Exempel

Låt

$$\begin{cases} x = t^2 \mathrm{e}^{\psi} \\ y = t\psi. \end{cases}$$

Bestäm funktionalmatrisen $\dfrac{\partial(x,y)}{\partial(t,\psi)}$ och funktionaldeterminanten $\dfrac{d(x,y)}{d(t,\psi)}$.

Visa även att $(t,\psi) \mapsto (x,y)$ har en $\mathcal{C}^1$ invers i någon omgivning till $(t,\psi) = (1,1)$.

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Eftersom funktionerna är av klass $\mathcal{C}^1$ och

$$\frac{d(x,y)}{d(t,\psi)}(1,1) = 2 \cdot 1^2 \cdot \mathrm{e}^1 - 1^2 \cdot 1 \cdot \mathrm{e}^1 = 2\mathrm{e} - \mathrm{e} = \mathrm{e} \neq 0$$

ger inversa funktionssatsen att avbildningen är lokalt inverterbar.