4 \geq 0$

$f(0,0) = 0 \Rightarrow$ min.

d)

$$f(x,y,z) = x + y^2 + y^2 + z^2$$

$\nabla f(0,0,0) = (1,0,0) \neq (0,0,0)$ ej extrempunkt

e)

$$f(x,y,z) = 2\cos(x+y+z) + e^{xy} + e^{yz} + e^{xz}$$

$$= 2\left(1 - \frac{(x+y+z)^2}{2} + O((x+y+z)^4)\right) +$$

$$+ \left(1 + xy + O(x^2y^2)\right) + \left(1 + xz + O(x^2z^2)\right) + \left(1 + yz + O(y^2z^2)\right)$$

$$= 5 - (x+y+z)^2 + xy + xz + yz + O(|(x,y,z)|^4)$$

$$= \ldots = 5 \underbrace{- \left(x + \frac{y}{2} + \frac{z}{2}\right)^2 - \frac{3}{4}\left(y + \frac{z}{3}\right)^2 - \frac{2z^2}{3}}_{\text{Neg. definit}} + O(|(x,y,z)|^4)$$

$\Rightarrow$ Lokalt max.

f)

$f(x,y,z) = \cos(x+y+z) + \cos x \leq 2 \quad f(0,0,0) = 2$

Max.

2.70)

a)

$f(x,y) = 3 + 4x - 4y - x^2 - 2y^2$

$\nabla f = (4 - 2x, -4 - 4y) = (0,0) \Leftrightarrow x = 2, y = -1$

$f''_{xx} = -2, \quad f''_{xy} = 0, \quad f''_{yy} = -4.$

$f(2+h, -1+k) = 9 \underbrace{-h^2 - 2k^2}_{\text{neg. definit}}$

Lokalt max i $(2,-1)$.

b) $f(x,y,z) = x^2 + y^2 + z^2 - xy + 2z + x$

$\nabla f = (2x - y + 1, 2y - x, 2z + 2) = (0,0,0)$

$$\Leftrightarrow \begin{cases} 2x - y = -1 \\ 2y - x = 0 \\ z = -1 \end{cases} \Leftrightarrow \begin{cases} 3y = -1 \Leftrightarrow y = -1/3 \\ x = 2y = -2/3 \\ z = -1 \end{cases}$$

$f''_{xx} = 2, \quad f''_{xy} = -1, \quad f''_{yy} = 2, \quad f''_{xz} = 0, \quad f''_{yz} = 0 \quad f''_{zz} = 2$

$h^2 - hk + k^2 + l^2 = (h - \frac{k}{2})^2 + \frac{3k^2}{4} + l^2$ pos. definit

Lokalt min i $(-2/3, -1/3, -1)$.

c)

$f(x,y) = x e^{-2x^2 - y^2}$

$\nabla f = (e^{-2x^2-y^2} - 4x^2 e^{-2x^2-y^2}, -2xy e^{-2x^2-y^2}) = (0,0)$

$$\Leftrightarrow \begin{cases} 1 - 4x^2 = 0 \\ -2xy = 0 \end{cases} \Leftrightarrow \begin{cases} x = \pm 1/2 \\ y = 0 \end{cases}$$

$f''_{xx} = -4x \cdot e^{-2x^2-y^2} - 8x e^{-2x^2-y^2} + 16x^3 e^{-2x^2-y^2}$

$f''_{xy} = -2y e^{-2x^2-y^2} + 8x^2 y e^{-2x^2-y^2}$

$f''_{yy} = -2x e^{-2x^2-y^2} + 4xy^2 e^{-2x^2-y^2}$

$(1/2, 0)$ $\quad f''_{xx}(1/2,0)h^2 + 2f''_{xy}(1/2,0)hk + f''_{yy}(1/2,0)k^2 =$

$= e^{-1/2}(-\overset{4}{\cancel{2}}h^2 - k^2)$ neg. def $\Rightarrow$ lokalt max

$(-1/2, 0)$ $\quad f''_{xx}(-1/2,0)h^2 + 2f''_{xy}(-1/2,0)hk + f''_{yy}(-1/2,0)k^2$

$= e^{-1/2}(\overset{4}{\cancel{2}}h^2 + k^2)$ pos. def $\Rightarrow$ lokalt min.

