Exempel

Taylorutveckla

$$f(x, y) = \mathrm{e}^{x+2y}$$

kring $(1, 1)$ till ordning 2 med felterm på ordoform.

Lösning

$$(*) \qquad f(1+h,1+k) = f(1,1) + f'_x(1,1)h + f'_y(1,1)k +$$
$$+ \quad \frac{f''_{xx}(1,1)}{2}h^2 + f''_{xy}(1,1)hk + \frac{f''_{yy}(1,1)}{2}k^2 + \mathcal{O}(|(h,k)|^3).$$

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$$f''_{yy} = 4\mathrm{e}^{x+2y}, \quad f''_{yy}(1,1) = 4\mathrm{e}^3.$$

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Insatt i $(*)$:

$$f(1+h,1+k) = \mathrm{e}^3+\mathrm{e}^3 h+2\mathrm{e}^3 k+\frac{\mathrm{e}^3}{2}h^2+2\mathrm{e}^3 hk+2\mathrm{e}^3 k^2+\mathcal{O}(|(h,k)|^3),$$

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alternativt om vi föredrar att svara med x, y:

$$\begin{aligned} f(x,y) &= \mathrm{e}^3 + \mathrm{e}^3(x-1) + 2\mathrm{e}^3(y-1) + \\ &+ \frac{\mathrm{e}^3}{2}(x-1)^2 + 2\mathrm{e}^3(x-1)(y-1) + 2\mathrm{e}^3(y-1)^2 \\ &+ \mathcal{O}(|(x-1,y-1)|^3). \end{aligned}$$