# Exempel

Bestäm Maclaurinutvecklingen av ordning 2 med ordorestterm till

$$f(x, y) = (x^2 - y + 1)\sin(x + y),$$

(a) utgående från formeln för Maclaurinutvecklingar,

(b) genom att utnyttja utvecklingen för $\sin t$ från envariabelanalysen.

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Insatt i $(*)$ får vi:

$$f(x,y) = x + y - xy - y^2 + \mathcal{O}(|(x,y)|^3).$$

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$$\begin{aligned} f(x,y) &= (x^2 - y + 1)((x+y) + \mathcal{O}((x+y)^3)) \\ &= x^3 + x^2y - xy - y^2 + x + y + \mathcal{O}(|(x,y)|^3) \end{aligned}$$

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&= x + y - xy - y^2 + \mathcal{O}(|(x,y)|^3).
\end{aligned}$$

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Vi använde att

$$\mathcal{O}((x+y)^3) = b(x+y)(x+y)^3$$

$$= b(x+y)\frac{(x+y)^3}{|(x,y)|^3}|(x,y)|^3 = k(x,y)|(x,y)|^3,$$

och $k(x,y)$ är begränsad nära origo.