TATM38: Mathematical Models in Biology

List of projects 2024

The projects are carried out in groups of 3 - 4 students. Each group must produce a written report on a chosen topic. More than one group may choose the same project. The deadline for handing in the report is 14/10. It must be clearly stated which student has written which part of the report. The project must also be presented at a 30-minute seminar, including 5 minutes for discussions, using a computer for the presentation. A 1-2 page summary of the project should then be provided to the other students. Any use of AI tools in any part of the project must be discussed with and approved by the examiner, and accounted for.

EK refers to the course book Edelstein-Keshet: *Mathematical Models in Biology*, other necessary material will be handed out. Britton refers to the book N.F.Britton: *Essential Mathematical Biology* (Springer).

Project 1: Population genetics with selection pressure

Here you assume that fitness depends on genotype. Study sections 4.3 and 4.7 of Britton. Explain the Fisher-Haldane-Wright equation. Solve problem 4.9 for a model of sickle-cell anaemia and protection from malaria. Analyze stability and draw cob-web diagrams for some initial values, the data from 4.9c may be used for this. Find the overall mean fitness for this model and analyze its maximum (problem 4.15), plot Wright's adaptive topography.

Project 2: Ventilation and blood CO_2 levels

Here you analyze both time discrete and time continuous models for ventilation volume and blood CO_2 levels. In EK, read pages 27-28 in section 1.9 and solve problems 1.18 a-c and 2.17 a-d (section 2.10 may also be interesting), when breathing takes place at constant intervals. Explain all equations and analyze solutions and stability. Then solve problem 5.22 a-c for time continuous models, modeling breathing over a longer time scales, explain and analyze the equations, stability and phase planes. Make some simulations (discrete and continuous time) and plot V and C as functions of time.

Project 3: Plankton ecosystem

Formulate and explain the model of phytoplankton, zooplankton and nitrogen in the ocean in section 2.6 of Britton. Write down a 2-dimensional dynamical system for P and Z. Analyze steady states for different c -values, and perform a detailed phase plane analysis (draw null clines) for the three cases. Plot (P,Z), (P,t) and (Z,t) diagrams. Try to find a time-varying $c(t)$ that produces a graph similar to figure 2.12 in Britton.

Project 4: Models for neural impulses

Formulate the Fitzhugh-Nagumo model for neural excitations (EK section 8.5, also Britton 6.4), and describe its relation to the Hodgkin-Huxley model. Analyze stability of the steady state (exercise 8.9 in EK and 6.7 in Britton). Analyze the system when a stimulation is introduced, what happens to the stationary state and trajectories? More detailed: Choose some values, e.g., $a = b = 0.75$, $c = 3$ [if several groups choose this project they will get different values of a, b, c]. Find (numerically) the steady state $(\bar{x}, \bar{y})$. Increase the impulse i_0 step by step, in each step analyze the new steady state, the Jacobian matrix and its eigenvalues, the phase plane (what happens to the trajectory starting in the original $(\bar{x}, \bar{y})$, which may be the state before the stimulation is introduced), plot both the phase plane and the curves x(t) and y(t) for some different initial values x(0), y(0) (including $(\bar{x}, \bar{y})$). For what value of i_0 occurs a drastic change in the solution?