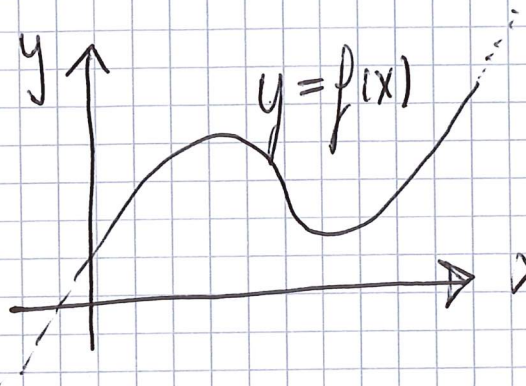
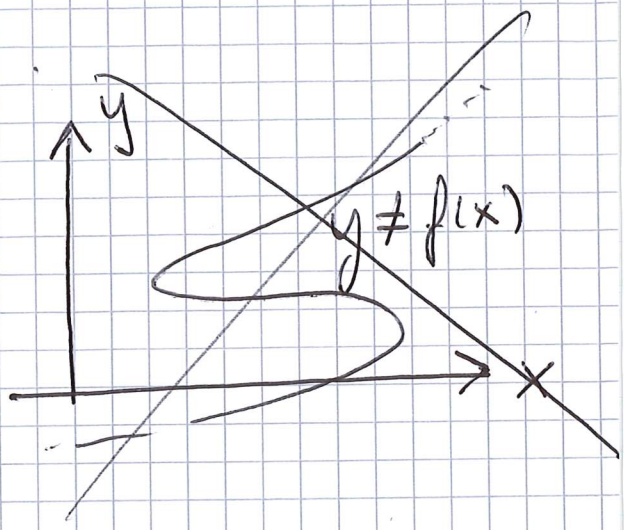
Fö 2 | Läs 2.1-2.3.

Funktioner.

$y = f(x)$ ordnar för varje tillåtet x **ett** bestämt y.





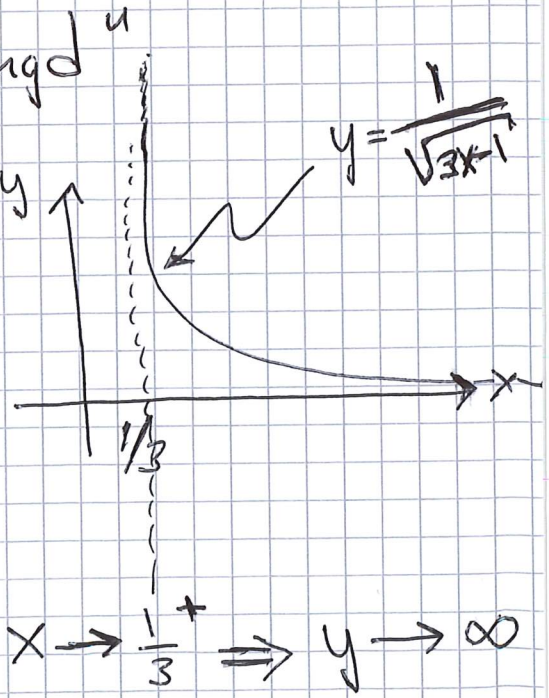
$D_f = \{$ tillåtna invärden $\}$

" definitionsmängd "

$V_f = \{$ möjliga utvärden $\}$

" värdemängd "

Ex. $f(x) = \dfrac{1}{\sqrt{3x-1}}$



D_f: $x > \frac{1}{3}$, $x \in \mathbb{R}$

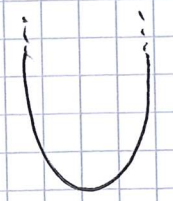
V_f: $y > 0$, $y \in \mathbb{R}$

$x \to \frac{1}{3}^{+} \Rightarrow y \to \infty$

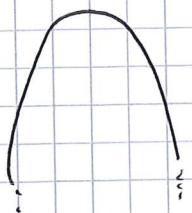
2:a gradsfunktion. Parabel.

