Exempel

Beräkna

$$\iint_D \frac{1}{x^2+y^2}\,dxdy,$$

där $D = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1\}$.

Lösning

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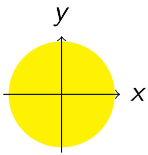
$$\iint_D \frac{1}{x^2+y^2} dxdy = \iint_{D\setminus\{(0,0)\}} \frac{1}{x^2+y^2} dxdy.$$



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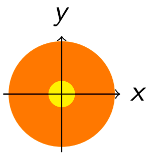

$$D_n = \{(x, y) \in \mathbb{R}^2 : 1/n^2 \leq x^2 + y^2 \leq 1\}$$

Lösning

$$\iint_D \frac{1}{x^2+y^2}\,dxdy = \lim_{n\to\infty} \iint_{D_n} \frac{1}{x^2+y^2}\,dxdy$$

Lösning

$$\iint_D \frac{1}{x^2+y^2}dxdy = \lim_{n\to\infty}\iint_{D_n}\frac{1}{x^2+y^2}dxdy =$$
$$\lim_{n\to\infty}\int_0^{2\pi}\left(\int_{1/n}^{1}\frac{1}{\rho^2}\rho d\rho\right)d\varphi$$

Lösning

$$\iint_D \frac{1}{x^2+y^2}\,dxdy = \lim_{n\to\infty} \iint_{D_n} \frac{1}{x^2+y^2}\,dxdy =$$
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$$\lim_{n\to\infty} 2\pi(\ln 1 - \ln(1/n)) = \infty.$$

Lösning

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Så integralen är divergent.