Metaheuristics for Multiobjective Optimization

Ch 4.1 – 4.2

Mina Niknafs

4.2

MULTIOBJECTIVE OPTIMIZATION CONCEPTS

Multiobjective optimization problems (MOP)

- There are many conflicting objectives to handle
- Pareto optimal solution
  - It is not possible to improve a given objective without deteriorating at least another objective
- Mostly focused on P-metaheuristics

MOP

$$\text{MOP} = \begin{cases} \min F(x) = (f_1(x), f_2(x), \ldots, f_n(x)) \\ \text{s.c. } x \in S \end{cases}$$

- $x = (x_1, \ldots, x_k)$: vector representing the decision variables
- S: the set of feasible solutions
- F(x): the vector of objectives to be optimized

Decision space and objective space

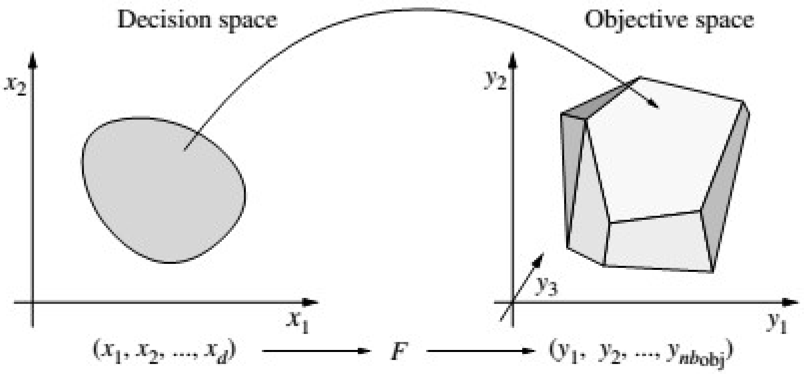
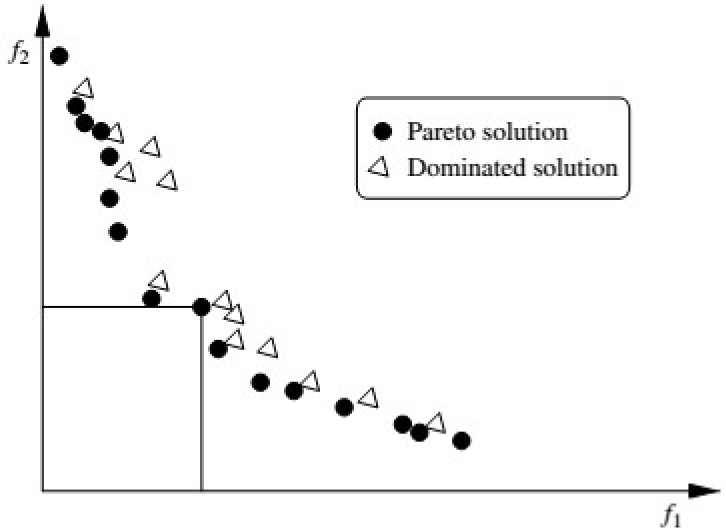


FIGURE 4.1 Decision space and objective space in a MOP.

Definitions

Definition 4.2 Pareto dominance. *An objective vector $u = (u_1, \ldots, u_n)$ is said to dominate $v = (v_1, \ldots, v_n)$ (denoted by $u \prec v$) if and only if no component of v is smaller than the corresponding component of u and at least one component of u is strictly smaller, that is,*

$$\forall i \in \{1, \ldots, n\} : \; u_i \leq v_i \;\; \wedge \;\; \exists i \in \{1, \ldots, n\} : \; u_i < v_i$$



Definitions

Definition 4.3 Pareto optimality. *A solution $x^* \in S$ is Pareto optimal*[4] *if for every $x \in S$, $F(x)$ does not dominate $F(x^*)$, that is, $F(x) \nprec F(x^*)$.*

Definition 4.4 Pareto optimal set. *For a given MOP (F, S), the Pareto optimal set is defined as $\mathcal{P}^* = \{x \in S / \nexists x' \in S, F(x') \prec F(x)\}$.*

Pareto front

Definition 4.5 Pareto front. *For a given MOP (F, S) and its Pareto optimal set $\mathcal{P}^*$, the Pareto front is defined as $\mathcal{PF}^* = \{F(x), x \in \mathcal{P}^*\}$.*