$f(x) = ax^2 + bx + c$

$a > 0$



$a < 0$



kvadreringsregel

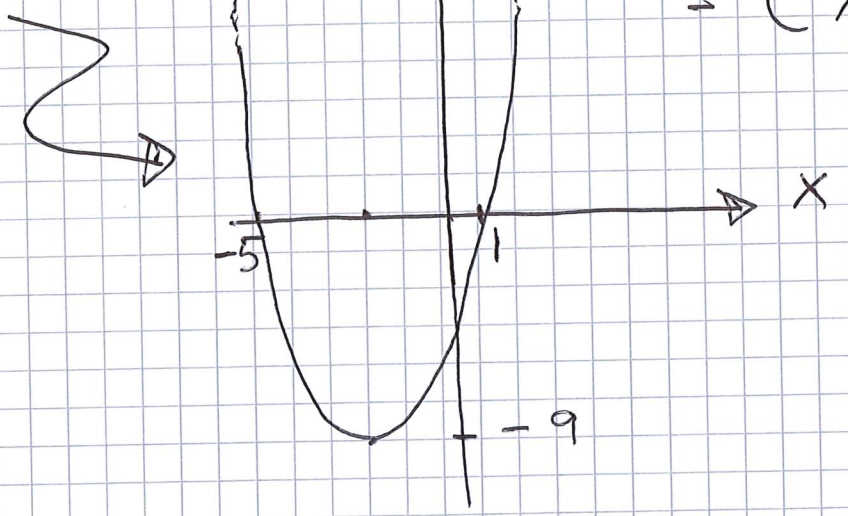
Ex. $f(x) = x^2 + 4x - 5 = (x+2)^2 - 4 - 5 =$

$= (x+2)^2 - 9 = (x+2)^2 - 3^2 =$

$= (x+2+3)(x+2-3) =$

$= (x+5)(x-1)$

$y = x^2 + 4x - 5$



Ex. Bestäm största värdet av

$g(x) = -2x^2 + 3x - 5$.

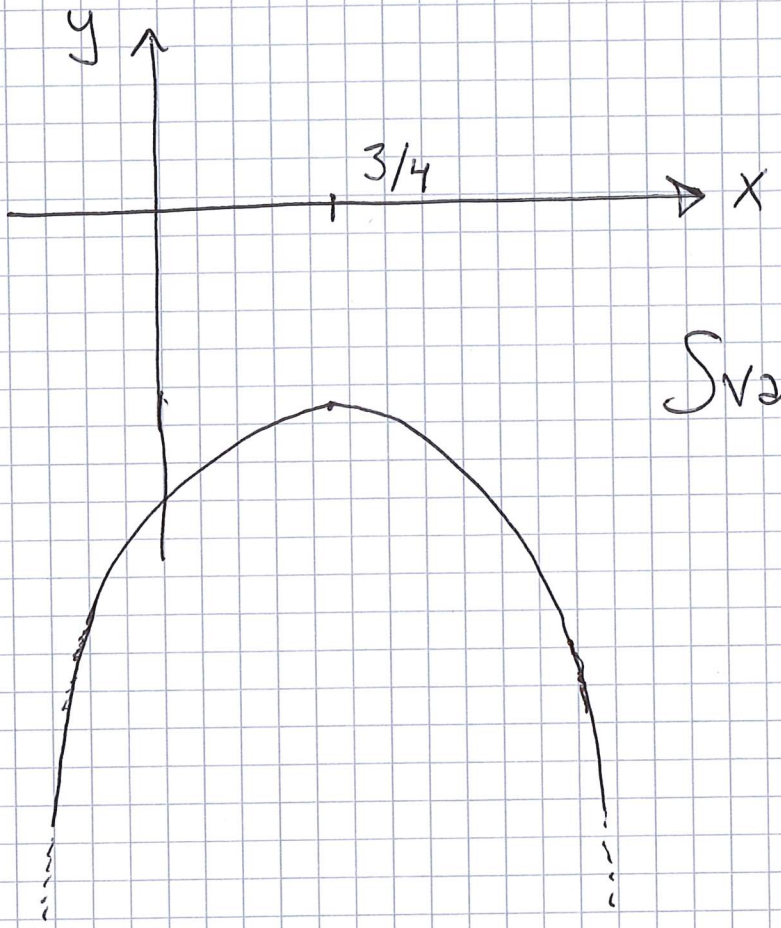
L:

$g(x) = -2\left(x^2 - \frac{3}{2}x + \frac{5}{2}\right) =$

$= -2\left(\left(x - \frac{3}{4}\right)^2 - \frac{9}{16} + \frac{5}{2}\right) =$

$= -2\left(\left(x - \frac{3}{4}\right)^2 + \frac{31}{16}\right)$.

$$g(x) = -2\left(x - \frac{3}{4}\right)^2 - \frac{31}{8}$$



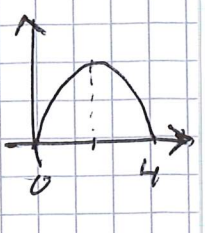
Svar: Största värde är $-\frac{31}{8}$ fås då $x = \frac{3}{4}$.

