Exempel

En yta i $\mathbb{R}^3$ parametriseras av

$$\begin{cases} x = s^2 + t^2, \\ y = st, \\ z = s^2 - t^2. \end{cases}$$

Bestäm tangentplanet genom den punkt som svarar mot $(s, t) = (1, 1)$.

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$$(1,-2,0) \bullet ((x,y,z) - (2,1,0)) = 0 \Leftrightarrow x - 2y = 0.$$