2.70)

d)

$f(x,y) = x + y - 3\ln(2+xy) \qquad x>0,\ y>0$

$\nabla f = \left(1 - \frac{3y}{2+xy},\ 1 - \frac{3x}{2+xy}\right) = (0,0)$

$\Leftrightarrow \begin{cases} 3y = 2+xy \\ 3x = 2+xy \end{cases} \Leftrightarrow \begin{cases} x = y \\ 3x = 2 + x^2 \end{cases} \Leftrightarrow \begin{cases} x = \frac{3}{2} \pm \sqrt{\frac{9}{4} - 2} = \frac{3}{2} \pm \frac{1}{2} \\ y = x \end{cases}$

$\Leftrightarrow \begin{cases} x = y = 2 \\ x = y = 1 \end{cases}$

$f''_{xx} = \frac{3y^2}{(2+xy)^2}, \quad f''_{xy} = -\frac{3(2+xy) - 3y \cdot x}{(2+xy)^2}, \quad f''_{yy} = \frac{3x^2}{(2+xy)^2}$

$(1,1): \quad f''_{xx}(1,1)h^2 + 2f''_{xy}(1,1)hk + f''_{yy}(1,1)k^2 =$

$= \frac{1}{3}h^2 - \frac{4}{3}hk + \frac{1}{3}k^2 = \frac{1}{3}(h^2 + 4hk + k^2) =$

$= \frac{1}{3}((h-2k)^2 - 3k^2)$ indefinit $\Rightarrow$ sadelpunkt.

$(2,2): \quad f''_{xx}(2,2)h^2 + 2f''_{xy}(2,2)hk + f''_{yy}(2,2)k^2 =$

$= \frac{1}{3}h^2 - \frac{1}{3}hk + \frac{1}{3}k^2 = \frac{1}{3}(h^2 + hk + k^2) =$

$= \frac{1}{3}\left(\left(h - \frac{k}{2}\right)^2 + \frac{3k^2}{4}\right)$ pos. definit

lokalt min.

e)

$f(x,y) = \ln(x^2+y^2) - x - 2y$

$\nabla f = \left(\frac{2x}{x^2+y^2} - 1,\ \frac{2y}{x^2+y^2} - 2\right) = (0,0)$

$\Leftrightarrow \begin{cases} x^2+y^2 = 2x \\ x^2+y^2 = y \end{cases} \Leftrightarrow \begin{cases} 2x = y \\ x^2 + 4x^2 = 2x \end{cases} \Leftrightarrow \begin{cases} (x = y = 0) \text{ ej i } D_f \\ x = 2/5,\ y = 4/5 \end{cases}$

$f''_{xx} = \frac{2(x^2+y^2) - 2x \cdot 2x}{(x^2+y^2)^2}, \quad f''_{xy} = \frac{-2x \cdot 2y}{(x^2+y^2)^2}, \quad f''_{yy} = \frac{2(x^2+y^2) - 2y \cdot 2y}{(x^2+y^2)^2}$

$f''_{xx}(2/5,4/5)h^2 + 2f''_{xy}(2/5,4/5)hk + f''_{yy}(2/5,4/5)k^2 =$

$= \frac{24}{20^2}h^2 - \frac{64hk}{20^2} - \frac{24}{20^2}k^2 = \frac{8}{20^2}(3h^2 - 8hk - 3k^2)$ indef

sadelpunkt

2.70) f)

$$f(x,y,z) = x^4 + y^4 + z^4 - 4xyz$$

$$\nabla f = (4x^3 - 4yz,\ 4y^3 - 4xz,\ 4z^3 - 4xy) = (0,0,0)$$

$$\Leftrightarrow \begin{cases} x^3 = yz \\ y^3 = xz \\ z^3 = xy \end{cases}$$

$x = 0 \Rightarrow y = z = 0$ o.s.v. så $(0,0,0)$ en punkt

$x, y, z \neq 0$ ger

$$\begin{cases} \frac{x^3}{y^3} = \frac{y}{x} \Leftrightarrow x^4 = y^4 \\ \frac{y^3}{z^3} = \frac{z}{y} \Leftrightarrow y^4 = z^4 \\ z^3 = xy \end{cases}$$