Ex. $f(x) = \sqrt{4x - x^2}$. D_f, V_f?

$\sqrt{\ }$ kräver?

$4x - x^2 \geq 0$

$\Longleftrightarrow \quad 0 \leq x \leq 4$



↗ strängt växande

$f(2) = \sqrt{4 \cdot 2 - 2^2} = 2$

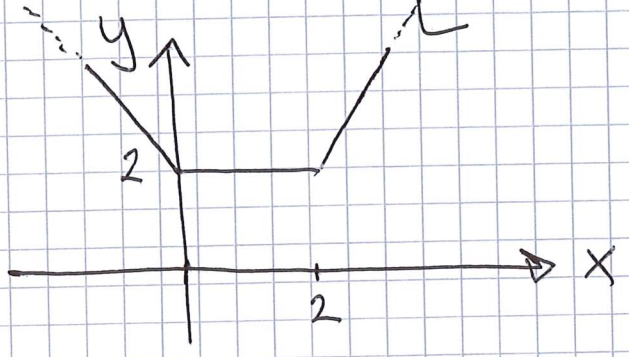
$D_f = [0, 4]$

Svar:

$V_f = [0, 2]$

Belopp

Ex. $y = |x-2| + |x| = \begin{cases} x-2+x = 2x-2, & x > 2 \\ -(x-2)+x = 2, & 0 < x \leq 2 \\ -(x-2)-x = -2x+2, & x < 0 \end{cases}$



Rep:

$y = kx + m$

Sammansättningar mm

$f(x) = x^2 + 2$, $g(x) = \sqrt{x-2}$

$f(7) = 7^2 + 2 = 51$

$f(2x) = (2x)^2 + 2 = 4x^2 + 2 \neq 2f(x)$

$g(x-1) \neq g(x) - 1$

$f \circ g = f(g(x)) = (\sqrt{x-2})^2 + 2 = x$, $x \geq 2$

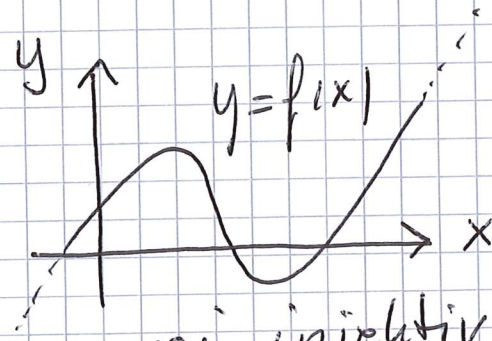
$g \circ f = g(f(x)) = \sqrt{f(x)-2} = \sqrt{x^2+2-2} = \sqrt{x^2} = |x|$

Omvändbara (injektiva) funktioner.

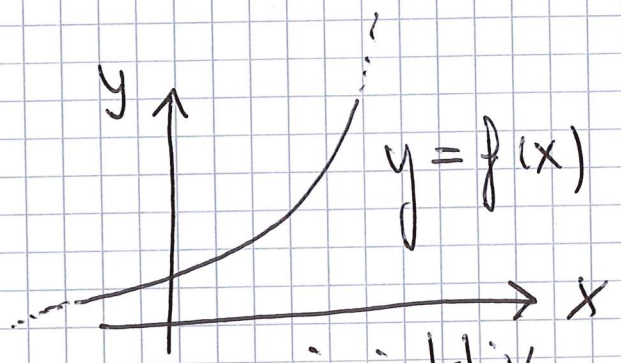
$y = f(x) \iff x = f^{-1}(y)$

f injektiv

Def: f är omvändbar ifall det till varje $y \in V_f$ bara finns ett $x \in D_f$ sådant att $y = f(x)$.



ej injektiv.



injektiv.

