Exempel

Beräkna

$$\iint_D \frac{1}{\sqrt{x^2+y^2}}\,dxdy,$$

där $D = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \le 1\}$.

Lösning

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$$\iint_D \frac{1}{\sqrt{x^2 + y^2}} dxdy = \iint_{D \setminus \{(0,0)\}} \frac{1}{\sqrt{x^2 + y^2}} dxdy.$$

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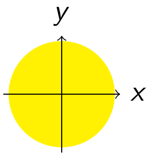

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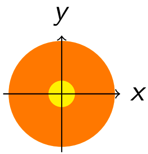

$$D_n = \{(x, y) \in \mathbb{R}^2 : 1/n^2 \le x^2 + y^2 \le 1\}$$

Lösning

$$\iint_D \frac{1}{\sqrt{x^2+y^2}}dxdy = \lim_{n\to\infty}\iint_{D_n} \frac{1}{\sqrt{x^2+y^2}}dxdy$$

Lösning

$$\iint_D \frac{1}{\sqrt{x^2+y^2}}dxdy = \lim_{n\to\infty}\iint_{D_n}\frac{1}{\sqrt{x^2+y^2}}dxdy =$$
$$\lim_{n\to\infty}\int_0^{2\pi}\left(\int_{1/n}^{1}\frac{1}{\rho}\rho d\rho\right)d\varphi$$

Lösning

$$\iint_D \frac{1}{\sqrt{x^2+y^2}}dxdy = \lim_{n\to\infty}\iint_{D_n}\frac{1}{\sqrt{x^2+y^2}}dxdy =$$

$$\lim_{n\to\infty}\int_0^{2\pi}\left(\int_{1/n}^{1}\frac{1}{\rho}\rho d\rho\right)d\varphi =$$

$$\lim_{n\to\infty} 2\pi(1-\frac{1}{n}) = 2\pi.$$