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Heuristic Search Methodologies

Report on the implementation
of a heuristic algorithm

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1 Introduction

In this report, a metaheuristic algorithm has been implemented in order to solve a number of instances. The instances are standard vehicle routing problems (VRPs), i.e. fixed demand, a single depot, homogeneous fleet and vehicle capacity. The objective is to minimize total travel distance while satisfying the demand.

The metaheuristic algorithm have some stochastic elements and in order to find some approximate performance on average, each instance has been solved ten times by the metaheuristic algorithm.

For comparison reason, each instance has been solved, either to optimality or estimate a lower bound of the objective function using a standard VRP formulation and constraint generation. The generated constraints are so called sub–tour elimination constraints, there are 2^n such constraints. The computational time for the optimal solutions are of the magnitude days – weeks. Two of the instances was terminated after a month runtime.

2 Metaheuristic algorithm

The meta heuristic algorithm used in this report is based on a set partitioning formulation (Section 2.1) of the VRP, where the decision making revolves which routes to use (and which not to).

There are exponentially many routes, and generating all is not reasonable. In this report, two heuristic methods have been used to generate a subset of routes, and a restricted problem of the master problem is solved. The two algorithms used are *Clarke and Wright* (CW) (Section 2.2) and *Ant Colony Optimization* (ACO) (Section 2.3).

2.1 Master problem

The master problem is a set partitioning problem (SPP) and is formulated as

$$\begin{array}{llllll} \text{(SPP)} & \min & \sum\limits_{r \in R} c_r x_r & & & \\ & \text{s.t.} & \sum\limits_{r \in R} a_{ir} x_r & \geqslant & 1 & \forall i \in I \\ & & x_r & & \text{binary} & \forall r \in R \end{array}$$

where R is the set of all routes, I is the set of customers, c_r is the cost (travel distance) of route r, a_{ir} is 1 if customer i is in route r (0 otherwise) and x_r is the binary variable indicating if route r is used (=1) or not (=0). The constraint set ($\forall i \in I$) states that each customer needs to be visited in at least one used route.

The restricted problem is the SPP but instead of R only a subset, R_S, of routes are used.

2.2 Clarke and Wright

The heuristic method CW evaluates the incremental cost $\bar{c_{ij}} = c_{ij} - c_{dj} - c_{id}$ for all pairs of customers i and j and depot d, which express the incremental cost for merging two existing routes where the two customers are the last and first customers, respectively, in two distinct routes. It is equivalent to adding edge (i, j) and removing edge (d, j) and edge (i, d).

The initial solution consist of singleton routes, i.e. $d - i - d$, for all customers i. The list of incremental costs is sorted in a non decreasing order. The routes are merged in the order given by the sorted list and with respect to vehicle capacity, if i and j are last and first in two distinct routes and if $\bar{c_{ij}} < 0$.

Exclusive for the metaheuristic algorithm used in this report, each time two routes merges, the new route is added to R_S.

2.3 Ant Colony Optimization

In ACO, ants (construction algorithms) are used to construct solutions. During the walk (construction phase), they are influenced by pheromones (probability) which affects the walk. The pheromones are updated over time in two ways. The whole system evaporates (decreases proportionally to the probability value) and the ants are reinforcing the parts of the system they have visited (increasing based on the quality of the solution found).

For the VRP, the ants can be seen as the modelling of trucks driving in the network constructing the routes. However, each ant may construct more than one route, i.e. it constructs a complete solution to the VRP.

The ACO used in this report is based on the algorithm 3.13 in Talbi (2009)[1] which is the ACO for TSP.

Algorithm 3.13 Ant colony algorithm for the TSP problem (ACO–TSP).