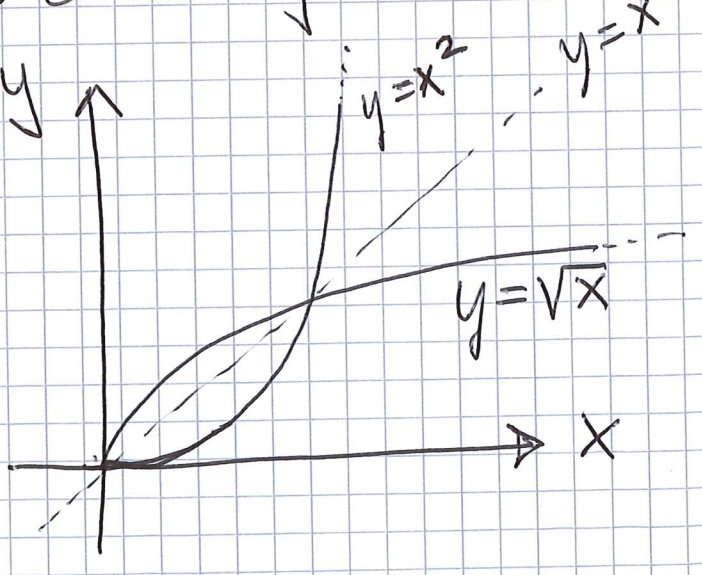
Ex. $y = x^2$, $x \geq 0$ injektiv.

$\iff$

$x = \pm\sqrt{y}$

byt x mot y.

Spegling i linjen $y = x$



$y = \sqrt{x}$

"inversen ogör vad funktionen gjort"

Ex. Sök invers till $f(x) = \frac{3x+2}{x-1}$, $x > 1$

L: $y = \frac{3x+2}{x-1}$

löst x i y

$y(x-1) = 3x+2$

$yx - y = 3x+2$

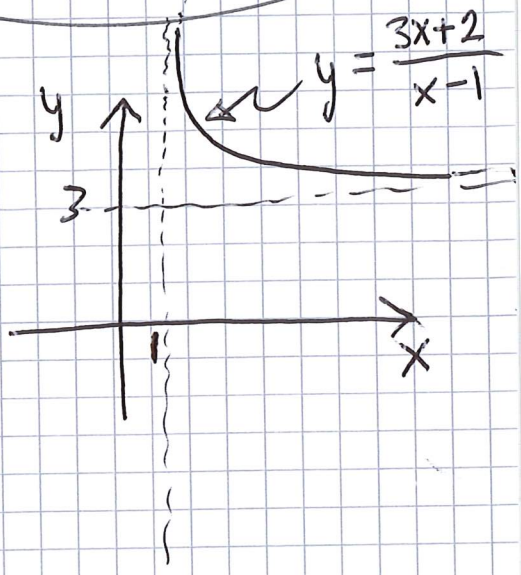
$yx - 3x = 2+y$

$x(y-3) = 2+y$

$x = \frac{2+y}{y-3} = f^{-1}(y)$

byt x mot y.

