

Fö 3 | Läs 2.1-2.3

Se även Fö 2
nollep.

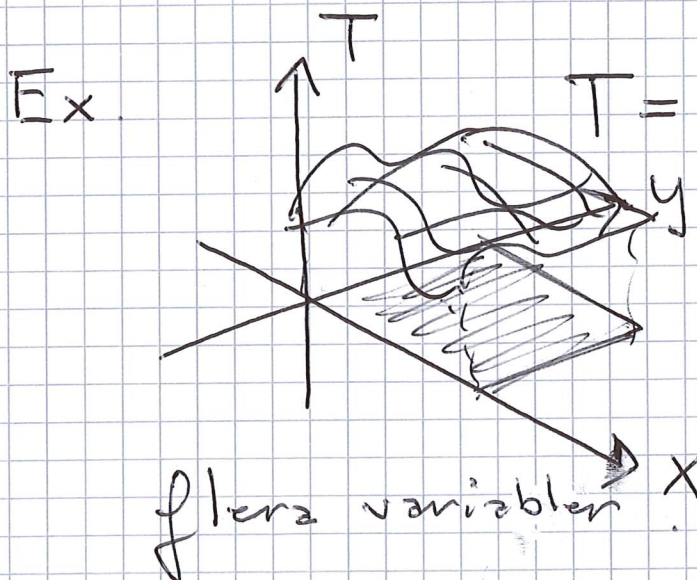
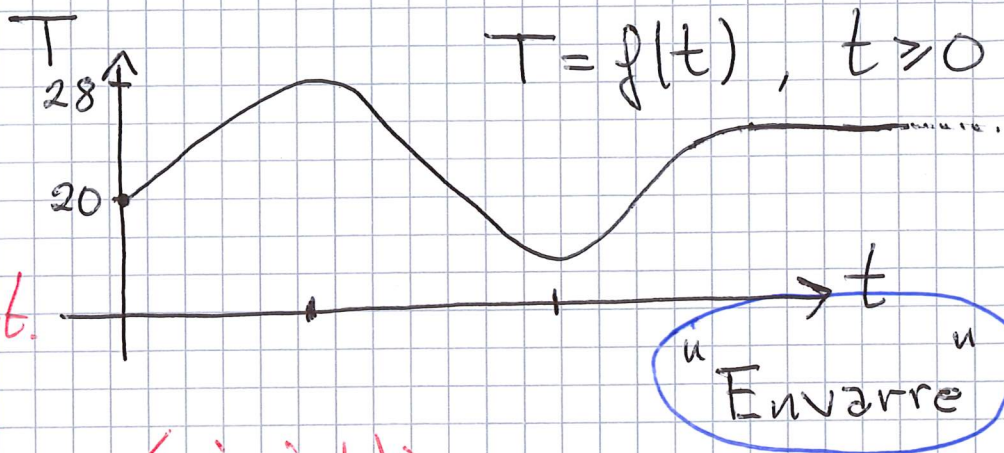
Mer om funktioner.

Viktiga begrepp.
Ex.
vaxande.
avtagande.

strängt vax./avt.

monoton

strängt monoton. / injektiv.



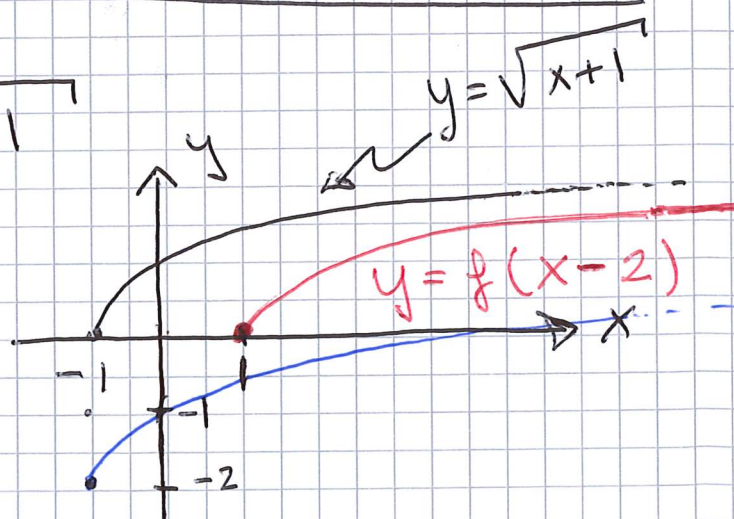
Reellvärde.