$x = \pm y;\ y = \pm z$ samt $z^3 = xy$ ger nu

möjligheterna $(1,1,1)$, $(1,-1,-1)$, $(-1,-1,1)$, $(-1,1,-1)$, $(0,0,0)$

$f''_{xx} = 12x^2$, $f''_{xy} = -4z$, $f''_{xz} = -4y$, $f''_{yy} = 12y^2$, $f''_{yz} = -4x$, $f''_{zz} = 12z^2$

$(0,0,0)$: Alla andra derivator noll, men

$f(x,x,x) = 3x^4 - 4x^3 < 0$ nära 0, $x \neq 0$

$f(x,0,0) = x^4 > 0$ nära 0 $x \neq 0$ så sadelpunkt

$$Q(h,k,l) = f''_{xx}h^2 + 2f''_{xy}hk + f''_{xz}hl + f''_{yy}k^2 + 2f''_{yz}kl + f''_{zz}l^2$$

i $(1,1,1)$ $Q(h,k,l) = 12h^2 - 8hk - 8hl + 12k^2 - 8kl + 12l^2$

$$= 4(3h^2 - 2hk - 2hl + 3k^2 - 2kl + 3l^2) =$$

$$= 4\left(3\left(h - \frac{k}{3} + \frac{l}{3}\right)^2 + \frac{8k^2}{3} - 4kl + \frac{8l^2}{3}\right)$$

$$= 4\left(3\left(h - \frac{k}{3} - \frac{l}{3}\right)^2 + \frac{8}{3}\left(k - \frac{12l}{16}\right)^2 + \frac{7l^2}{6}\right)$$

Pos. definit. Lokalt min i $(1,1,1)$.

Övriga punkter behandlas analogt, och alla

dessa är lokala min.

2.70)

a)

$f(x,y) = x^3 + 3xy^2 - 15x - 12y$

$\nabla f = (3x^2 + 3y^2 - 15,\ 6xy - 12) = (0,0)$

$\Leftrightarrow \begin{cases} x^2 + y^2 = 5 \\ xy = 2 \end{cases} \Leftrightarrow \begin{cases} x^2 + 4/x^2 = 5 \\ y = 2/x \end{cases} \Leftrightarrow \begin{cases} x^4 - 5x^2 + 4 = 0 \\ y = 2/x \end{cases}$

$\Leftrightarrow \begin{cases} x^2 = \frac{5}{2} \pm \sqrt{\frac{25}{4} - 4} = \frac{5}{2} \pm \frac{3}{2} \\ y = 2/x \end{cases} \Leftrightarrow \begin{cases} x = 2, y = 1, \quad x = -2, y = -1 \\ x = 1, y = 2, \quad x = -1, y = -2. \end{cases}$

$f''_{xx} = 6x, \quad f''_{xy} = 6y, \quad f''_{yy} = 6x$

$Q(h,k) = f''_{xx} h^2 + 2 f''_{xy} hk + f''_{yy} k^2$

i $(2,1)$ $\quad Q(h,k) = 12h^2 + 12hk + 12k^2 = 12(h^2 + hk + k^2) = 12\left(\left(h - \frac{k}{2}\right)^2 + \frac{3k^2}{4}\right)$

pos. def. lok. min.

i $(-2,-1)$ $\quad Q(h,k) = -12h^2 - 12hk - 12k^2 \Rightarrow$ neg. def. lok. max.

i $(1,2)$: $\quad Q(h,k) = 6h^2 + 24hk + 6k^2 = 6(h^2 + 4hk + k^2) =$

$= 6\left((h + 2k)^2 - 3k^2\right)$ indef. sadel.

i $(-1,-2)$: $\quad Q(h,k) = -6h^2 - 24hk - 6k^2 = -6(h^2 + 4hk + k^2)$

indef. m, sadel.

