Exempel

Beräkna

$$\iint_D \frac{dxdy}{(xy^2)^{1/4}},$$

där $D = \{(x, y) \in \mathbb{R}^2 : 0 < y \leq x \leq 1\}$.

Lösning

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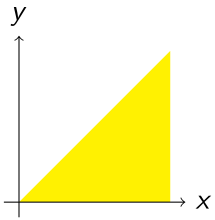

Lösning

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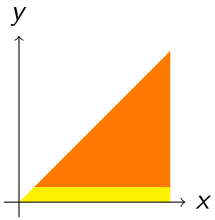

$$D_n = \{(x, y) \in \mathbb{R}^2 : 1/n \le x \le 1, 1/n \le y \le x\}$$

Lösning

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$$\lim_{n\to\infty} \left[\frac{8x^{5/4}}{5} - \frac{8(1/n)^{1/2}x^{3/4}}{3} \right]_{x=1/n}^{1}$$

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$$\lim_{n\to\infty} \left(\frac{8}{5} - \frac{8(1/n)^{1/2}}{3} - \frac{8(1/n)^{5/4}}{5} + \frac{8(1/n)^{1/2}(1/n)^{3/4}}{3} \right)$$

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