Initialize the pheromone information ;
Repeat
  For each ant **Do**
    Solution construction using the pheromone trails:
      $S = \{1, 2, \ldots, n\}$ /* Set of potentially selected cities */
      Random selection of the initial city i ;
      Repeat
        Select new city j with probability $p_{ij} = \frac{\tau_{ij}^{\alpha} \times \eta_{ij}^{\beta}}{\sum_{k \in S} \tau_{ik}^{\alpha} \times \eta_{ik}^{\beta}}$;
        $S = S - \{j\}$; $i = j$;
      Until $S = \emptyset$
  End For
  Update the pheromone trail:
    For $i, j \in [1, n]$ **Do**
      $\tau_{ij} = (1 - \rho)\tau_{ij}$ /* Evaporation */ ;
    For $i \in [1, n]$ **Do**
      $\tau_{i\pi(i)} = \tau_{i\pi(i)} + \Delta$ /* π: best found solution */ ;
Until Stopping criteria
Output: Best solution found or a set of solutions.

For the metaheuristic algorithm used in this report, the initial pheromones are $\tau_{ij} = 1$ and $\eta_{ij} = \frac{1}{d_{ij}+1}$ where d_{ij} is the distance between vertex (depot or customer) i and vertex j. The addition of 1 is to avoid a 0 in the denominator, in case the distance is 0. The stopping criteria is set to a number of exploration phases (repeat until stopping criteria).

In the city selection stage, if the vehicle capacity is violated by including a selected city, then the ant is instead forced to go back to the depot.

Only a subset of the ants are allowed to reinforce the pheromones. Each edge visited by the best ants (whom found the solutions with least objective function values) are reinforced. The value $\Delta = \frac{1}{f(x_k)}$ where $f(x_k)$ is the objective function value (total distance) of the solution found by ant k. The best ants are also updating R_S with the routes (part of solutions) constructed, as well as certain parts of their routes. Parts of routes are routes constructed by the m first customers, e.g. if the best ant finds route $0-3-4-5-0$, it will update R_S with the routes $0-3-0$, $0-3-4-0$ and $0-3-4-5-0$.

[1] El-Ghazali Talbi, *Metaheuristics: From Design to Implementation*, Wiley Publishing.

3 Instances

The instances originates from a pilot case in Linköping where three freight forwarders did cooperate regarding the distribution of goods to the central area of the city. Data regarding the deliveries (weight, owner, receiver etc.) were collected during the pilot. The instances used in this report are synthesised from the data collected. Here, all three freight forwarders are seen as one stakeholder. All deliveries during a certain day is the total demand of the instances. Each delivery address is represented by a (customer) vertex in the network. In reality, several distinct customers have the same delivery address, e.g. if they are a part of a mall. But for comparison reason, in order to find an optimal solution, the demand is aggregated to delivery address for the synthesized instances. There are 65 distinct customers and 34 distinct delivery addresses. There are three depots, one for each freight forwarder, i.e. there are three instances, one for each depot. It is not possible to do any relevant comparisons between the results of the different instances. One of the depots is located much closer to the (customer) vertices, compared to the other depots. Which results in an unfair advantage relative to the vehicle routing problem. The distance matrix is evaluated using Google API with a resolution of 1 meter, i.e. the distances are based on Google's road network data which include one–way direction roads. Thus, the distance matrix is asymmetric.

4 Results

The results of the heuristic algorithm are based on the following parameter values.

Parameter	Value	Comment
α	1	Ant Colony Pheromone exponent
β	2	Distance Pheromone exponent
$(1-\rho)$	0.95	Evaporation
nr_ants	30	Number of ants each exploration phase
nr_best	3	Number of the ants' solutions saved each exploration phase
nr_iter	1000	Number of exploration phases

Table 1: Parameter values for the ant colony heuristic used.

In average, the results from the heuristic algorithm is the following.

	Routes	Time to		Distance		
Depot	generated	generate	solve SPP	Heuristic	Optimal	Gap
0	21315.7	34.74s	1.53s	24677.5	24443.0	1.0%
1	20908.6	35.01s	1.50s	30274.5	29953.0	1.1%
2	19079.3	32.80s	451.4ms	15801.6	15067.0	4.9%

Table 2: The average results for 10 runs. Red color values are lower bounds, the objective function values are based on infeasible solutions.