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b)

$f(x,y) = (x^2 + y^2 - 4)e^{-x-y}$

$\nabla f = \left(2xe^{-x-y} - (x^2 + y^2 - 4)e^{-x-y},\ 2ye^{-x-y} - (x^2 + y^2 - 4)e^{-x-y}\right) = (0,0)$

$\Leftrightarrow \begin{cases} x^2 + y^2 - 2x = 4 \\ x^2 + y^2 - 2y = 4 \end{cases} \Leftrightarrow \begin{cases} x = y \\ 2x^2 - 2x = 4 \end{cases} \Leftrightarrow \begin{cases} x = y \\ x^2 - x - 2 = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x = y \\ x = \frac{1}{2} \pm \frac{3}{2} \end{cases}$ $\quad (-1,-1)$ och $(2,2)$ stat. punkter

$f''_{xx} = \left(2 - 2x - 2x + (x^2 + y^2 - 4)\right)e^{-x-y}, \quad f''_{xy} = \left(-2x - 2y + (x^2 + y^2 - 4)\right)e^{-x-y}$

$f''_{yy} = \left(2 - 2y - 2y + (x^2 + y^2 - 4)\right)e^{-x-y}$

i $(-1,-1)$: $\quad e^2(4h^2 + 4hk + 4k^2) = 4e^2(h^2 + hk + k^2) = 4e^2\left(\left(h + \frac{k}{2}\right) + \frac{3k^2}{4}\right)$

pos. def. $\Rightarrow$ lok. min

i $(2,2)$ $\quad e^{-4}(-2h^2 - 8hk - 2k^2) = -2e^{-4}\left((h + 2k)^2 - 3k^2\right)$ indef. sadel.

2.70 i)

$f(x,y) = 4x^2 + 4xy^2 + y^4 + y^6$

$\nabla f = (8x + 4y^2,\ 8xy + 4y^3 + 6y^5) = (0,0) \Leftrightarrow$

$$\begin{cases} 2x + y^2 = 0 \\ 4xy + 2y^3 + 3y^5 = 0 \end{cases} \Leftrightarrow y = 0 \text{ eller } 4x + 2y^2 + 3y^4 = 0$$

$$(x,y) = (0,0) \text{ eller } \begin{cases} y^2 = -2x \\ 4x + 2(-2x) + 3(-2x)^2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} y^2 = -2x \\ 12x^2 = 0 \end{cases}$$

(0,0) enda stationära punkten.

Andraderivatatest. ger inget då kv. formen blir positivt semidefinit.

$4x^2 + 4xy^2 + y^4 + y^6 = (2x + y^2)^2 + y^6 \geq 0 = f(0,0)$

så minimum i (0,0).

j)

$f(x,y,z) = 4x + y^2/x + 4z^2/y + 8/z \qquad (x,y,z \neq 0)$

$\nabla f = (4 - y^2/x^2,\ 2y/x - 4z^2/y^2,\ 8z/y - 8/z^2) = (0,0,0)$

$$\Leftrightarrow \begin{cases} 4x^2 = y^2 \\ 2y^3 = 4xz^2 \\ 8z^3 = 8y \end{cases}$$

Andra ekv. ger att om x är negativt måste y vara negativt.

$$\text{så} \begin{cases} y = 2x \\ 8x^3 = 2xz^2 \\ 8z^3 = 2x \end{cases} \Leftrightarrow \begin{cases} y = 2x \\ z^2 = 4x^2 \\ z^3 = 2x \end{cases} \Leftrightarrow \begin{cases} y = 2x \\ z = \pm 2x \\ \pm 8x^3 = 2x \end{cases}$$

$$\begin{cases} x = 1/2 \\ y = 1 \\ z = 1 \end{cases} \text{ eller } \begin{cases} x = -1/2 \\ y = -1 \\ z = -1. \end{cases}$$

$f''_{xx} = \frac{2y^2}{x^3}$, $f''_{xy} = -\frac{2y}{x^2}$, $f''_{xz} = 0$, $f''_{yy} = \frac{2}{x} + \frac{8z^2}{y^3}$, $f''_{yz} = -\frac{8z}{y^2}$, $f''_{zz} = \frac{8}{y} + \frac{16}{z^3}$

i $(1/2,1,1)$: $16h^2 - 16hk + 9k^2 - 16kl + 24l^2 = 16(h - \frac{k}{2})^2 + 5(k - \frac{8l}{5})^2 + \frac{56l^2}{5}$

pos def. $\Rightarrow$ lokalt min

i $(-1/2,1,1)$: $-16h^2 + 16hk - 9k^2 + 16kl - 24l^2 \Rightarrow$ neg. definit. lok. max.

