Exempel

Uttryck w'_u, w'_v och w''_{uu} i x, y om

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$$w'_v = w'_x - 2x w'_u$$

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$$w'_v = w'_x - 2x w'_u = \ldots = \frac{2y w'_x - 2x w'_y}{2x + 2y}.$$

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$$\begin{aligned} w'_u &= \frac{1}{2x+2y}(w'_x + w'_y), \\ w'_v &= w'_x - 2xw'_u = \ldots = \frac{2yw'_x - 2xw'_y}{2x+2y}. \end{aligned}$$

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\frac{1}{2x+2y}\left(\left(\frac{1}{2x+2y}(w'_x + w'_y)\right)'_x + \left(\frac{1}{2x+2y}(w'_x + w'_y)\right)'_y\right)
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$$
\frac{1}{(2x+2y)^2}\left(w''_{xx} + w''_{yx} + w''_{xy} + w''_{yy}\right) - \frac{4}{(2x+2y)^3}\left(w'_x + w'_y\right)
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\frac{1}{(2x+2y)^2}\left(w''_{xx} + 2w''_{xy} + w''_{yy} - \frac{4(w'_x + w'_y)}{2x+2y}\right).
$$

Alternativt skrivsätt med differentialoperatorer

Påståendet

$$w'_u = \frac{1}{2x+2y}(w'_x + w'_y)$$

som gäller för alla w, kan alternativt skrivas

$$\frac{\partial}{\partial u} = \frac{1}{2x+2y}\frac{\partial}{\partial x} + \frac{1}{2x+2y}\frac{\partial}{\partial y}.$$

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Alltså gäller

$$\frac{\partial^2 w}{\partial u^2} = \left(\frac{1}{2x+2y}\frac{\partial}{\partial x} + \frac{1}{2x+2y}\frac{\partial}{\partial y}\right)\left(\frac{1}{2x+2y}\frac{\partial}{\partial x} + \frac{1}{2x+2y}\frac{\partial}{\partial y}\right) w =$$

$$\left(\frac{1}{2x+2y}\frac{\partial}{\partial x} + \frac{1}{2x+2y}\frac{\partial}{\partial y}\right)\left(\frac{1}{2x+2y}\frac{\partial w}{\partial x} + \frac{1}{2x+2y}\frac{\partial w}{\partial y}\right) = \ldots$$

$$= \frac{1}{(2x+2y)^2}\left(\frac{\partial^2 w}{\partial x^2} + 2\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2} - \frac{4}{2x+2y}\frac{\partial w}{\partial x} - \frac{4}{2x+2y}\frac{\partial w}{\partial y}\right).$$