And the best solution found by the heuristic algorithm for each instance are

```
        Best heuristic solution when 0 is the depot
Route 1: 0 - 14 - 0, cost 6321.0
Route 2: 0 - 6 - 16 - 3 - 4 - 30 - 34 - 13 - 15 - 35 - 25 - 5 - 24 - 8 - 20 - 32 - 31 - 0, cost 7889.0
Route 3: 0 - 21 - 22 - 23 - 36 - 24 - 5 - 30 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 19 - 26 - 12 - 28 - 27 - 9 - 0, cost 10337.0
Total cost: 24547.0
Gap to optimum: 0.425%
```

```
        Best heuristic solution when 1 is the depot
Route 1: 1 - 14 - 1, cost 8187.0
Route 2: 1 - 21 - 22 - 23 - 36 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 16 - 3 - 4 - 19 - 26 - 12 - 28 - 27 - 9 - 1, cost 12525.0
Route 3: 1 - 6 - 30 - 25 - 13 - 35 - 15 - 34 - 5 - 24 - 8 - 20 - 32 - 31 - 1, cost 9393.0
Total cost: 30105.0
Gap to optimum: 0.507%

        Best heuristic solution when 2 is the depot
Route 1: 2 - 27 - 14 - 2, cost 3642.0
Route 2: 2 - 21 - 22 - 23 - 36 - 8 - 24 - 5 - 13 - 25 - 35 - 15 - 34 - 30 - 17 - 29 - 10 - 18 - 11 - 2, cost 5088.0
Route 3: 2 - 26 - 6 - 16 - 3 - 4 - 33 - 7 - 19 - 28 - 12 - 27 - 9 - 32 - 31 - 20 - 2, cost 6884.0
Total cost: 15614.0
Gap to optimum: 3.630%
```

compared to the optimal solutions

```
        Optimal solution when 0 is the depot
0 - 6 - 16 - 30 - 25 - 34 - 35 - 15 - 13 - 5 - 24 - 8 - 20 - 31 - 32 - 0, cost 7528.0
0 - 14 - 0, cost 6321.0
0 - 21 - 22 - 23 - 36 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 3 - 4 - 19 - 26 - 12 - 28 - 27 - 9 - 0, cost 10594.0
Total cost: 24443.0

        Infeasible solution when 1 is the depot
1 - 6 - 16 - 17 - 29 - 10 - 21 - 22 - 23 - 36 - 8 - 24 - 5 - 30 - 15 - 34 - 35 - 4 - 19 - 1, cost 10949.0
1 - 14 - 1, cost 8187.0
1 - 26 - 27 - 9 - 1, cost 8229.0
3 - 33 - 7 - 3, cost 859.0
11 - 18 - 11, cost 1313.0
12 - 28 - 12, cost 24.0
13 - 25 - 13, cost 0.0
20 - 32 - 31 - 20, cost 392.0
Total cost: 29953.0

        Infeasible solution when 2 is the depot
2 - 14 - 2, cost 3685.0
2 - 21 - 22 - 23 - 36 - 20 - 8 - 34 - 13 - 30 - 17 - 29 - 10 - 18 - 11 - 2, cost 5306.0
2 - 26 - 6 - 16 - 33 - 7 - 3 - 4 - 19 - 9 - 2, cost 5802.0
5 - 24 - 5, cost 40.0
12 - 28 - 27 - 12, cost 144.0
15 - 35 - 25 - 15, cost 0.0
31 - 32 - 31, cost 90.0
Total cost: 15067.0
```

5 Discussion and Conclusion

Achieving a 1% gap between the objective function value of the optimal solution compared to the average solution given by the metaheuristic algorithm might be good. At least in comparison to the difference in computational time, going from days to less than a min.

Originally, the best ants only updated R_S with complete routes, and not also certain parts of their routes. The number of routes generated was significantly smaller (a tenth). However, the quality of the complete routes were much higher than the corresponding parts. It is evident since the heuristic solutions given are quite similar.