Ex.

$$f(x) = \sqrt{x+1}$$

$$D_f: x \geq -1$$

$$V_f: y \geq 0$$

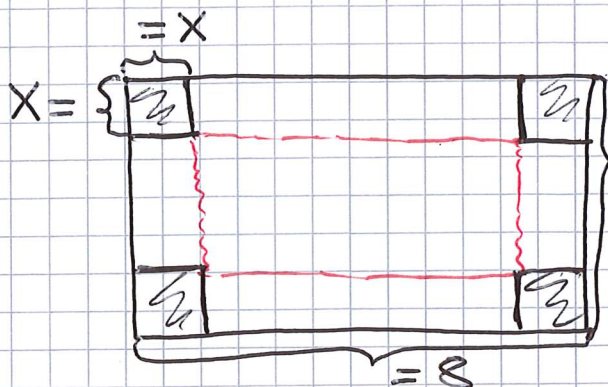


jfr. $y = f(x) - 2$

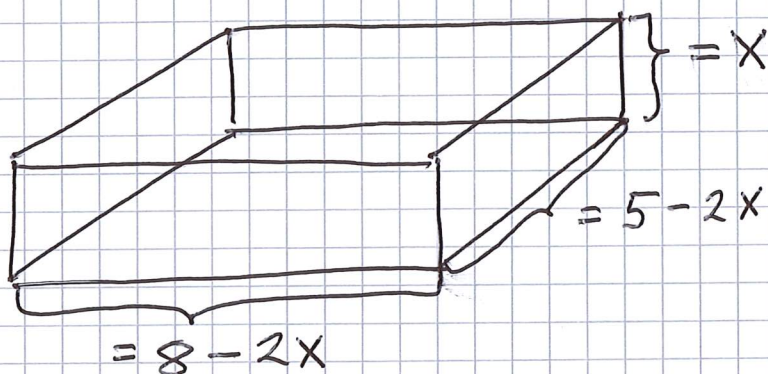
$y = f(2x)$
"trycker ihop."

Ex. Konstruera öppen rätvinklig låda av en plåt, 5×8 dm.

Maximera volymen.

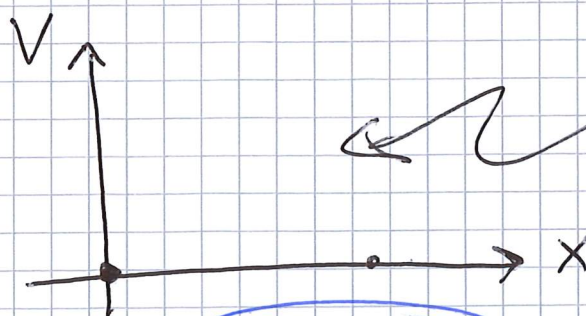


klipp bort 4 st
 $= 5$ kvadrater. Se figur.
Vik ihop till låda.



$$V(x) = (8 - 2x)(5 - 2x)x = 4x^3 - 26x^2 + 40x$$

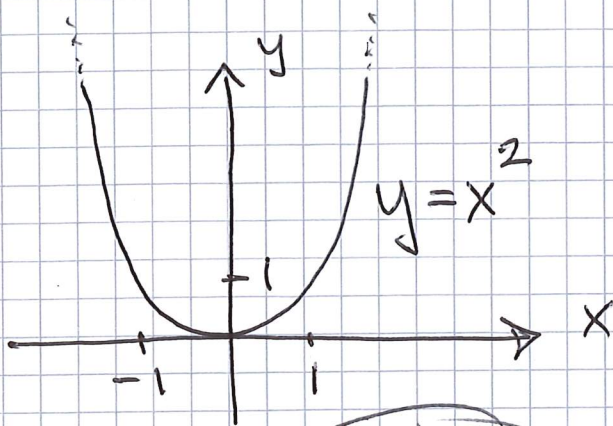
$$D_V: 0 \leq x \leq \frac{5}{2}$$



Graf men
senare i
kursen.