Svar: $f^{-1}(x) = \frac{2+x}{x-3}$, $x > 3$

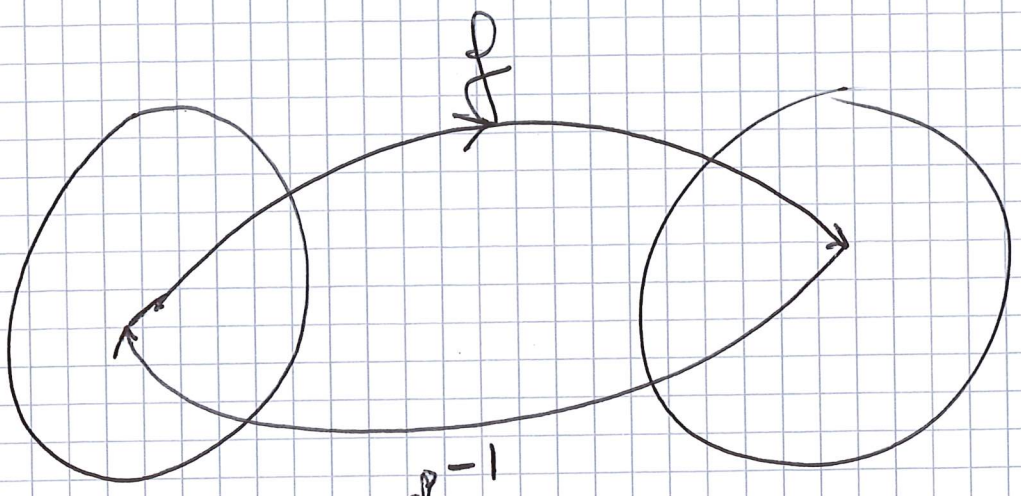


test: $f(2) = \frac{3 \cdot 2 + 2}{2-1} = 8$

$f^{-1}(8) = \frac{2+8}{8-3} = \frac{10}{5} = 2$

$f^{-1}(f(x)) = x$

$f(f^{-1}(x)) = x$

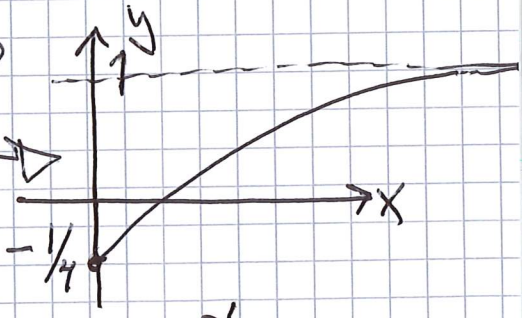


f^{-1} utläses f:s invers.

Ex. Sök ev. invers till $f(x) = \frac{\sqrt{x}-1}{\sqrt{x}+4}$

L: $y = \frac{\sqrt{x}-1}{\sqrt{x}+4}$, $x \geq 0$

/ Sub. $t = \sqrt{x} \geq 0$ /



$y(t+4) = t-1$

$1+4y = t-yt$

$1+4y = t(1-y)$

$t = \frac{1+4y}{1-y}$

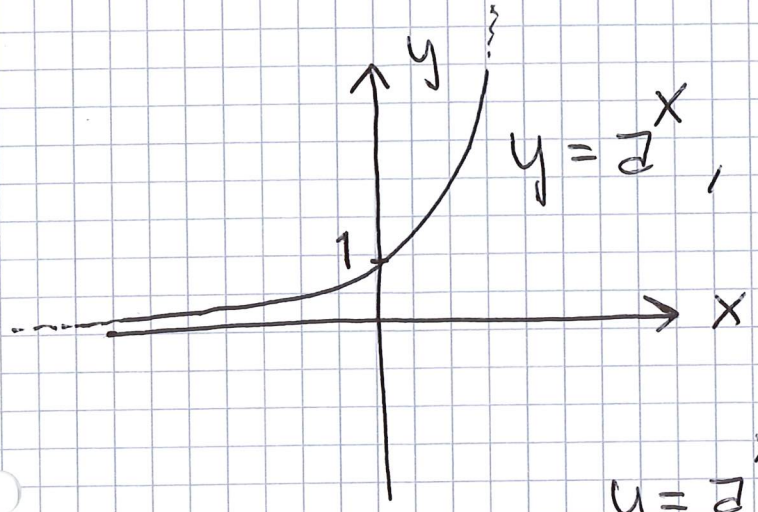
$x = \left(\frac{1+4y}{1-y}\right)^2 = f^{-1}(y)$

byt x mot y.

Svar: $f^{-1}(x) = \left(\frac{1+4x}{1-x}\right)^2$, $-\frac{1}{4} \leq x < 1$

där

Exp. / Log. funktioner.