2.63)

$f(x,y) = (y-x^2)(y-3x^2)$

a) $f(x,kx) = (kx-x^2)(kx-3x^2) =$

$= k^2x^2 + O(x^3)$ så om $k \neq 0$ lok. min i $x=0$

$f(x,0) = 3x^4$ har lokalt min. i $x=0$

$f(0,y) = y^2$ har lokalt min i $y=0$.

b) $f(x,2x^2) = (2x^2-x^2)(2x^2-3x^2) = -x^4$ har lokalt max i $x=0$.

2.67)

$h^2 + 2ahk + k^2 = (h+ak)^2 + (1-a^2)k^2$

$1-a^2 > 0 \Rightarrow$ pos. definit

$1-a^2 = 0 \Rightarrow$ pos. semidefinit

$1-a^2 < 0 \Rightarrow$ indefinit

2.71) $f(x,y) = 4x^2e^y - 2x^4 - e^{4y}$

a) $\nabla f = (8xe^y - 8x^3, 4x^2e^y - 4e^{4y}) = (0,0)$

$$\Leftrightarrow \begin{cases} 8xe^y = 8x^3 \\ 4x^2e^y = 4e^{4y} \Rightarrow x \neq 0 \end{cases} \Leftrightarrow \begin{cases} e^y = x^2 \\ e^{2y} = e^{4y} \end{cases} \Leftrightarrow \begin{cases} x = \pm 1 \\ y = 0 \end{cases}$$

b)

$f''_{xx} = 8e^y - 24x^2$, $f''_{xy} = 8xe^y$, $f''_{yy} = 4x^2e^y - 16e^{4y}$

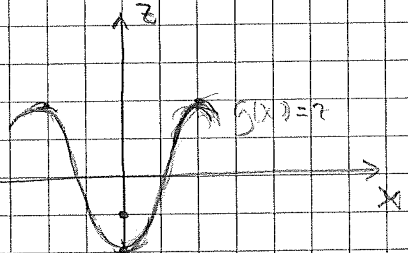
i $(1,0)$: $-16h^2 + 16hk - 12k^2 = -4(4h^2 - 4hk + 3k^2) =$

$= -4(4(h-\frac{k}{2})^2 + 2k^2)$ neg. definit $\Rightarrow$ lok. max

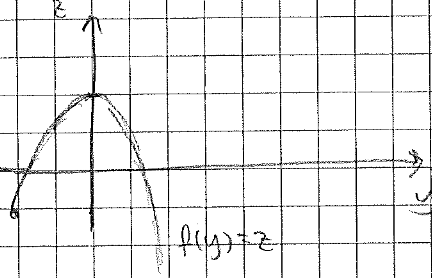
i $(-1,0)$: $-16h^2 - 16hk - 12k^2 = -4(4h^2 + 4hk + 3k^2) =$

