Exempel

Beräkna

$$\iint_D (6x^2y + 2y)\, dxdy$$

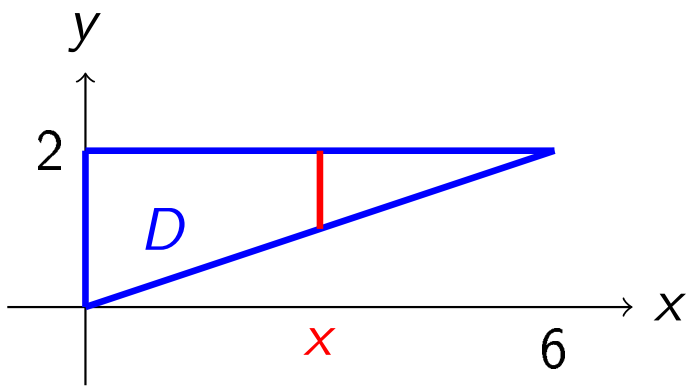
där $D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}$.

Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$

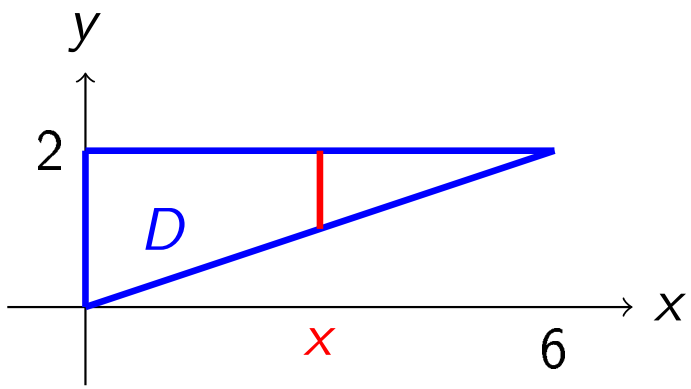
Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$



Lösning

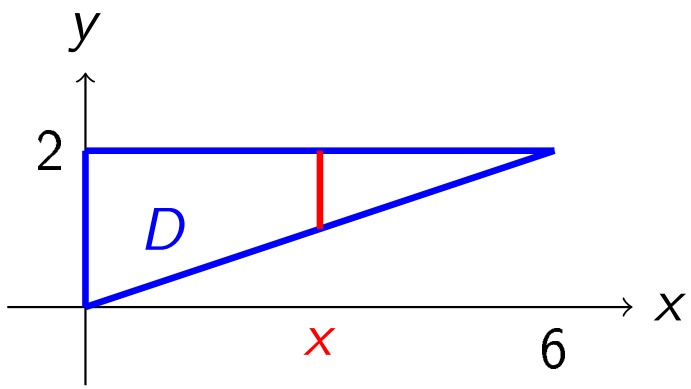
$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$



$$D = \{(x, y) : 0 \leq x \leq 6, \frac{x}{3} \leq y \leq 2\}.$$

Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$

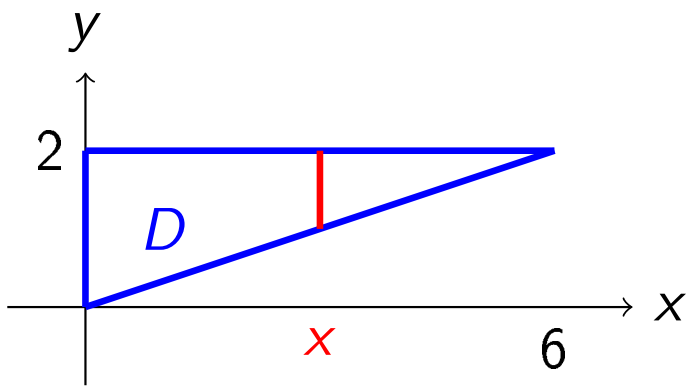


$$D = \{(x, y) : 0 \leq x \leq 6, \frac{x}{3} \leq y \leq 2\}.$$

$$\iint_D (6x^2y + 2y)dxdy = \int_0^6 \left(\int_{x/3}^2 (6x^2y + 2y)dy \right) dx$$

Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$



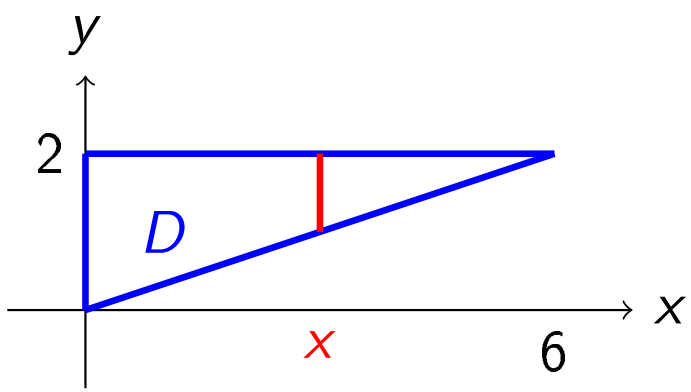
$$D = \{(x, y) : 0 \leq x \leq 6, \frac{x}{3} \leq y \leq 2\}.$$

$$\iint_D (6x^2y + 2y)dxdy = \int_0^6 \left(\int_{x/3}^2 (6x^2y + 2y)dy \right) dx =$$

$$\int_0^6 \left[3x^2y^2 + y^2 \right]_{y=x/3}^2 dx$$

Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$



$$D = \{(x, y) : 0 \leq x \leq 6, \frac{x}{3} \leq y \leq 2\}.$$

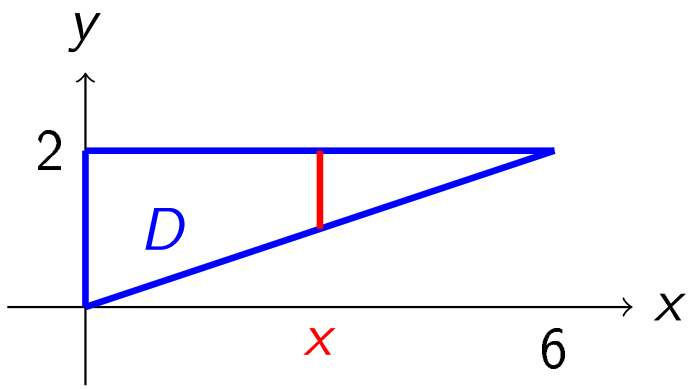
$$\iint_D (6x^2y + 2y)dxdy = \int_0^6 \left(\int_{x/3}^2 (6x^2y + 2y)dy \right) dx =$$

$$\int_0^6 \left[3x^2y^2 + y^2 \right]_{y=x/3}^2 dx =$$

$$\int_0^6 \left(12x^2 + 4 - \frac{x^4}{3} - \frac{x^2}{9} \right) dx$$

Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$



$$D = \{(x, y) : 0 \leq x \leq 6, \frac{x}{3} \leq y \leq 2\}.$$

$$\iint_D (6x^2y + 2y)dxdy = \int_0^6 \left(\int_{x/3}^2 (6x^2y + 2y)dy \right) dx =$$

