Metaheuristics

4.3 Main design issues of multiobjective metaheuristics
4.4 Fitness assignment strategies

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4.3 Main design issues of multiobjective metaheuristics

4.3 Issues

- Algorithms for solving MOPs
  - Exact
    - Useful for small problem sizes
  - Approximate
    - Needed if we have more than two criteria or large scale
    - Design and solve with:
      - Concepts from monoobjective metaheuristics
      - **Fitness assignment**
      - Diversity preserving
      - Elitism

Optimization algorithms

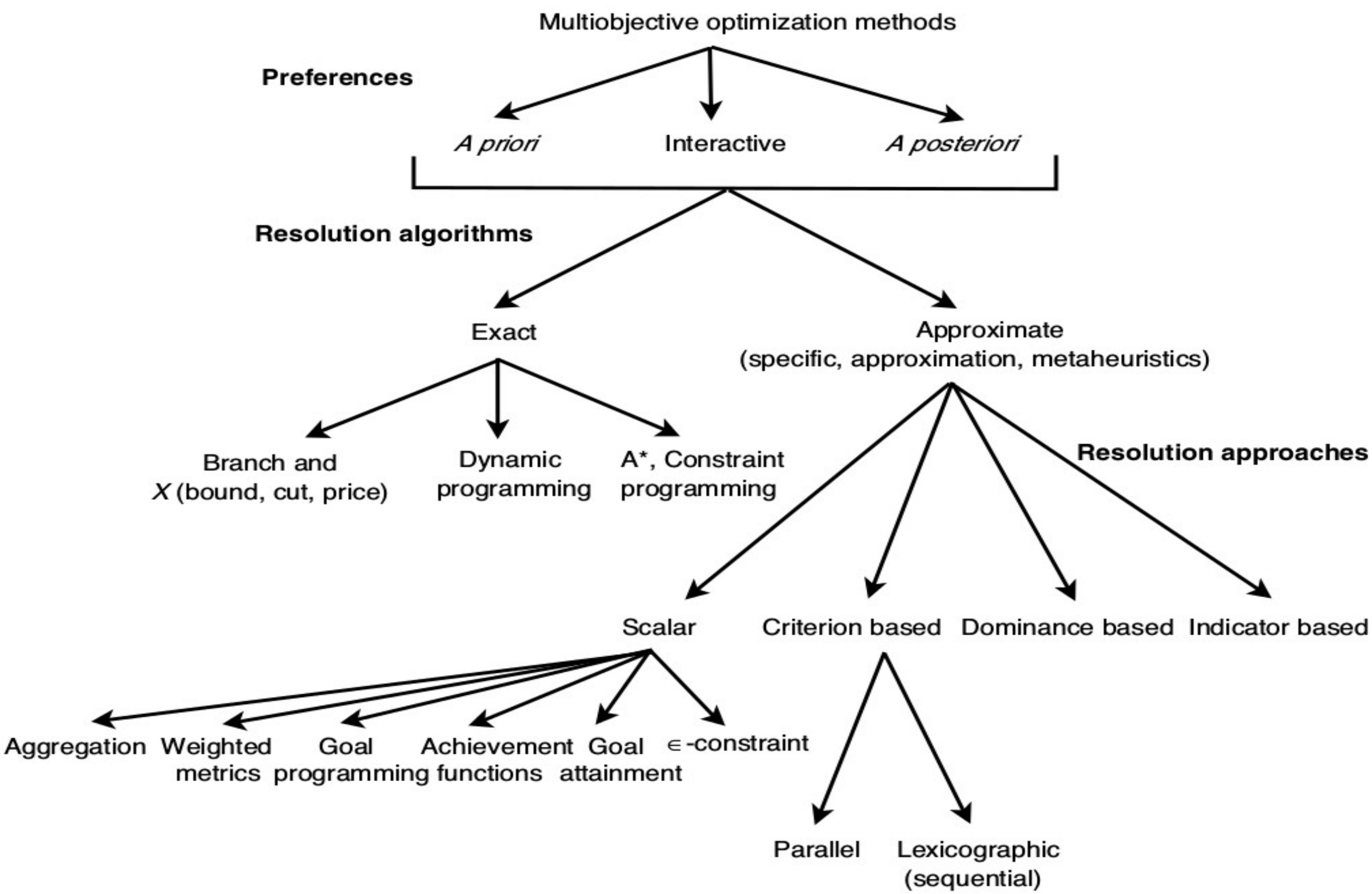

FIGURE 4.9 Classification of multiobjective optimization algorithms.