$= -4(4(h+\frac{k}{2})^2 + 2k^2)$ neg definit $\Rightarrow$ lok. max

c) $g(x) = f(x,0) = 4x^2 - 2x^4 - 1$



d) $h(y) = f(\pm 1, y) = 4e^y - 2 - e^{4y}$

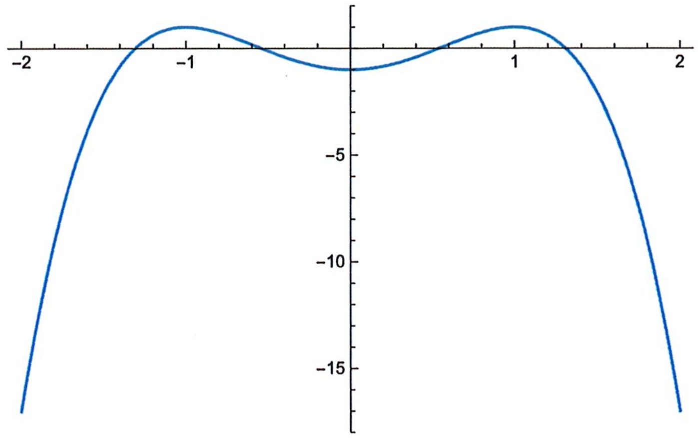


2.71

c)

```
In[1]:= Plot[4 * x^2 - 2 * x^4 - 1, {x, -2, 2}]
```

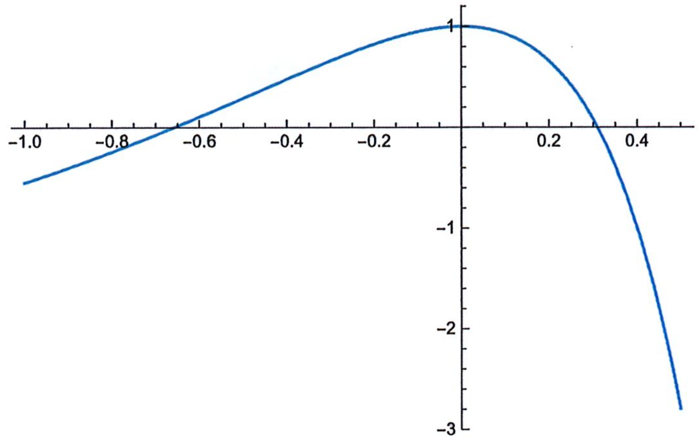
Out[1]=



d)

```
In[2]:= Plot[4 * Exp[y] - 2 - Exp[4 * y], {y, -1, 0.5}]
```

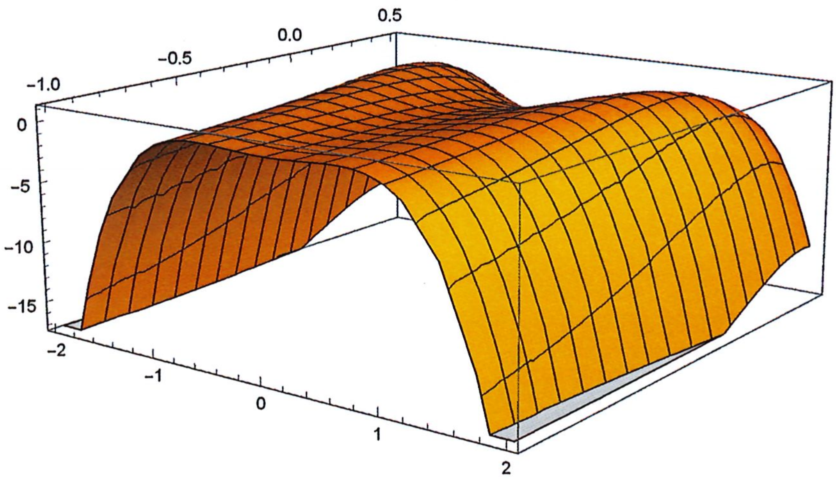
Out[2]=



e)

```
In[3]:= Plot3D[4 * x^2 * Exp[y] - 2 * x^4 - Exp[4 * y], {x, -2, 2}, {y, -1, 0.5}]
```

Out[3]=



2.73)

$$f(x,y) = \begin{cases} \dfrac{x^4+y^4}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Eftersom $f(x,y) \geq 0$ med likhet om och endast om $x=y=0$ har funktionen ett (globalt) strängt min i $(x,y)=(0,0)$.

Om $(x,y) \neq (0,0)$ kan vi skriva

$$f(\rho,\theta) = \frac{\rho^4(\cos^4\theta + \sin^4\theta)}{\rho^2} = \rho^2(\cos^4\theta + \sin^4\theta).$$

(Om $f(x,y)$ har ett lokalt max/min i punkten (a,b) har $f(\rho,\theta)$ det i motsvarande punkt i (ρ,θ)-planet).

Eftersom $f'_\rho = 2\rho(\cos^4\theta + \sin^4\theta) > 0$ om $\rho > 0$ har funktionen inga stationära punkter i $(x,y) \neq (0,0)$.

2.74) Med $(a,b) = (0,0)$

$$f(x,y) = x^2 + 100xy + y^2$$

gäller $f''_{xx} = 2$, $f''_{xy} = 100$, $f''_{yy} = 2$

$f(x,x) = 102x^2 > 0 \quad x \neq 0$

$f(x,-x) = -98x^2 < 0, \ x \neq 0$ så sadelpunkt.