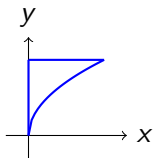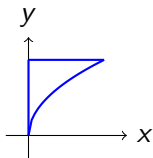Beräkna

$$\int_0^1 \left(\int_{\sqrt{x}}^1 \sqrt{1+y^3}\, dy \right) dx.$$

$$\{(x,y) : 0 \le x \le 1, \sqrt{x} \le y \le 1\} = \{(x,y) : 0 \le y \le 1, 0 \le x \le y^2\}.$$

$$\int_0^1 \left(\int_{\sqrt{x}}^1 \sqrt{1+y^3}\, dy \right) dx = \int_0^1 \left(\int_0^{y^2} \sqrt{1+y^3}\, dx \right) dy$$

$$\int_0^1 \left(\int_{\sqrt{x}}^1 \sqrt{1+y^3} \, dy \right) dx = \int_0^1 \left(\int_0^{y^2} \sqrt{1+y^3} \, dx \right) dy =$$

$$\int_0^1 \left[x \sqrt{1+y^3} \right]_{x=0}^{y^2} dy$$

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$$\int_0^1 \left[x\sqrt{1+y^3} \right]_{x=0}^{y^2} dy = \int_0^1 y^2 \sqrt{1+y^3}\,dy$$

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$$\int_0^1 \left[x\sqrt{1+y^3} \right]_{x=0}^{y^2} dy = \int_0^1 y^2 \sqrt{1+y^3}\,dy =$$

$$\left/ \begin{array}{l} t = y^3 \\ dt = 3y^2\,dy \\ 0 \mapsto 0,\ 1 \mapsto 1 \end{array} \right/$$

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$$\left/ \begin{array}{l} t = y^3 \\ dt = 3y^2\, dy \\ 0 \mapsto 0,\ 1 \mapsto 1 \end{array} \right/ = \int_0^1 \frac{1}{3}\sqrt{1+t}\, dt$$

$$\int_0^1 \left(\int_{\sqrt{x}}^1 \sqrt{1+y^3}\, dy \right) dx = \int_0^1 \left(\int_0^{y^2} \sqrt{1+y^3}\, dx \right) dy =$$

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$$\left[\frac{2}{9}(1+t)^{3/2} \right]_0^1 = \frac{4\sqrt{2}-2}{9}.$$