Uttryck z_x' och z_y' med hjälp av z_u' och z_v' (och ev. x och y) om

$$\begin{cases} u = xy, \\ v = x - y. \end{cases}$$

Lösning

$u = xy,\ v = x - y$

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$$z'_x = \frac{\partial z}{\partial x}$$

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$$z'_x = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial x}$$

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$$= y\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}$$

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$$= x\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}$$

$u = xy$, $v = x - y$

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$$z'_y = \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y}$$

$$= x\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = xz'_u - z'_v.$$