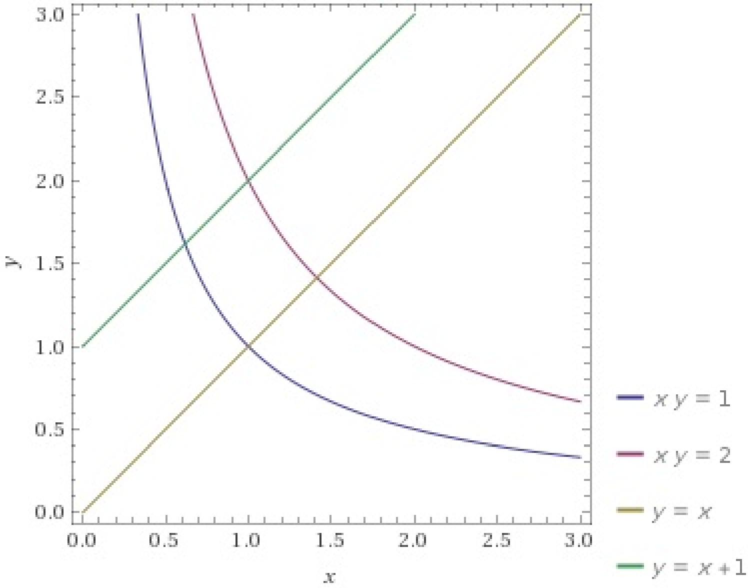
# Exempel

Beräkna arean av det begränsade området i första kvadranten mellan kurvorna $xy = 1$, $xy = 2$, $y = x$ och $y = 1 + x$.

# Område



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$$\frac{d(u, v)}{d(x, y)} = \begin{vmatrix} y & x \\ -1 & 1 \end{vmatrix} = y + x = \ldots = \sqrt{v^2 + 4u}.$$

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$$\frac{1}{4}\left(3-\sqrt{5}+4\ln\frac{4}{1+\sqrt{5}}\right) \approx 0.4.$$