$$\int_0^6 \left[3x^2y^2 + y^2 \right]_{y=x/3}^2 dx =$$

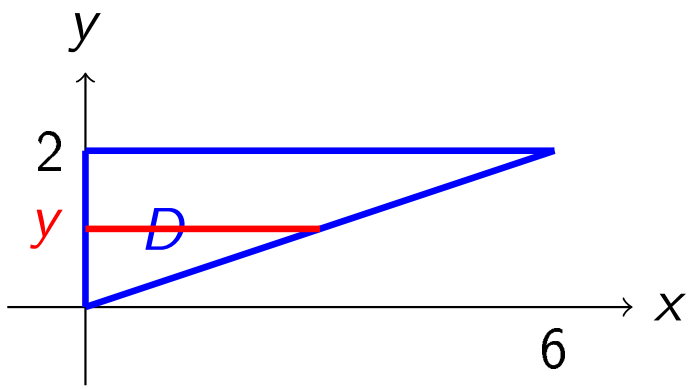
$$\int_0^6 \left(12x^2 + 4 - \frac{x^4}{3} - \frac{x^2}{9} \right) dx = \left[4x^3 + 4x - \frac{x^5}{15} - \frac{x^3}{27} \right]_0^6 = \frac{1808}{5}.$$

Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$

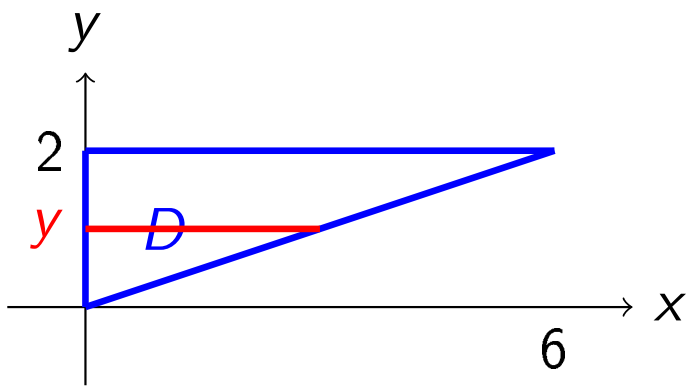
Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$



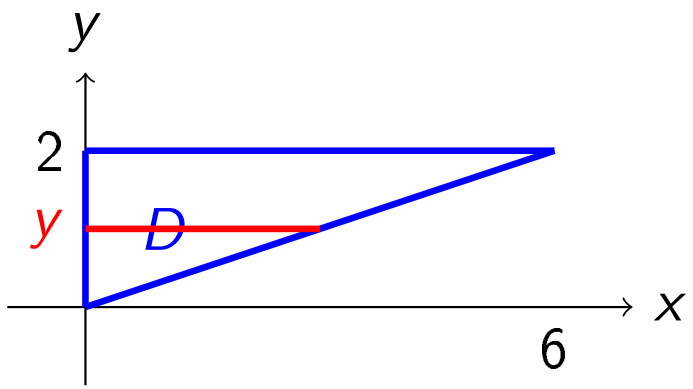
Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$



Lösning

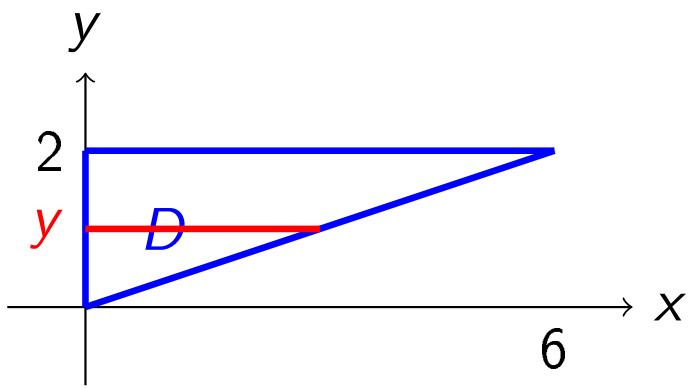
$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$