- The image of the Pareto optimal set in the objective space
- The main goal of MOP

Desired Pareto front

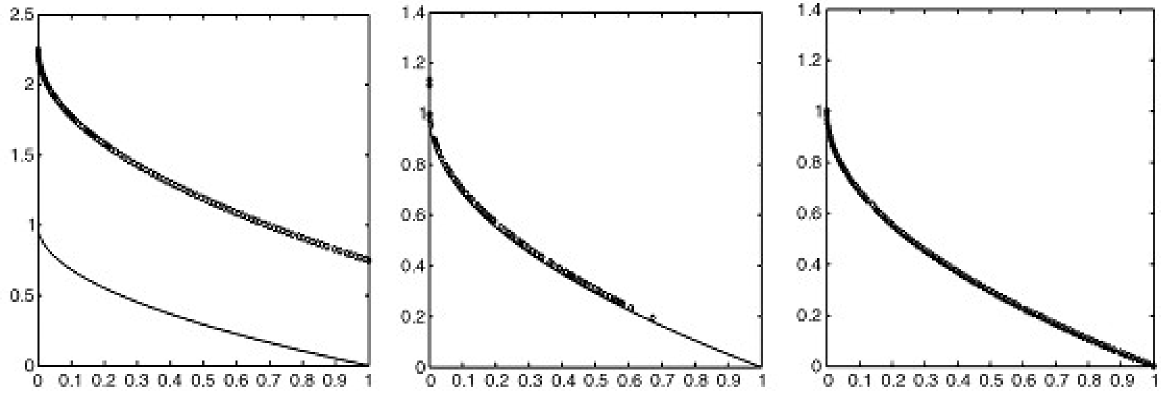


FIGURE 4.3 Examples of Pareto fronts: bad convergence and good diversity (left), good convergence and bad diversity (center), good convergence and diversity (right).

Definitions

Definition 4.6 Ideal vector. *A point* $y^* = (y_1^*, y_2^*, \ldots, y_n^*)$ *is an ideal vector if it minimizes each objective function* f_i *in* $F(x)$, *that is,* $y_i^* = \min(f_i(x)), x \in S, i \in [1, n]$.

Definition 4.7 Reference point. *A reference point* $z^* = [\overline{z}_1, \overline{z}_2, \ldots, \overline{z}_n]$ *is a vector that defines the aspiration level (or goal)* $\overline{z}_i$ *to reach for each objective* f_i.

Definition 4.8 Nadir point. *A point* $y^* = (y_1^*, y_2^*, \ldots, y_n^*)$ *is the nadir point if it maximizes each objective function* f_i *of* F *over the Pareto set, that is,* $y_i^* = \max(f_i(x)), \quad x \in \mathcal{P}^*, i \in [1, n]$.

The ideal and nadir points give some information on the ranges of the Pareto optimal front

Definitions

Definition 4.9 Utility function. *A utility (or value) function v, which represents the preferences of the decision maker, maps the objective vector to a scalar-valued function:* $v : \mathcal{R}^n \longrightarrow \mathcal{R}$.

Definition 4.10 Locally Pareto optimal solution. *A solution x is locally Pareto optimal if and only if $\forall w \in N(x)$, $F(w)$ does not dominate $F(x)$, and $N(x)$ represents the neighborhood of the solution x.*

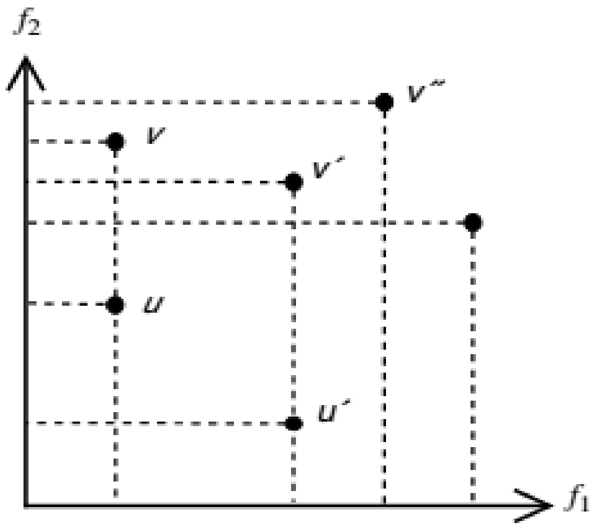
$$(\mathrm{MOP}_\lambda) \begin{cases} \min F(x) = \sum_{i=1}^{n} \lambda_i f_i(x) \\ \text{s.c. } x \in S \end{cases}$$

