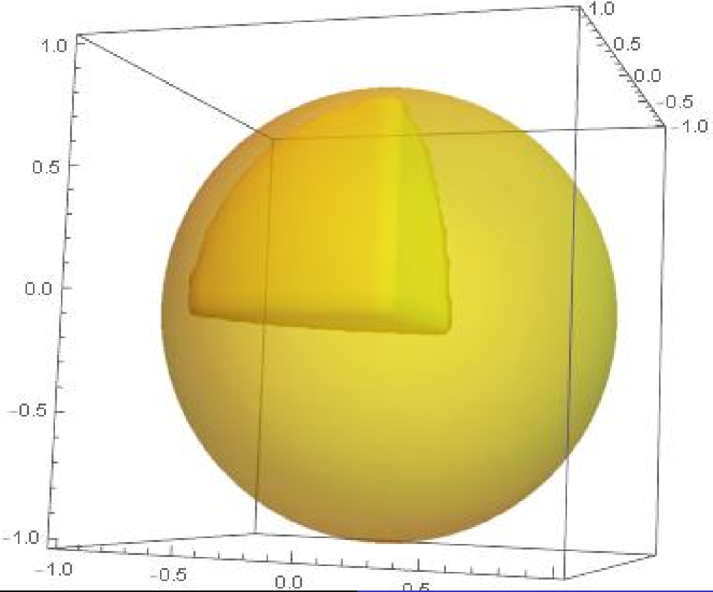
Exempel

Beräkna

$$\iiint_D xzdxdydz$$

där D är den del av enhetsklotet som ligger i $x \leq 0$, $y \leq 0$, $z \geq 0$.

Bild av D



Lösning

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$$\int_0^{\pi/2} \left(\int_{\pi}^{3\pi/2} \left(\int_0^1 r\cos\varphi\sin\theta \cdot r\cos\theta \cdot r^2 \sin\theta dr \right) d\varphi \right) d\theta$$

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$$\int_\pi^{3\pi/2}\cos\varphi d\varphi \cdot \int_0^{\pi/2}\sin^2\theta\cos\theta d\theta \cdot \int_0^1 r^4 dr$$

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$$\int_\pi^{3\pi/2}\cos\varphi d\varphi\cdot\int_0^{\pi/2}\sin^2\theta\cos\theta d\theta\cdot\int_0^1 r^4 dr = \ldots = \frac{-1}{15}.$$