Derivata osv.

Rita grafer.



Rita.

Rep.:

$$y = kx + m$$

$$y = ax^2 + bx + c$$

Ex. $f(x) = 3x^2 - 2x + 5$

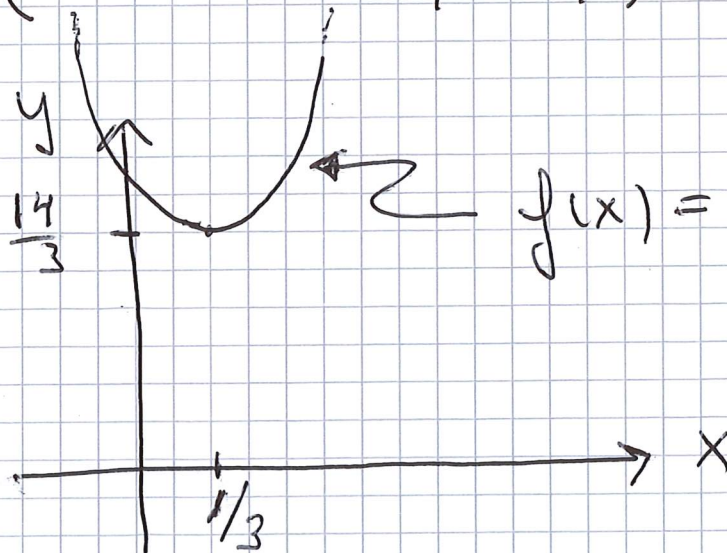
lös dubbla
produkten.

$$f(x) = 3x^2 - 2x + 5 = 3\left(x^2 - \frac{2x}{3} + \frac{5}{3}\right) =$$

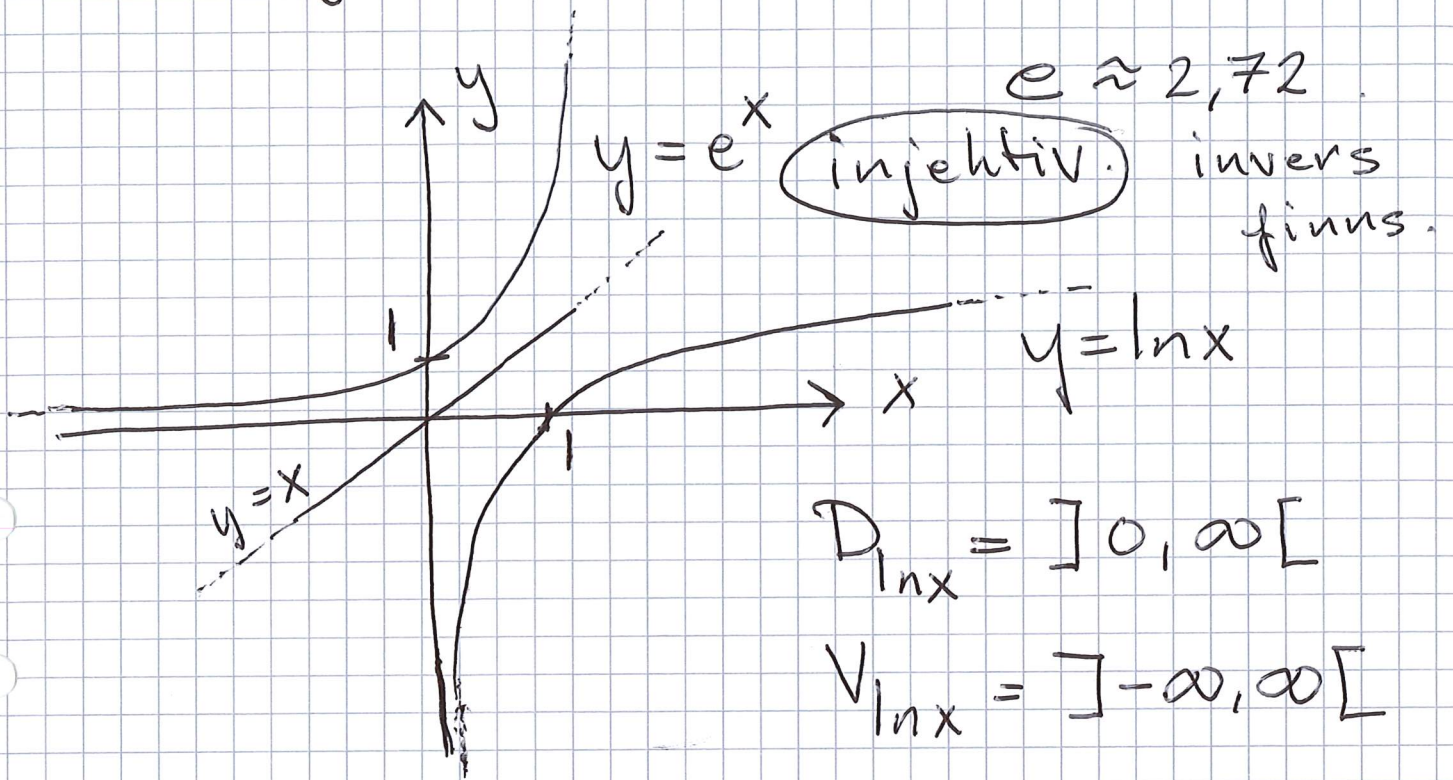
$$= 3\left(\left(x - \frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right)^2 + \frac{5}{3}\right) =$$

