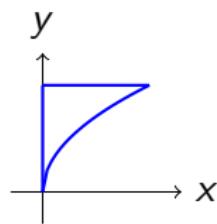
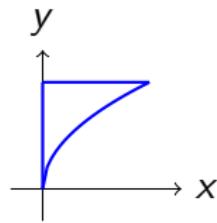


Exempel

Beräkna

$$\int_0^1 \left(\int_{\sqrt{x}}^1 \sqrt{1+y^3} dy \right) dx.$$





$$\{(x, y) : 0 \leq x \leq 1, \sqrt{x} \leq y \leq 1\} = \{(x, y) : 0 \leq y \leq 1, 0 \leq x \leq y^2\}.$$

$$\int_0^1 \left(\int_{\sqrt{x}}^1 \sqrt{1+y^3} dy \right) dx = \int_0^1 \left(\int_0^{y^2} \sqrt{1+y^3} dx \right) dy$$

Lösning

$$\int_0^1 \left(\int_{\sqrt{x}}^1 \sqrt{1+y^3} dy \right) dx = \int_0^1 \left(\int_0^{y^2} \sqrt{1+y^3} dx \right) dy =$$
$$\int_0^1 \left[x \sqrt{1+y^3} \right]_{x=0}^{y^2} dy$$

$$\int_0^1 \left(\int_{\sqrt{x}}^1 \sqrt{1+y^3} dy \right) dx = \int_0^1 \left(\int_0^{y^2} \sqrt{1+y^3} dx \right) dy =$$
$$\int_0^1 \left[x \sqrt{1+y^3} \right]_{x=0}^{y^2} dy = \int_0^1 y^2 \sqrt{1+y^3} dy$$

$$\int_0^1 \left(\int_{\sqrt{x}}^1 \sqrt{1+y^3} dy \right) dx = \int_0^1 \left(\int_0^{y^2} \sqrt{1+y^3} dx \right) dy =$$
$$\int_0^1 \left[x\sqrt{1+y^3} \right]_{x=0}^{y^2} dy = \int_0^1 y^2 \sqrt{1+y^3} dy =$$
$$\begin{cases} t = y^3 \\ dt = 3y^2 dy \\ 0 \mapsto 0, 1 \mapsto 1 \end{cases}$$

$$\int_0^1 \left(\int_{\sqrt{x}}^1 \sqrt{1+y^3} dy \right) dx = \int_0^1 \left(\int_0^{y^2} \sqrt{1+y^3} dx \right) dy =$$
$$\int_0^1 \left[x\sqrt{1+y^3} \right]_{x=0}^{y^2} dy = \int_0^1 y^2 \sqrt{1+y^3} dy =$$
$$\begin{cases} t = y^3 \\ dt = 3y^2 dy \\ 0 \mapsto 0, 1 \mapsto 1 \end{cases} = \int_0^1 \frac{1}{3} \sqrt{1+t} dt$$

$$\begin{aligned} \int_0^1 \left(\int_{\sqrt{x}}^1 \sqrt{1+y^3} dy \right) dx &= \int_0^1 \left(\int_0^{y^2} \sqrt{1+y^3} dx \right) dy = \\ \int_0^1 \left[x\sqrt{1+y^3} \right]_{x=0}^{y^2} dy &= \int_0^1 y^2 \sqrt{1+y^3} dy = \\ \left/ \begin{array}{l} t = y^3 \\ dt = 3y^2 dy \\ 0 \mapsto 0, 1 \mapsto 1 \end{array} \right/ &= \int_0^1 \frac{1}{3} \sqrt{1+t} dt = \\ \left[\frac{2}{9}(1+t)^{3/2} \right]_0^1 &= \frac{4\sqrt{2}-2}{9}. \end{aligned}$$