Definitions

Definition 4.11 Weak dominance. *An objective vector $u = (u_1, \dots, u_n)$ is said to weakly dominate $v = (v_1, \dots, v_n)$ (denoted by $u \preccurlyeq v$) if all components of u are smaller than or equal to the corresponding components of v, that is, $\forall i \in \{1, \dots, n\},\ u_i \leq v_i$ (Fig. 4.5).*

Definition 4.12 Strict dominance. *An objective vector $u = (u_1, \dots, u_n)$ is said to strictly dominate $v = (v_1, \dots, v_n)$ (denoted by $u \prec\prec v$) if all components of u are smaller than the corresponding components of v, that is, $\forall i \in \{1, \dots, n\},\ u_i < v_i$*

Definition 4.13 ϵ-Dominance. *An objective vector $u = (u_1, \dots, u_n)$ is said to ϵ-dominate $v = (v_1, \dots, v_n)$ (denoted by $u \prec_\epsilon v$) if and only if no component of v is smaller than the corresponding component of $u - \epsilon$ and at least one component of $u - \epsilon$ is strictly better, that is, $\forall i \in \{1, \dots, n\} :\ u_i - \epsilon_i \leq v_i\ \wedge\ \exists i \in \{1, \dots, n\} :\ u_i - \epsilon_i < v_i$ (Fig. 4.6).*



Weak dominance and strict dominance concepts. Solution u weakly dominate solution v; solution u´ weakly dominates solution v´; solution u strictly dominates solutions v´ and v˝.

4.2

MULTIOBJECTIVE OPTIMIZATION PROBLEM

Categories and applications

- Two categories:
  - continuous
  - discrete
- Academic applications
- Real-Life applications

Academic applications

- Multiobjective Continuous Problems

$$\min F(x) = (f_1(x), f_2(x), \ldots, f_n(x))$$

subject to

$$g_i(x) \leq 0, \quad i = 1, \ldots, m$$

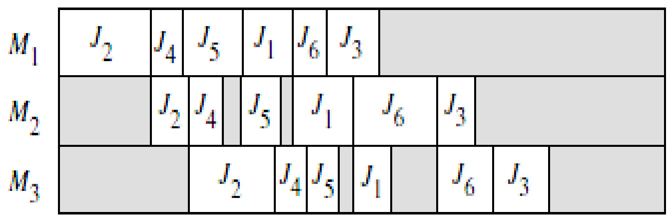
$$h_i(x) = 0, \ i = 1, \ldots, p$$

- Well-known test problems (standard benchmarks):
  - ZDT
  - DTLZ

Academic applications

- Multiobjective Combinatorial Problems
  - lack of “standard” benchmarks
  - Two complexity categories:
    - **Polynomial problems:** shortest path, spanning tree problems, assignment problems
    - **NP-hard problems:** scheduling problems, routing problems

Multiobjective scheduling problems



$C_{\max}$	Makespan (total completion time): $\max\{C_i \mid i \in [1 \ldots n]\}$
$\bar{C}$:	Mean value of jobs completion time
$T_{\max}$:	Maximum tardiness: $\max\{[\max(0, C_i - d_i)] \mid i \in [1 \ldots n]\}$
T:	Total tardiness: $\sum_{i=1}^{n} [\max(0, C_i - d_i)]$
U:	Number of jobs delayed with regard to their due date d_i
$F_{\max}$:	Maximum job flow-time: $\max\{C_i - r_i \mid i \in [1 \ldots n]\}$
$\bar{F}$:	Mean job flow-time

Real-life applications

- Engineering design
- Environment and energetics
- Telecommunications
- Control
- Computational biology and bioinformatics
- Transportation and logistics

Multicriteria decision making

- There is a need for interaction between the decision maker and the problem solver:
  - A priori
  - A posteriori
  - Intercative

?