2:2 kvadreringsregeln.

$$= 3\left(\left(x - \frac{1}{3}\right)^2 - \frac{1}{9} + \frac{15}{9}\right) = 3\left(x - \frac{1}{3}\right)^2 + \frac{14}{3}$$



Logaritm - exponential funktioner

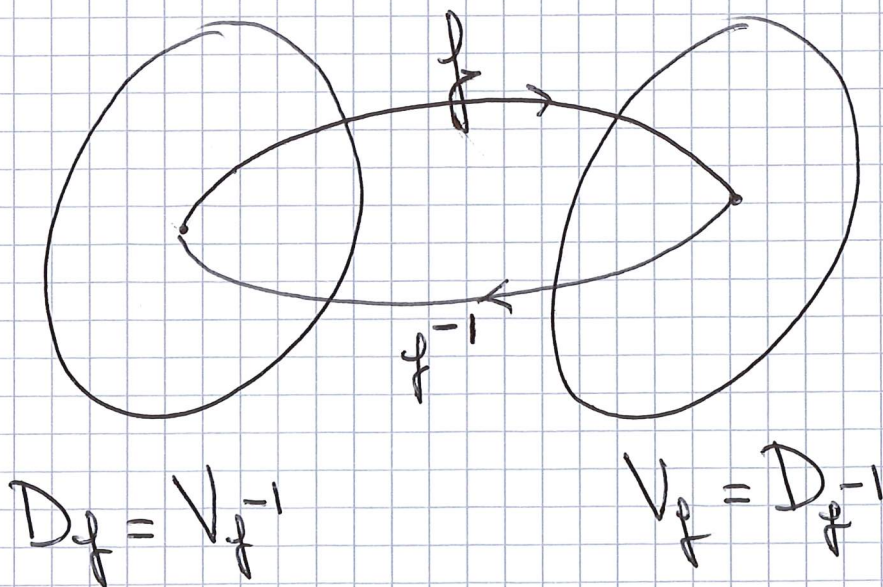


$$y = e^x \iff x = \ln y \quad \text{byt } x \text{ mot } y$$