$$D = \{(x, y) : 0 \leq y \leq 2, 0 \leq x \leq 3y\}.$$

Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$

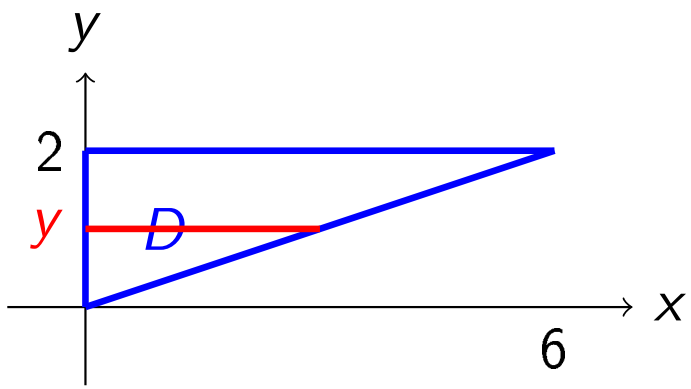


$$D = \{(x, y) : 0 \leq y \leq 2, 0 \leq x \leq 3y\}.$$

$$\iint_D (6x^2y + 2y)dxdy = \int_0^2 \left(\int_0^{3y} (6x^2y + 2y)dx \right) dy$$

Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$



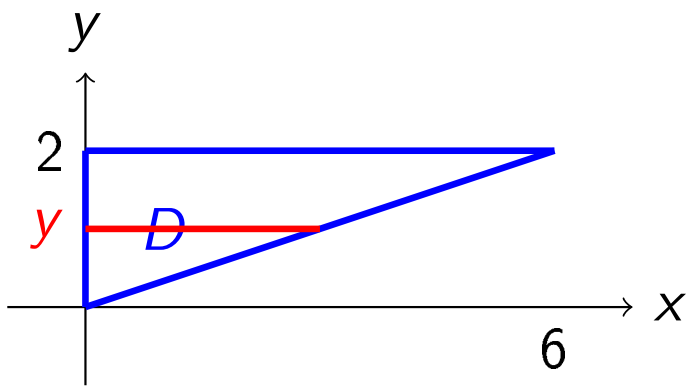
$$D = \{(x, y) : 0 \leq y \leq 2, 0 \leq x \leq 3y\}.$$

$$\iint_D (6x^2y + 2y)dxdy = \int_0^2 \left(\int_0^{3y} (6x^2y + 2y)dx \right) dy =$$

$$\int_0^2 \left[2x^3y + 2xy \right]_{x=0}^{3y} dy$$

Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$



$$D = \{(x, y) : 0 \leq y \leq 2, 0 \leq x \leq 3y\}.$$

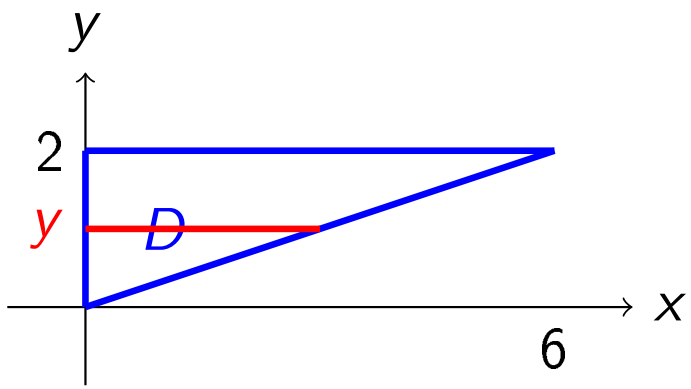
$$\iint_D (6x^2y + 2y)dxdy = \int_0^2 \left(\int_0^{3y} (6x^2y + 2y)dx \right) dy =$$

$$\int_0^2 \left[2x^3y + 2xy \right]_{x=0}^{3y} dy =$$

$$\int_0^2 \left(54y^4 + 6y^2 \right) dy$$

Lösning

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 3y \leq 6\}.$$



$$D = \{(x, y) : 0 \leq y \leq 2, 0 \leq x \leq 3y\}.$$

$$\iint_D (6x^2y + 2y)dxdy = \int_0^2 \left(\int_0^{3y} (6x^2y + 2y)dx \right) dy =$$

$$\int_0^2 \left[2x^3y + 2xy\right]_{x=0}^{3y} dy =$$

$$\int_0^2 \left(54y^4 + 6y^2\right) dy = \left[\frac{54y^5}{5} + 2y^3\right]_0^2 = \frac{1808}{5}.$$