Homework 11

Homework 11.1 *Let (M, g) be a 2-dimensional Riemannian manifold, and let R be the (Riemannian) curvature tensor. Moreover, let g_{ij} and R_{ijkl} denote the components of the metric and the curvature tensor in the chart $(U, \vec{x})$.*

(a) *Show that*

$$S = \frac{2R_{1212}}{\det(g)},$$

where S denotes the scalar curvature.

(b) *Show that*

$$R_{ijkl} = \frac{S}{2}\big(g_{ik}g_{jl} - g_{il}g_{jk}\big).$$

Homework 11.2 *The* helicoid Σ *is a surface in $\mathbb{R}^3$ given by the image of*

$$\vec{x} : (u, v) \to (u\cos(v), u\sin(v), cv)$$

for $u > 0$ and $v \in \mathbb{R}$, where $c > 0$ is a fixed parameter.

(a) *Show that $\vec{x}$ is injective and that $\vec{x}'_u$ and $\vec{x}'_v$ are linearly independent at every point in the domain of $\vec{x}$.*

(b) *Compute the mean curvature and the Gaussian curvature of Σ.*

(c) *The map $\vec{x}$ defines a chart $(\vec{x}(U), \vec{x}^{-1})$ on Σ, where*

$$U = \{(u, v) : u > 0, v \in \mathbb{R}\}.$$

Compute the scalar curvature of Σ (with respect to the induced metric) in the chart $(\vec{x}(U), \vec{x}^{-1})$ and compare it to the Gaussian curvature.

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