Exempel

Beräkna (om den är konvergent)

$$\iint_D \frac{1}{(x^2+y^2)^2}\,dxdy,$$

där $D = \{(x,y) \in \mathbb{R}^2 : 1 \leq x^2+y^2, x \geq 0, y \geq 0\}$.

Lösning

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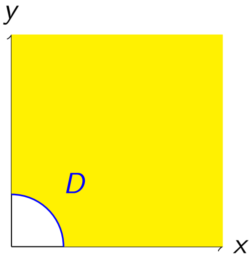

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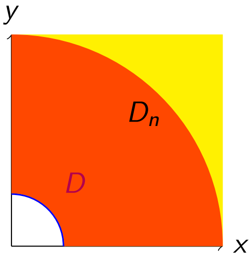

$$D_n = \{(x, y) : 1 \le x^2 + y^2 \le n^2, x \ge 0, y \ge 0\}.$$

Lösning

$$\iint_D \frac{1}{(x^2+y^2)^2}\,dxdy = \lim_{n\to\infty} \iint_{D_n} \frac{1}{(x^2+y^2)^2}\,dxdy$$

Lösning

$$\iint_D \frac{1}{(x^2+y^2)^2}\,dxdy = \lim_{n\to\infty} \iint_{D_n} \frac{1}{(x^2+y^2)^2}\,dxdy =$$

$$\lim_{n\to\infty} \int_0^{\pi/2} \left(\int_1^n \frac{1}{\rho^4}\,\rho d\rho \right) d\varphi$$

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$$\lim_{n\to\infty}\frac{\pi}{2}\left[\frac{-1}{2\rho^2}\right]_1^n = \lim_{n\to\infty}\frac{\pi}{2}(\frac{1}{2}-\frac{1}{2n^2}) = \frac{\pi}{4}.$$