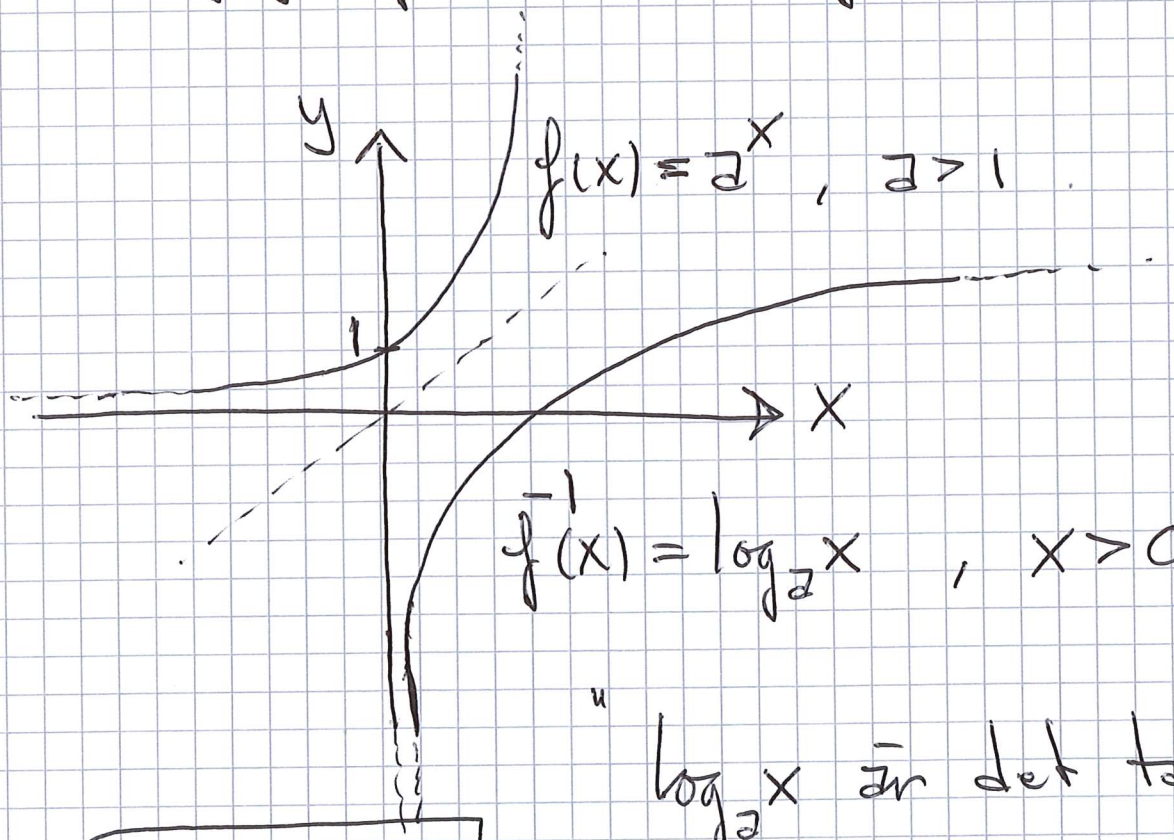
$y = a^x$, $a > 1$. strängt växande.

injektiv

$y = a^x \iff x = \log_a y$

byt x mot y.

"spegling i linjen $y = x$"



$f(x) = a^x$, $a > 1$.

$f^{-1}(x) = \log_a x$, $x > 0$.

"$\log_a x$ är det tal som a skall upphöjas till för att få x."

$a^{\log_a x} = x$

$\log_a a^x = x$

Ex. $\log_2 8 = \log_2 2^3 = 3$

$\log_e$ skrivs $\ln$

$\log_{10}$ skrivs $\lg$

---

log. lagar:

$$\ln(xy) = \ln x + \ln y \quad \ldots (1)$$

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y \quad \ldots (2)$$

$$\ln x^{\alpha} = \alpha \ln x \quad \ldots (3)$$

"bevis" av (1).

$$s = \ln x \Leftrightarrow e^s = x$$

$$t = \ln y \Leftrightarrow e^t = y$$

$$xy = e^s \cdot e^t = e^{s+t}$$

$$\ln(xy) = \ln e^{s+t} = s + t = \ln x + \ln y$$

Ex. $\ln x^3 = \ln(x \cdot x \cdot x) = \ln x + \ln x + \ln x = 3 \ln x$

Basbyte.

$$\log_a x = \frac{\ln x}{\ln a}$$

bevis:

$$\log_a x = t$$

/ "tänk" $a^{\log_a x} = a^t$ /

$$x = a^t$$

$$\ln x = \ln a^t$$

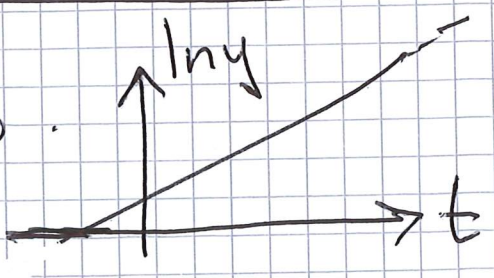
$$\ln x = t \ln a$$

$$t = \frac{\ln x}{\ln a}$$ v.s.b.

$$a^x = e^{\ln a^x} = e^{x \ln a}$$

Skriver om på basen e.