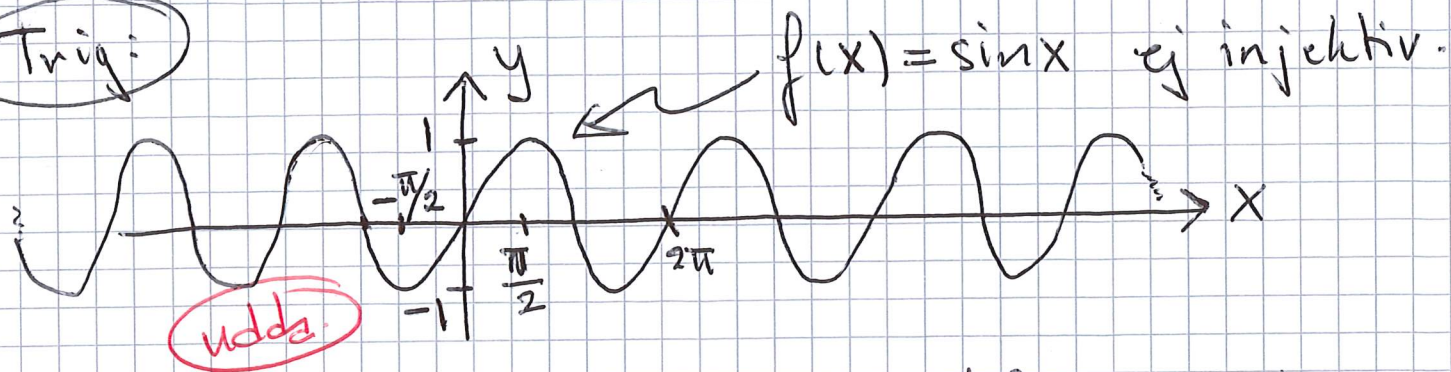
$y > 0$
 $x \in \mathbb{R}$

Spegling i linjen
 $y = x$

Kom ihåg!

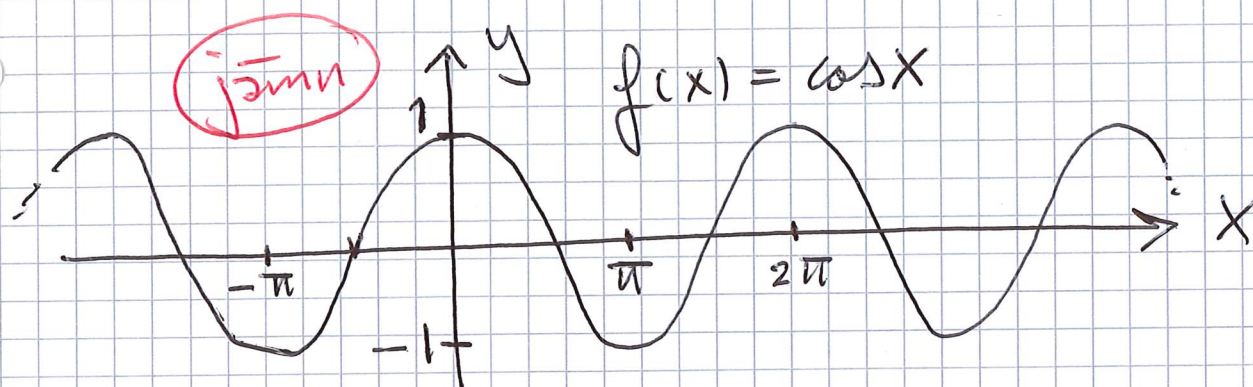


Trig:



Obs! Restriktion.

men $f(x) = \sin x$, $-\pi/2 \leq x \leq \pi/2$
är injektiv.

