Exempel

Beräkna

$$\iint_D \mathrm{e}^{-x+y}\, dxdy,$$

där $D = \{(x, y) \in \mathbb{R}^2 : x \geq 1, y \leq -1\}$.

Lösning

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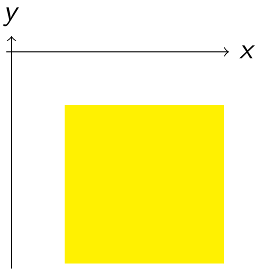

Lösning

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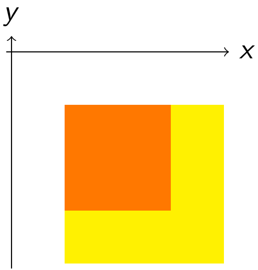

$$D_n = \{(x, y) \in \mathbb{R}^2 : 1 \leq x \leq n, -n \leq y \leq -1\}$$

Lösning

$$\iint_D \mathrm{e}^{-x+y}\,dxdy = \lim_{n\to\infty} \iint_{D_n} \mathrm{e}^{-x+y}\,dxdy$$

Lösning

$$\iint_D e^{-x+y}\,dxdy = \lim_{n\to\infty} \iint_{D_n} e^{-x+y}\,dxdy =$$
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$$\lim_{n\to\infty} (e^{-2} - e^{-n-1} - e^{-1-n} + e^{-n-n})$$

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$$\lim_{n\to\infty} \left[e^{-1+y} - e^{-n+y} \right]_{y=-n}^{-1} =$$

$$\lim_{n\to\infty} (e^{-2} - e^{-n-1} - e^{-1-n} + e^{-n-n}) = e^{-2}.$$