Ex. $y = Ce^{-\lambda t}$, $\lambda > 0$.



$$\ln y = \ln(Ce^{-\lambda t})$$

$$\ln y = \ln C + \ln e^{-\lambda t} = -\lambda t + \ln C$$

Ex. Beräkna

$$\lg 1000 = \lg 10^3 = 3$$

$$\ln 1 = \ln e^0 = 0$$

$$\lg 10 = 1$$

Ex. Lös ekv. $3\ln x = 2$.

$$3\ln x = 2$$

$$\ln x = \frac{2}{3} \quad / \; e^{\ln x} = e^{2/3} /$$

$$x = e^{2/3}$$

Ex. Lös $2^x = 6$

$$2^x = 6$$

$$\ln 2^x = \ln 6$$

$$x \ln 2 = \ln 6$$

$$x = \frac{\ln 6}{\ln 2}$$

Ex. Lös ekv. $e^{3x} - 3e^{2x} - e^x + 3 = 0$

L: $e^{3x} - 3e^{2x} - e^x + 3 = 0$

$$(e^x)^3 - 3(e^x)^2 - e^x + 3 = 0$$

Sub. $t = e^x$, $t > 0$.

$$t^3 - 3t^2 - t + 3 = 0$$

$$(t-1)(t^2 + 2t - 3) = 0$$

$$(t-1)(t+1)(t-3) = 0$$

$t_1 = 1$, ~~$t_2 = -1$~~, $t_3 = 3$

$e^x = 1 \Leftrightarrow x = 0$

$e^x = 3 \Leftrightarrow x = \ln 3$

Svar: $\begin{cases} x_1 = 0 \\ x_2 = \ln 3 \end{cases}$

Ex. Lös ekv. $\ln x + \ln(x-2) = \ln 3$

L: $\ln x + \ln(x-2) = \ln 3$

KRAV: ?? $\quad x > 2$.

/ log. lag $\ln x + \ln y = \ln(xy)$ /

$\Longleftrightarrow$ $\ln(x(x-2)) = \ln 3$ $\quad x > 2$

$\Longleftrightarrow$ $\ln(x^2 - 2x) = \ln 3$ $\quad x > 2$.

/ ln strängt växande.

$x^2 - 2x = 3$

$x^2 - 2x - 3 = 0$

$x = 1 \pm \sqrt{1+3} = 1 \pm 2 = \begin{cases} 3 \\ -1 \end{cases}$

~~$x = -1$~~ krav. $x > 2$.

Prövning: $\ln 3 + \ln(3-2) = \ln 3 + \underbrace{\ln 1}_{=0} = \ln 3$

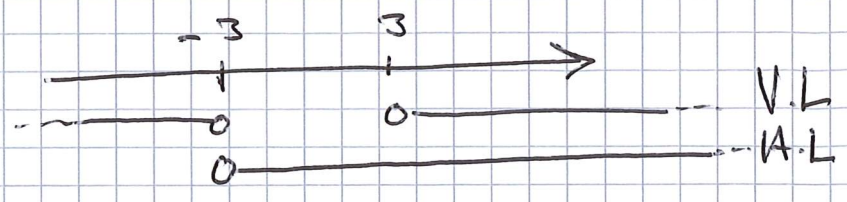
Svar: $x = 3$.

Ex. Bestäm alla $x \in \mathbb{R}$ sådana att

$$\ln(x^2-9) \leq \ln(x+3) .$$

L: KRAV

$x > 3$



$$\ln(x^2-9) \leq \ln(x+3)$$

$\Leftrightarrow \quad x^2-9 \leq x+3$ | $\ln x$ är strängt växande |

$\Leftrightarrow \quad (x+3)(x-3) \leq x+3$

$\Leftrightarrow \quad x-3 \leq 1$

$\Leftrightarrow \quad x \leq 4$

Svar: $3 < x \leq 4$.