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Project information

Optimization of realistic complex systems

Project 1: Modeling and solving a complex planning problem

1 Information

The task in project 1 is modeling and solving a complex planning problem. You will use the package GLPK, which contains the modeling language GMPL and the LP-solver glpsol. You should formulate the problem as a mathematical optimization model, which requires definition of variables, constraints and objective function. After that the model is written in GMPL format in a file.

2 Preparations

1. Study problem description.
2. Study information about GMPL and GLPK.
3. Formulate the problem as a mathematical optimization model.
4. Solve the problem as described below.
5. Demonstrate your results by writing a report which answers all questions and can be read both by a person who is not familiar with optimization and a person who knows optimization and interested how you solved the problem. Include your mathematical model.
6. Optionally, prepare a presentation of the result/report.

3 Problem Description

The general situation at Returpack AB is presented separately.

In this project you should make a mathematical formulation of a simplified problem. We consider only the goods flow, not the money flow. Moreover, although the flow actually consists of different types of packages (metal cans and plastic bottles), we consider the flow of empty packages as *one* type of flow. Their transformation into raw material is made at raw material manufacturers, and a transformation from raw material to the new packages at packaging manufacturers. There are some tranformation factors, one unit of incoming flow is not exactly one unit of outgoing flow. Similar transformation factors arise when Returpack compress empty packages.

More precisely we model the following situation. There is a certain demand of drinks for each store. The corresponding amount of empty packages is delivered to the stores, minus those thrown away ("spill"). Empty packages are sent to wholesalers or breweries, and wholesalers and breweries send empty packages to the main plant of Returpack. There everything is compressed and sent on to the raw material manufacturers. Raw material manufacturers produce new material and send it to package manufacturers, who produce new packages and send them to the breweries, where they are filled with drinks and sent further to the stores.

One is considering building new warehouses, which may replace wholesalers and/or breweries, so that empty packages are sent directly from the stores to these warehouses and then on to Returpack. Several possible locations for warehouses have been proposed, and the question is which of them to use. Transportation through them will become cheaper and shorter, which is good for the environment.

Thus we have a circulating flow, which, however, is broken by empty packages that are not returned, and by the transformation factors. Because of this the raw material manufacturers need to buy new material in order to fulfill the demand.

Wholesalers, breweries and warehouses have limited capacities for storing empty packages, as well as fixed costs for the handling. The fixed costs are not proportional to the stored amount, but are zero if no empty packages are stored there. Moreover, we work with linear costs on the flow, proportional to the distance between the sites, to minimize environmental disadvantages of the transports.

The main task is to compare the situation with and without warehouses. We assume that shipments are always planned so as to minimize the costs. We want to know which warehouses should be built, therefore there is an element of location problem.

It is also interesting to know how sensitive the solution is to changes in the deposit netto. Should one invest in trying to reduce this spill in recycling?

There are several instances of different sizes to be solved, from small examples that can be used to verify the model, to large, very difficult ones. (All of them are however smaller than the real problem.)

First you solve the problem without warehouses (by setting their capacities to zero in the model file), which gives a picture of the current situation (since we assume that the goods are sent so as to minimize the costs). Then the problem with possible warehouses is solved and the obtained solution determines which warehouses should be built.

The most important result is the gain from usage of warehouses and which of them to build, as well as the environmental gain, given by the optimal value of the linear part of the objective function (i.e. without the fixed costs).

The second issue concerns how changes in the retrun of empties (spill) alter the total cost and the purchase of raw material. To gain some knowledge about this, one can solve the problem for the given value of the deposit netto, and then the problem without the deposit netto. (The latter is easily done by multiplying the netto in the model by zero.)

3.1 Input data

Input data in the data files is given in GMPL format. The following parameters are used: `nbutik`: number of stores, `ngross`: number of wholesalers, `nbrygg`: number of breweries, `nmatr`: number of raw material manufacturers, `nfpack`: number of package manufacturers, `nlager`: number of possible warehouses.

`tomkost`: factor to multiply the distance in order to get the cost to transport an empty package (i.e. flow from stores to Returpack), `fullkost`: factor to multiply the distance in order to get the cost to transport a full package or a unit of new material (i.e. flow from Returpack back to the stores), `lagkost`: factor to multiply the distance in order to get the cost to transport an empty package from stores via new warehoses to Returpack, (the new cars give cheaper transports, which can be seen in the difference between `lagkost` and `tomkost`), `bristkost`: cost (per unit) when demand is not met (not used), `matrkop`: cost for purchasing one unit of new material, `maxlager`: the maximal number of warehouses that may be built.