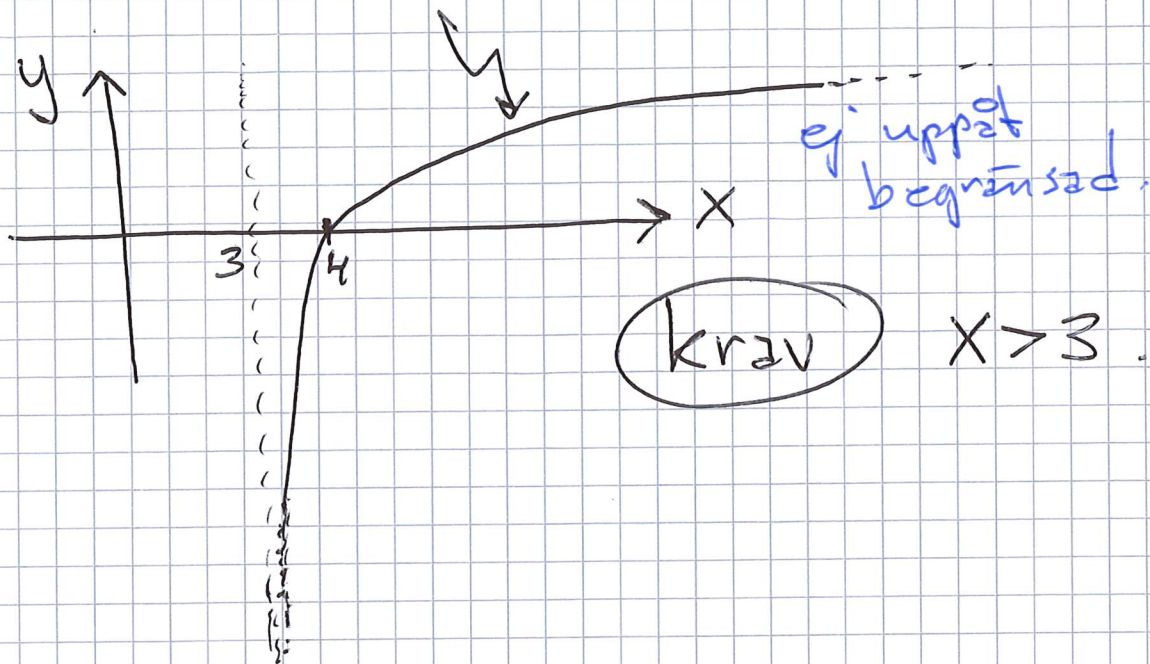


$f(x) = \cos x$, $0 \leq x \leq \pi$ är injektiv
obs!

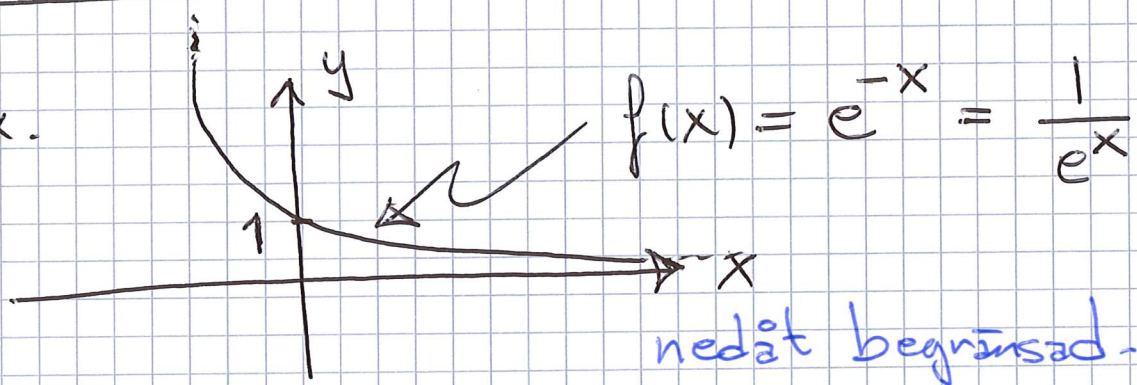
Allmänt: Om $f(-x) = f(x)$ för alla $x \in D_f$
så är f jämn.

Om $f(-x) = -f(x)$ för alla $x \in D_f$
så är f udda.