Optimization algorithms

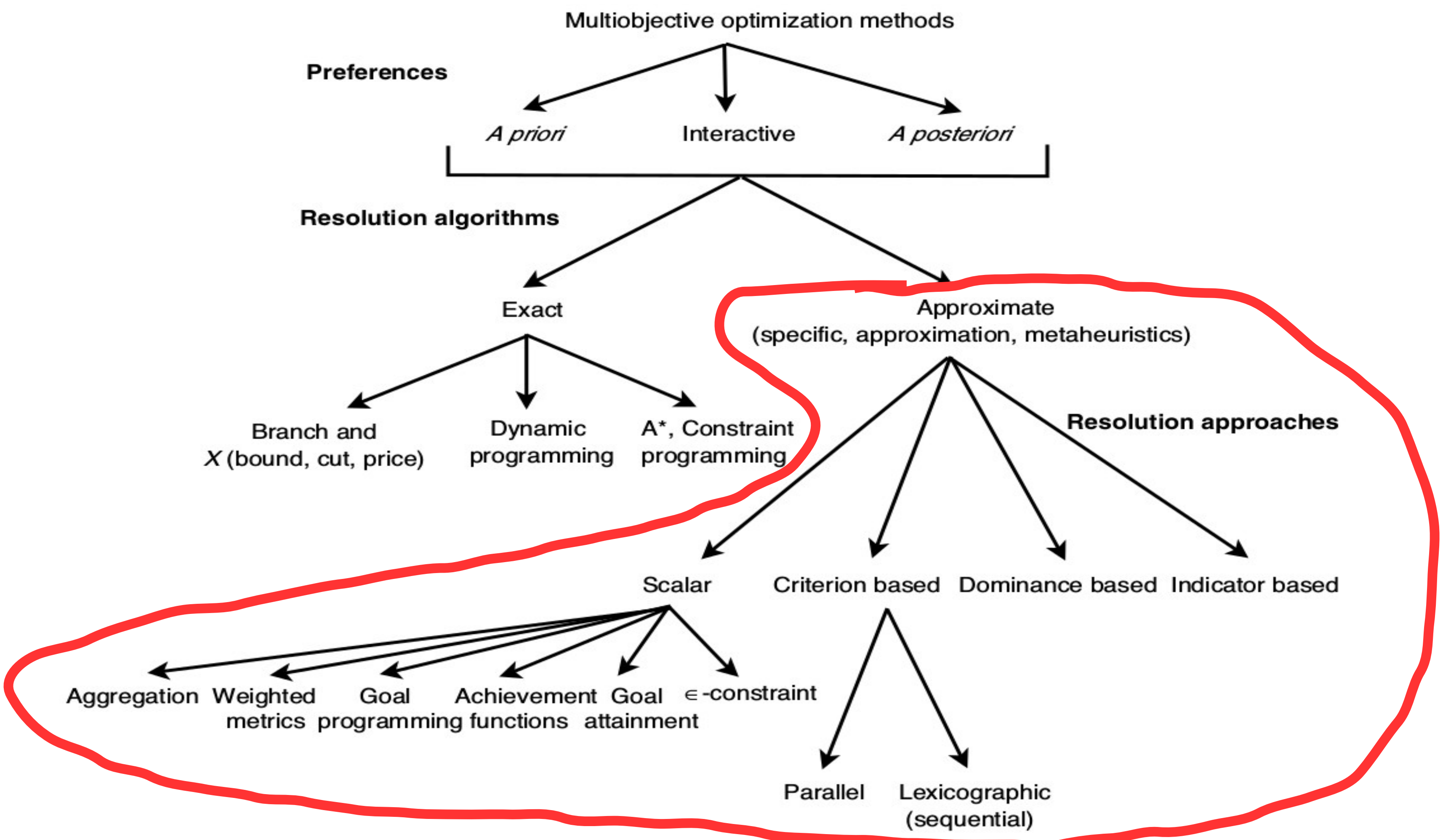

FIGURE 4.9 Classification of multiobjective optimization algorithms.

4.4 Fitness assignment strategies

Classification

- Scalar approaches
- Criterion-based approaches
- Dominance-based approaches
- Indicator-based approaches

Scalar approaches

- Transform MOP problem into monobjective one
- Many methods
  - Aggregation method
  - Weighted metrics
  - Goal programming
  - Achievement functions
  - Goal attainment
  - ϵ-constraint

Aggregation method

- Aggregation function to transform into monoobjective function
- $$f(x) = \sum_{i=1}^{n} \lambda_i f_i(x), \qquad x \in S$$
- Selection of weights λ
  - A priori single weight
  - A priori multiple weights
  - Dynamic multiple weights
  - Adaptive multiple weights
- Not working with nonconvex Pareto borders

Weighted metrics

- Define reference point ***z*** to attain → minimize distance between solution and ***z***
- Lp-metric
  - $1 \leq p \leq \infty$
  - $$(\mathrm{MOP}(\lambda, z)) \begin{cases} \min \left(\sum_{j=1}^{n} \lambda_j |f_j(x) - z_j|^p \right)^{\frac{1}{p}} \\ \text{s.c. } x \in S \end{cases}$$

Goal programming

- Decision maker defines aspiration levels for each objective function → minimize the deviations associated with the objective functions