	Routes	Time to		Distance		
Depot	generated	generate	solve SPP	Heuristic	Optimal	Gap
0	1966.3	19.23s	310.6ms	24798.6	24443.0	1.5%
1	1995.1	18.93s	329.2ms	30347.0	29953.0	1.3%
2	1982.5	18.89s	166.8ms	15828.7	15067.0	5.1%

Table 3: The average results for 10 runs.

The metaheuristic algorithm could be improved in a number of ways. Tuning the parameter values for instance. The metaheuristic algorithm is sensitive in regards of the pheromones. If the pheromones converge quickly to a state where certain edges are much more likely to be used than others, then we are lacking diversification. On the other hand, if the pheromones are converging slowly, then we are spending time generating a lot of redundant (bad quality) routes. One way to improve the diversification is to reset the pheromones every now and than, or to use different types and combinations of pheromones.

An upper bound for the objective function for the case when 1 or 2 is the depot can be found using the optimal solution when 0 is the depot. In that case the objective function values are 30040 and 15752 respectively, providing a 0.8% and 0.3% gap between the heuristic solution and the upper bound.

A Appendix

A.1 All solutions when 0 is the depot

```
Route 1: 0 - 14 - 0, cost 6321.0
Route 2: 0 - 6 - 16 - 3 - 4 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 19 - 0, cost 9976.0
Route 3: 0 - 26 - 12 - 28 - 27 - 9 - 32 - 31 - 20 - 8 - 24 - 5 - 30 - 35 - 15 - 34 - 13 - 25 - 23 - 22 - 21 - 36 - 0, cost 8438.0
Total cost: 24735.0

Route 1: 0 - 14 - 0, cost 6321.0
Route 2: 0 - 21 - 10 - 18 - 11 - 33 - 7 - 3 - 4 - 16 - 30 - 17 - 29 - 23 - 22 - 36 - 0, cost 9623.0
Route 3: 0 - 6 - 19 - 26 - 12 - 28 - 27 - 9 - 32 - 31 - 20 - 8 - 24 - 5 - 13 - 15 - 34 - 35 - 25 - 0, cost 8976.0
Total cost: 24920.0

Route 1: 0 - 14 - 0, cost 6321.0
Route 2: 0 - 21 - 22 - 23 - 36 - 24 - 5 - 30 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 19 - 26 - 12 - 28 - 27 - 9 - 0, cost 10337.0
Route 3: 0 - 6 - 16 - 3 - 4 - 13 - 34 - 35 - 25 - 15 - 5 - 24 - 8 - 20 - 32 - 31 - 0, cost 7896.0
