Exempel

Beräkna

$$\iiint_D \sin(x+y+z)dxdydz$$

där D är den parallellepiped som ges av olikheterna
$0 \leq x+y+z \leq \pi/2$, $0 \leq 2x+y+z \leq 2$ och $0 \leq x+y+3z \leq 1$.

Lösning

$0 \leq x + y + z \leq \pi/2$, $0 \leq 2x + y + z \leq 2$ och $0 \leq x + y + 3z \leq 1$.

Lösning

$0 \leq x + y + z \leq \pi/2$, $0 \leq 2x + y + z \leq 2$ och $0 \leq x + y + 3z \leq 1$.

$$\begin{cases} u = x + y + z \\ v = 2x + y + z \\ w = x + y + 3z \end{cases}$$

Lösning

$0 \leq x+y+z \leq \pi/2$, $0 \leq 2x+y+z \leq 2$ och $0 \leq x+y+3z \leq 1$.

$$\begin{cases} u = x+y+z \\ v = 2x+y+z \\ w = x+y+3z \end{cases}$$

I *uvw*-koordinaterna ges området av olikheterna $0 \leq u \leq \pi/2$, $0 \leq v \leq 2$ och $0 \leq w \leq 1$.

Lösning

$0 \leq x + y + z \leq \pi/2$, $0 \leq 2x + y + z \leq 2$ och $0 \leq x + y + 3z \leq 1$.

$$\begin{cases} u = x + y + z \\ v = 2x + y + z \\ w = x + y + 3z \end{cases}$$

I *uvw*-koordinaterna ges området av olikheterna $0 \leq u \leq \pi/2$, $0 \leq v \leq 2$ och $0 \leq w \leq 1$.

$$\frac{d(u,v,w)}{d(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

Lösning

$0 \leq x+y+z \leq \pi/2$, $0 \leq 2x+y+z \leq 2$ och $0 \leq x+y+3z \leq 1$.

$$\begin{cases} u = x+y+z \\ v = 2x+y+z \\ w = x+y+3z \end{cases}$$

I uvw-koordinaterna ges området av olikheterna $0 \leq u \leq \pi/2$, $0 \leq v \leq 2$ och $0 \leq w \leq 1$.

$$\frac{d(u,v,w)}{d(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = \ldots = -2.$$

Lösning

$0 \leq x+y+z \leq \pi/2$, $0 \leq 2x+y+z \leq 2$ och $0 \leq x+y+3z \leq 1$.

$$\begin{cases} u = x+y+z \\ v = 2x+y+z \\ w = x+y+3z \end{cases}$$

I uvw-koordinaterna ges området av olikheterna $0 \leq u \leq \pi/2$, $0 \leq v \leq 2$ och $0 \leq w \leq 1$.

$$\frac{d(u,v,w)}{d(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = \ldots = -2.$$

$$dxdydz = \left| \frac{d(x,y,z)}{d(u,v,w)} \right| dudvdw$$

Lösning

$0 \leq x + y + z \leq \pi/2$, $0 \leq 2x + y + z \leq 2$ och $0 \leq x + y + 3z \leq 1$.

$$\begin{cases} u = x + y + z \\ v = 2x + y + z \\ w = x + y + 3z \end{cases}$$

I uvw-koordinaterna ges området av olikheterna $0 \leq u \leq \pi/2$, $0 \leq v \leq 2$ och $0 \leq w \leq 1$.

$$\frac{d(u,v,w)}{d(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = \ldots = -2.$$

$$dxdydz = \left| \frac{d(x,y,z)}{d(u,v,w)} \right| dudvdw = \left| \frac{d(u,v,w)}{d(x,y,z)} \right|^{-1} dudvdw = \frac{1}{2} dudvdw.$$

lösning

$$\iiint_D \sin(x+y+z)dxdydz =$$

$$\int_0^1 \left(\int_0^2 \left(\int_0^{\pi/2} \sin u \frac{1}{2} du \right) dv \right) dw$$

lösning

$$\iiint_D \sin(x+y+z)dxdydz =$$

$$\int_0^1 \left(\int_0^2 \left(\int_0^{\pi/2} \sin u \frac{1}{2} du \right) dv \right) dw =$$

$$\frac{1}{2} \int_0^1 dw \cdot \int_0^2 dv \cdot \int_0^{\pi/2} \sin u du = \ldots = 1.$$