- Goals are easy to define by decision maker
- $$(\mathrm{MOP}(\bar{z})) \begin{cases} \min\left(\sum_{j=1}^{n} \lambda_j \delta_j\right) \\ \text{s.c. } f_j(x) - \delta_j \leq \bar{z}_j, \quad j \in [1, n] \\ \delta_j \geq 0, \quad j \in [1, n] \\ x \in S \end{cases}$$

Achievement functions

- No need to choose reference point carefully

$$w_j = \frac{1}{z_i^{nadir} - z_i^{ideal}}$$

$$(\mathrm{MOP}(\lambda, z)) \begin{cases} \min \max_{j \in [1,n]} [w_j (f_j(x) - \bar{z}_j)] + \rho \sum_{j=1}^{n} (f_j(x) - \bar{z}_j) \\ \text{s.c. } x \in S \end{cases}$$

Goal attainment

- Define the weight vector and the goals
- Find the best compromise solution
- $$\begin{cases} \min \alpha \\ \text{s.c. } x \in S \\ f_i(x) \leq z_i^* + \alpha\lambda_i, \quad i = 1, \ldots, n \\ \sum_{i=1}^{n} \lambda_i = 1 \end{cases}$$

ϵ-constraint

- Optimize one objective function (k) to constraint the rest
- $$(\mathrm{MOP}_k(\epsilon)) \begin{cases} \min \ f_k(x) \\ x \in S \\ \text{s.c. } f_j(x) \leq \epsilon_j, \quad j = 1, \ldots, n, \ j \neq k \end{cases}$$

About scalar methods

- You need a priori knowledge of the problem
- Low computational cost
- Pareto optimality is guaranteed but finds only one solution
- Sensitive to convexity, discontinuity, etc.