Total cost: 24554.0

Route 1: 0 - 14 - 0, cost 6321.0
Route 2: 0 - 6 - 4 - 16 - 3 - 30 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 19 - 0, cost 10066.0
Route 3: 0 - 26 - 12 - 28 - 27 - 9 - 32 - 31 - 20 - 8 - 24 - 5 - 13 - 15 - 35 - 34 - 25 - 23 - 22 - 21 - 36 - 0, cost 8346.0
Total cost: 24733.0

Route 1: 0 - 14 - 0, cost 6321.0
Route 2: 0 - 6 - 30 - 15 - 25 - 34 - 13 - 35 - 5 - 24 - 8 - 20 - 32 - 31 - 0, cost 7528.0
Route 3: 0 - 21 - 22 - 23 - 36 - 30 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 16 - 3 - 4 - 19 - 26 - 12 - 28 - 27 - 9 - 0, cost 10766.0
Total cost: 24615.0

Route 1: 0 - 14 - 0, cost 6321.0
Route 2: 0 - 21 - 22 - 23 - 36 - 24 - 5 - 30 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 19 - 26 - 12 - 28 - 27 - 9 - 0, cost 10337.0
Route 3: 0 - 6 - 16 - 3 - 4 - 34 - 13 - 15 - 35 - 25 - 5 - 24 - 8 - 20 - 32 - 31 - 0, cost 7896.0
Total cost: 24554.0

Route 1: 0 - 14 - 0, cost 6321.0
Route 2: 0 - 21 - 10 - 18 - 11 - 33 - 7 - 3 - 4 - 16 - 30 - 17 - 29 - 23 - 22 - 36 - 0, cost 9623.0
Route 3: 0 - 6 - 19 - 26 - 12 - 28 - 27 - 9 - 32 - 31 - 20 - 8 - 24 - 5 - 35 - 13 - 25 - 34 - 15 - 0, cost 8976.0
Total cost: 24920.0

Route 1: 0 - 14 - 31 - 0, cost 6493.0
Route 2: 0 - 6 - 16 - 3 - 4 - 17 - 29 - 10 - 18 - 11 - 21 - 22 - 23 - 36 - 0, cost 9277.0
Route 3: 0 - 32 - 20 - 8 - 24 - 5 - 35 - 25 - 13 - 15 - 34 - 30 - 33 - 7 - 19 - 26 - 12 - 28 - 27 - 9 - 0, cost 8812.0
Total cost: 24582.0

Route 1: 0 - 14 - 0, cost 6321.0
Route 2: 0 - 6 - 30 - 35 - 13 - 34 - 25 - 15 - 5 - 24 - 8 - 20 - 32 - 31 - 0, cost 7528.0
Route 3: 0 - 21 - 22 - 23 - 36 - 30 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 16 - 3 - 4 - 19 - 26 - 12 - 28 - 27 - 9 - 0, cost 10766.0
Total cost: 24615.0

Route 1: 0 - 14 - 0, cost 6321.0
Route 2: 0 - 6 - 16 - 3 - 4 - 30 - 34 - 13 - 15 - 35 - 25 - 5 - 24 - 8 - 20 - 32 - 31 - 0, cost 7889.0
Route 3: 0 - 21 - 22 - 23 - 36 - 24 - 5 - 30 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 19 - 26 - 12 - 28 - 27 - 9 - 0, cost 10337.0
Total cost: 24547.0
```

