Exempel

Betrakta variabelbytet

$$\begin{cases} u = -x + y, \\ v = 2x - y. \end{cases}$$

Uttryck z'_x och z'_y med hjälp av z'_u och z'_v. Använd sedan detta för att bestämma alla $z \in \mathcal{C}^1(\mathbb{R}^2)$ som löser

$$z'_x + 2z'_y = 0.$$

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SVAR: Lösningarna ges av alla funktioner på formen $z = h(2x - y)$ där $h \in \mathcal{C}^1(\mathbb{R})$.