`behov`: demand at every store, `netto`: number of disappearing empty packages, i.e., that are not returned back to the stores ("spill").

`gkaptom`: capacity for storing empty packages at each wholesaler, `gkapfull`: capacity for storing full packages at each wholesaler (not used), `bkaptom`: capacity for storing empty packages at each brewery, `bkapfull`: capacity for storing full packages at each brewery (not used), `lkaptom`: capacity for storing empty packages at each possible warehouse. (Capacities for transportation of full packages are not included to the model.)

`fkostG`: fixed cost for storing empty packages at each wholesaler, `fkostB`: fixed cost for storing empty packages at each brewery, `fkostL`: fixed cost for storing empty packages at each warehose, which includes the cost for building the warehouse (discounted). As everything becomes more expensive, `fkostL` should be multiplied with 10.

`faktR`: compression factor at Returpack (outflow is `faktR` times inflow), `faktM`: conversion factor at raw material manufacturers (outflow is `faktM` times inflow), `faktF`: conversion factor at empty package manufacturers (outflow is `faktF` times inflow).

`distBuG`: distance between store and wholesaler, `distBuB`: distance between store and brewery, `distGR`: distance from wholesaler to Returpack, `distBR`: distance from brewery to Returpack, `distTB`: distance from empty package manufacturer to brewery, `distGB`: distance between wholesaler and brewery, `distRM`: distance from Returpack to raw material manufacturer, `distMT`: distance from raw material manufacturer to empty package manufacturer, `distBuL`:distance from store to warehouse, `distLR`: distance from warehouse to Returpack.

Study the data file *ex1.dat* to see the format used for these data.

In the file *initmodel.mod*, you will find definitions of index sets and the format of all input data, as a start of the model file.

It is left to you to define variables, objective function and constraints.

3.2 Instances

Below a number of instances of the problem are given, with $|N|$ the number of nodes in the the network, $|A|$ the number of arcs and n the number of variables in the problem.

Problem	`nbutik`	`ngross`	`nbrygg`	`nmatr`	`nfpack`	`nlager`	$\|N\|$	$\|A\|$	n
ex1	4	3	2	2	2	3			90
ex3	5	3	2	2	2	4	19	70	111
ex5	20	5	3	3	3	4	39	288	483
ex6	100	10	5	3	3	10	132	2602	4230
ex7	500	30	5	3	5	20	564	27 748	45 806
ex9	1000	30	5	3	5	100	1144	135 328	171 466
ex10	1000	30	5	3	5	100	1144	135 328	171 466

The example *ex1.dat* does not come from a geographical map, therefore the distances are not natural. In the examples *ex9.dat* and *ex10.dat*, there are significantly more alternatives for warehouses than one needs, in order to have better possibilities to choose from. (However, this makes the problem more difficult to solve.)

Practical tips: The largest instances are difficult to solve. Do not let the program to run for too long. A speed-up can be achieved by solving an LP-relaxation (using *--nomip*) and rounding down the result, or by allowing approximate cuts in the tree-search (using, e.g., *--mipgap 0.01*). You cannot then, however, expect to get the exact optimum.

4 Assignments

1. Solve the small test problem to check that everything works correctly. (Optimal value of the objective function is 104850.6467, and warehouse 2 should be built.)
2. Solve the larger instances. Do the following for each example.

   Note the number of search tree nodes and the number of simplex iterations, both for the first LP-problem and for the total solution. Note the time it took. Note the number of breweries, wholesalers and warehouses which are used for storing empty packages. Note the objective function value (with and without fixed costs).

   a) Solve the problem without warehouses (set capacity to zero in the model file).

   b) Solve the problem with warehouses. Note which warehouses are built and which wholesalers and breweries are used for storing empty packages. Note also how much new raw material is purchased.

   c) State how much one earns by building warehouses, with respect to the total gain and the environmental gain.

   d) Solve problem with no spill (all empties are returned) (with warehouses). Note the objective function value and how much new raw material is purchased.

   e) State how much much one earns if all packages were returned, with respect to the total gain and the environmental gain. Does the conclusion change if the cost of raw materials is multiplied by 100?

   f) If it takes too long to solve the problem exactly, indicate how you achieved a speed-up and how reliable the result is.