A.2 All solutions when 1 is the depot

```
Route 1: 1 - 14 - 1, cost 8187.0
Route 2: 1 - 26 - 12 - 28 - 27 - 9 - 32 - 31 - 20 - 8 - 24 - 5 - 30 - 17 - 29 - 10 - 18 - 11 - 21 - 22 - 23 - 36 - 1, cost 11465.0
Route 3: 1 - 6 - 16 - 3 - 4 - 35 - 34 - 25 - 15 - 13 - 33 - 7 - 19 - 1, cost 10713.0
Total cost: 30365.0

Route 1: 1 - 14 - 1, cost 8187.0
Route 2: 1 - 21 - 10 - 18 - 11 - 33 - 7 - 16 - 3 - 4 - 30 - 17 - 29 - 22 - 23 - 36 - 1, cost 11553.0
Route 3: 1 - 6 - 19 - 26 - 12 - 28 - 27 - 9 - 32 - 31 - 20 - 8 - 24 - 5 - 35 - 15 - 34 - 13 - 25 - 1, cost 10841.0
Total cost: 30581.0

Route 1: 1 - 31 - 14 - 1, cost 8358.0
Route 2: 1 - 21 - 22 - 23 - 36 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 19 - 26 - 12 - 28 - 27 - 9 - 1, cost 12106.0
Route 3: 1 - 6 - 16 - 3 - 4 - 30 - 25 - 35 - 34 - 13 - 15 - 5 - 24 - 8 - 20 - 32 - 1, cost 9664.0
Total cost: 30128.0

Route 1: 1 - 14 - 1, cost 8187.0
```

```
Route 2: 1 - 21 - 22 - 23 - 36 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 16 - 3 - 4 - 19 - 26 - 12 - 28 - 27 - 9 - 1, cost 12525.0
Route 3: 1 - 6 - 30 - 25 - 13 - 35 - 15 - 34 - 5 - 24 - 8 - 20 - 32 - 31 - 1, cost 9393.0
Total cost: 30105.0

Route 1: 1 - 31 - 14 - 1, cost 8358.0
Route 2: 1 - 21 - 22 - 23 - 36 - 24 - 5 - 30 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 16 - 3 - 4 - 19 - 28 - 12 - 27 - 9 - 1, cost 12298.0
Route 3: 1 - 26 - 6 - 34 - 25 - 13 - 35 - 15 - 8 - 20 - 32 - 1, cost 9833.0
Total cost: 30489.0

Route 1: 1 - 14 - 1, cost 8187.0
Route 2: 1 - 6 - 30 - 15 - 13 - 34 - 35 - 25 - 5 - 24 - 8 - 20 - 32 - 31 - 1, cost 9393.0
Route 3: 1 - 26 - 12 - 28 - 27 - 9 - 36 - 23 - 22 - 21 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 16 - 3 - 4 - 19 - 1, cost 12790.0
Total cost: 30370.0

Route 1: 1 - 14 - 1, cost 8187.0
Route 2: 1 - 6 - 34 - 13 - 35 - 25 - 15 - 5 - 24 - 8 - 20 - 32 - 31 - 1, cost 9401.0
Route 3: 1 - 21 - 22 - 23 - 36 - 30 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 16 - 3 - 4 - 19 - 26 - 12 - 28 - 27 - 9 - 1, cost 12632.0
Total cost: 30220.0

Route 1: 1 - 14 - 1, cost 8187.0
Route 2: 1 - 21 - 22 - 23 - 36 - 30 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 16 - 3 - 4 - 19 - 26 - 12 - 28 - 27 - 9 - 1, cost 12632.0
Route 3: 1 - 6 - 35 - 25 - 15 - 13 - 34 - 5 - 24 - 8 - 20 - 32 - 31 - 1, cost 9401.0
Total cost: 30220.0

Route 1: 1 - 14 - 1, cost 8187.0
Route 2: 1 - 21 - 22 - 23 - 36 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 16 - 3 - 4 - 19 - 26 - 12 - 28 - 27 - 9 - 1, cost 12525.0
Route 3: 1 - 6 - 30 - 25 - 15 - 34 - 35 - 13 - 5 - 24 - 8 - 20 - 32 - 31 - 1, cost 9393.0
Total cost: 30105.0

Route 1: 1 - 14 - 1, cost 8187.0
Route 2: 1 - 21 - 22 - 23 - 36 - 24 - 5 - 30 - 17 - 29 - 10 - 18 - 11 - 33 - 7 - 19 - 26 - 12 - 28 - 27 - 9 - 1, cost 12203.0
Route 3: 1 - 6 - 16 - 3 - 4 - 34 - 15 - 13 - 35 - 25 - 8 - 20 - 32 - 31 - 1, cost 9772.0
Total cost: 30162.0
```

