

8.4.24

$$U = \left[e \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}, e \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}, e \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, e \begin{pmatrix} 2 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right] \text{ i } \mathbb{R}^4$$

Bestäm $\dim U$ och ange om

$$\bar{u} = e \begin{pmatrix} -2 \\ 3 \\ 1 \\ 6 \end{pmatrix} \text{ tillhör } U.$$

Lösning Bra ekr

$$\begin{array}{l} \text{H) (-2)} \\ \downarrow \\ \left(\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 0 \\ 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & -1 & 0 \end{array} \begin{array}{l} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right) \end{array} \begin{array}{l} \text{H) (-2)} \\ \downarrow \\ \left(\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 2 & -1 & -3 & 0 \end{array} \begin{array}{l} x_1 \\ -2x_1+x_2 \\ -x_1+x_3 \\ -x_1+x_4 \end{array} \right) \end{array}$$

$$\begin{array}{l} \Rightarrow \\ \text{(-3)} \\ \downarrow \text{(-2)} \\ \left(\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 3 & 3 & 0 \end{array} \begin{array}{l} x_1 \\ -2x_1+x_2 \\ 2x_1-x_2-x_1+x_3 \\ 4x_1-2x_2-x_1+x_4 \end{array} \right) \end{array}$$

$$\begin{array}{l} \Rightarrow \\ \left(\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \begin{array}{l} x_1 \\ -2x_1+x_2 \\ x_1-x_2+x_3 \\ -3x_1+3x_2-3x_3+6x_1-4x_2+2x_4 \end{array} \right) \end{array}$$

$$\dim U = 3, \quad U = \left[e \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}, e \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}, e \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right]$$

$$3x_1 - x_2 - 3x_3 + 2x_4 = 0 \text{ beskriver } U.$$

$$\bar{u} \text{ i ekr: } 3 \cdot (-2) - 3 - 3 \cdot 1 + 2 \cdot 6 = 0$$

Ja, $\bar{u} \in U$.