Criterion based methods

- Mainly based on P-metaheuristics
- Parallel approach
  - All objectives are handled in parallel
  - Ex.: split populations and use different objective function for each subgroup (VEGA alg.)
- Sequential or Lexicographic approach
  - Order the objective functions by priority
  - Solve one at the time

Dominance based approaches

- Dominance in the fitness assignment
- Ranking methods
  - Dominance rank
    - Rank – number of solutions in the population that dominate the considered solution
  - Dominance depth
    - Compose solution fronts starting from the nondominating ones
  - Dominance count
    - Number of solutions dominated by the solution
  - Other: guided dominance, fuzzy dominance, cone dominance

Dominance based approaches - continued

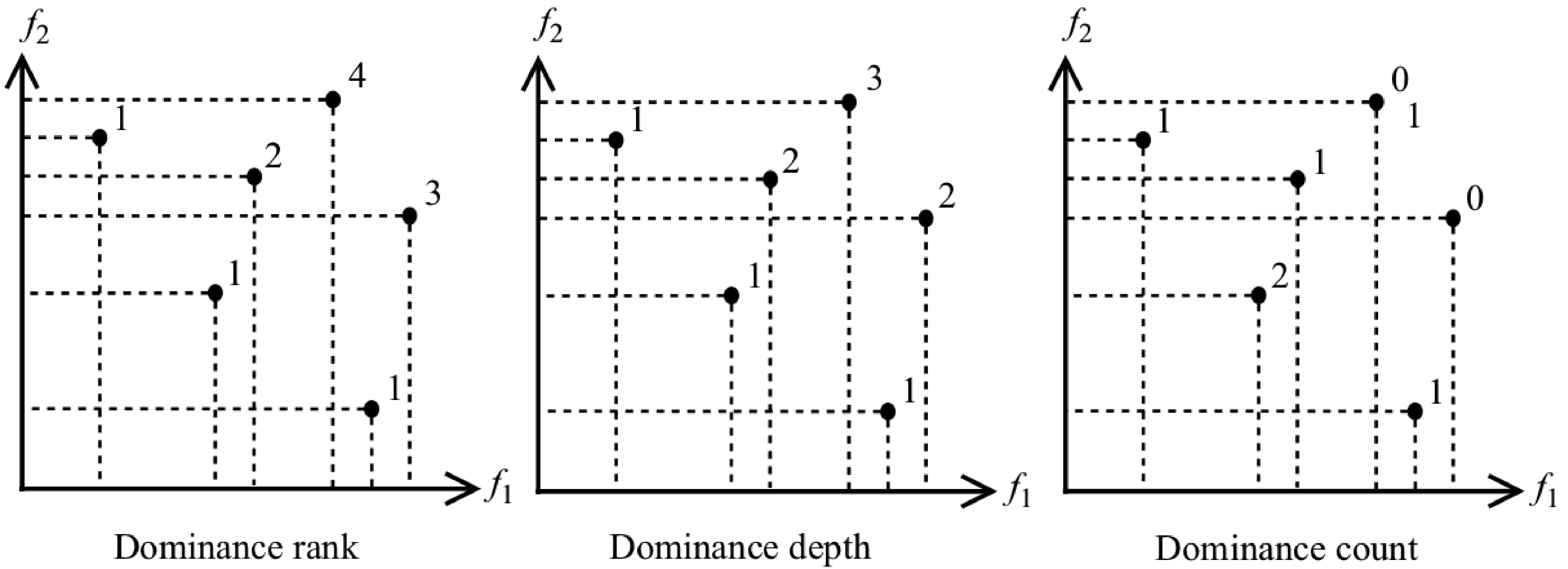

FIGURE 4.16 Fitness assignment: some dominance-based ranking methods.

Indicator based approaches

- Search is guided by performance quality indicator
- Optimization goal given by binary indicator “I”
- I(A, B) → difference in quality between two sets
- R → reference set
- Ω → space of all efficient set approximations
- Optimization goal: $\arg\min_{A \in \Omega} I(A, R)$

Indicator based approaches - advantages

- The decision maker preference may be easily incorporated into the optimization algorithm
- No diversity maintenance; it is implicitly taken into account in the performance indicator definition.
- Small sensitivity of the landscape associated with the Pareto front
- Only few parameters are defined in the algorithm

The end :)