A.3 All solutions when 2 is the depot

```
Route 1: 2 - 27 - 14 - 2, cost 3642.0
Route 2: 2 - 6 - 16 - 3 - 4 - 33 - 7 - 19 - 26 - 12 - 28 - 27 - 9 - 36 - 24 - 5 - 8 - 20 - 32 - 31 - 2, cost 7217.0
Route 3: 2 - 21 - 22 - 23 - 36 - 25 - 35 - 34 - 15 - 13 - 30 - 17 - 29 - 10 - 18 - 11 - 2, cost 5025.0
Total cost: 15884.0

Route 1: 2 - 27 - 14 - 2, cost 3642.0
Route 2: 2 - 26 - 6 - 19 - 28 - 12 - 27 - 9 - 32 - 31 - 20 - 8 - 24 - 5 - 30 - 17 - 29 - 10 - 18 - 21 - 2, cost 6745.0
Route 3: 2 - 11 - 33 - 7 - 3 - 4 - 16 - 25 - 13 - 35 - 15 - 34 - 23 - 22 - 36 - 2, cost 5544.0
Total cost: 15931.0

Route 1: 2 - 27 - 14 - 2, cost 3642.0
Route 2: 2 - 26 - 6 - 19 - 28 - 12 - 27 - 9 - 32 - 31 - 20 - 8 - 24 - 5 - 30 - 17 - 29 - 10 - 18 - 21 - 2, cost 6745.0
Route 3: 2 - 11 - 33 - 7 - 3 - 4 - 16 - 35 - 34 - 15 - 13 - 25 - 23 - 22 - 36 - 2, cost 5544.0
Total cost: 15931.0

Route 1: 2 - 27 - 14 - 2, cost 3642.0
Route 2: 2 - 21 - 22 - 23 - 36 - 8 - 24 - 5 - 13 - 25 - 35 - 15 - 34 - 30 - 17 - 29 - 10 - 18 - 11 - 2, cost 5088.0
Route 3: 2 - 26 - 6 - 16 - 3 - 4 - 33 - 7 - 19 - 28 - 12 - 27 - 9 - 32 - 31 - 20 - 2, cost 6884.0
Total cost: 15614.0

Route 1: 2 - 27 - 14 - 2, cost 3642.0
Route 2: 2 - 21 - 22 - 23 - 36 - 24 - 5 - 34 - 25 - 35 - 15 - 13 - 30 - 17 - 29 - 10 - 18 - 11 - 2, cost 5014.0
Route 3: 2 - 26 - 6 - 16 - 3 - 4 - 33 - 7 - 19 - 12 - 28 - 27 - 9 - 32 - 31 - 20 - 8 - 2, cost 7092.0
Total cost: 15748.0

Route 1: 2 - 31 - 14 - 2, cost 3856.0
Route 2: 2 - 21 - 22 - 23 - 36 - 8 - 24 - 5 - 35 - 15 - 13 - 25 - 34 - 30 - 17 - 29 - 10 - 18 - 11 - 2, cost 5088.0
Route 3: 2 - 6 - 16 - 3 - 4 - 33 - 7 - 19 - 26 - 12 - 28 - 27 - 9 - 32 - 20 - 2, cost 6814.0
Total cost: 15758.0

Route 1: 2 - 31 - 14 - 2, cost 3856.0
Route 2: 2 - 21 - 22 - 23 - 36 - 24 - 5 - 34 - 25 - 35 - 13 - 15 - 30 - 17 - 29 - 10 - 18 - 11 - 2, cost 5014.0
Route 3: 2 - 26 - 6 - 4 - 16 - 3 - 33 - 7 - 19 - 28 - 12 - 27 - 9 - 32 - 20 - 8 - 2, cost 7006.0
Total cost: 15876.0

Route 1: 2 - 27 - 14 - 2, cost 3642.0
Route 2: 2 - 26 - 6 - 16 - 3 - 4 - 33 - 7 - 19 - 28 - 12 - 27 - 9 - 31 - 32 - 20 - 2, cost 6884.0
Route 3: 2 - 21 - 22 - 23 - 36 - 8 - 24 - 5 - 35 - 34 - 13 - 25 - 15 - 30 - 17 - 29 - 10 - 18 - 11 - 2, cost 5088.0
Total cost: 15614.0
```

```
Route 1: 2 - 27 - 14 - 2, cost 3642.0
Route 2: 2 - 26 - 6 - 19 - 28 - 12 - 27 - 9 - 32 - 31 - 20 - 8 - 24 - 5 - 30 - 17 - 29 - 10 - 18 - 21 - 2, cost 6745.0
Route 3: 2 - 11 - 33 - 7 - 3 - 4 - 16 - 15 - 13 - 34 - 35 - 25 - 23 - 22 - 36 - 2, cost 5544.0
Total cost: 15931.0

Route 1: 2 - 27 - 14 - 2, cost 3642.0
Route 2: 2 - 21 - 22 - 23 - 8 - 24 - 5 - 15 - 34 - 35 - 13 - 25 - 30 - 17 - 29 - 10 - 18 - 11 - 2, cost 4942.0
Route 3: 2 - 26 - 6 - 16 - 3 - 4 - 33 - 7 - 19 - 12 - 28 - 27 - 9 - 36 - 20 - 31 - 32 - 2, cost 7145.0
Total cost: 15729.0
```