

2.67c) Ange alla enhetsvektorer som bildar

45° vinkel mot

(ON-bas) $\bar{u} = \underline{e} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ och $\bar{v} = \underline{e} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

$$\bar{x} = \underline{e} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad |\bar{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2} = 1$$

$$\bar{u} \cdot \bar{x} = |\bar{u}| \cdot |\bar{x}| \cos 45^\circ = \sqrt{1^2 + 1^2 + 0^2} \cdot 1 \cdot \frac{1}{\sqrt{2}} = 1$$

$$\bar{u} \cdot \bar{x} = \underline{e} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \underline{e} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_1 + x_2, \quad x_1 + x_2 = 1$$

$$\bar{v} \cdot \bar{x} \quad \text{ger ekv} \quad x_2 + x_3 = 1$$

Så
$$\begin{cases} x_1 + x_2 = 1 \\ x_2 + x_3 = 1 \end{cases} \Leftrightarrow \begin{cases} x_1 = t \\ x_2 = 1 - t \\ x_3 = t \end{cases}, \quad t \in \mathbb{R}$$

$$|\bar{x}| = 1 \text{ ger } x_1^2 + x_2^2 + x_3^2 = 1$$

$$\Leftrightarrow t^2 + (1-t)^2 + t^2 = 1 \Leftrightarrow 3t^2 - 2t = 0$$

$$\Leftrightarrow t(3t-2) = 0 \Leftrightarrow t=0 \text{ et } t = \frac{2}{3}$$

$$t=0 \text{ ger } \begin{cases} x_1 = 0 \\ x_2 = 1 \\ x_3 = 0 \end{cases}, \quad t = \frac{2}{3} \text{ ger } \begin{cases} x_1 = \frac{2}{3} \\ x_2 = \frac{1}{3} \\ x_3 = \frac{2}{3} \end{cases}$$

Svar $\bar{x} = \underline{e} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \bar{e}_2$

el $\bar{x} = \underline{e} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{pmatrix} = \frac{2}{3} \bar{e}_1 + \frac{1}{3} \bar{e}_2 + \frac{2}{3} \bar{e}_3$