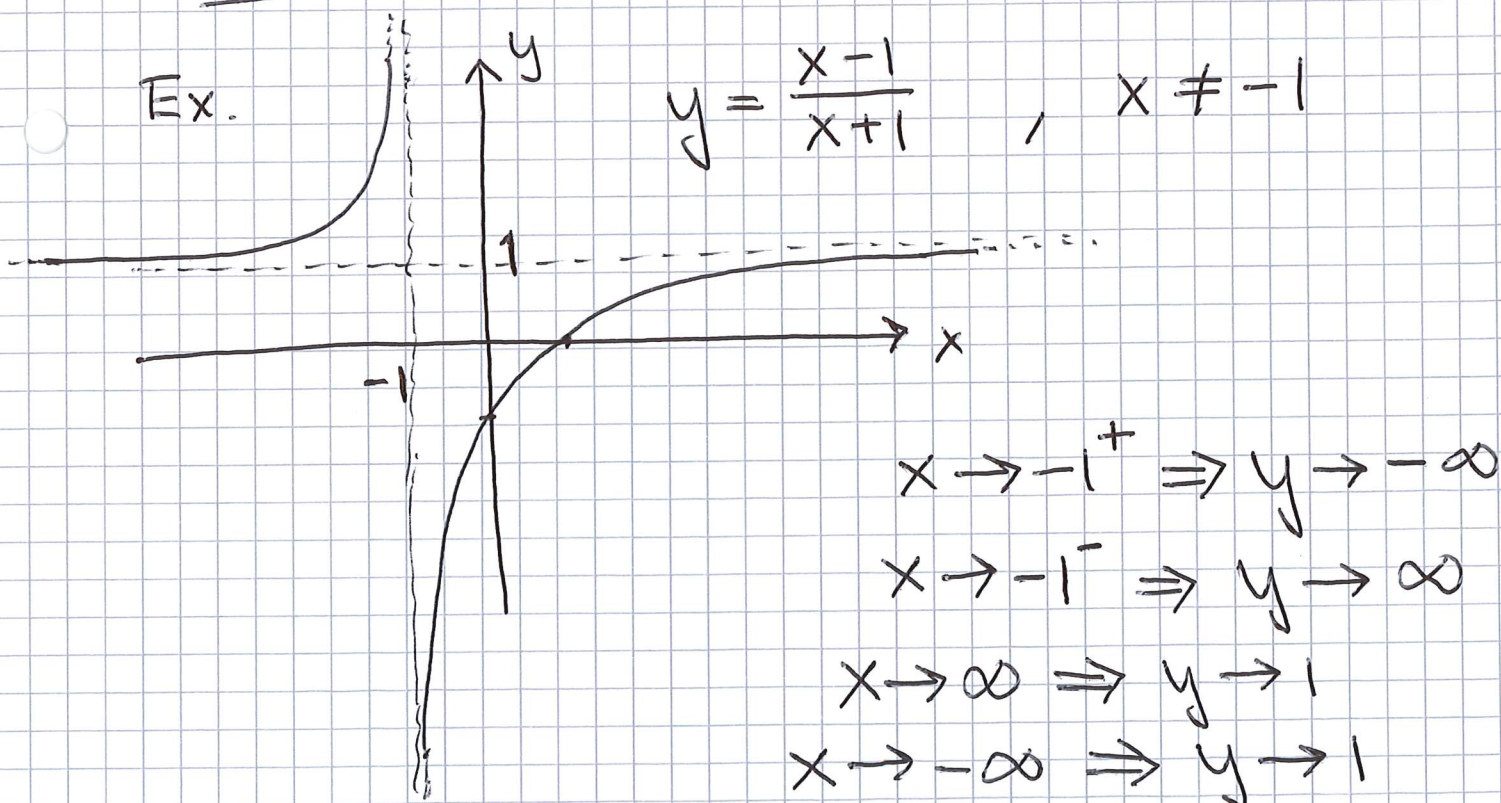
Ex. Rita $f(x) = \ln(x-3)$



Ex. $f(x) = e^{-x} = \frac{1}{e^x}$



Ex. $y = \frac{x-1}{x+1}, x \neq -1$



Mer om injektiva funktioner

Ex. Bestäm inversen till

$$f(x) = \frac{2x+1}{x-1}, \quad x \neq 1$$

L: $y = \frac{2x+1}{x-1}$ "lös ut x i y ".

$$y(x-1) = 2x+1$$

$$yx - y = 2x + 1$$

$$yx - 2x = y + 1$$

$$x(y-2) = y+1$$

$$x = \frac{y+1}{y-2}$$

$$f^{-1}(y) = \frac{y+1}{y-2}$$

byt x mot y

Svar: $f^{-1}(x) = \frac{x+1}{x-2}, \quad x \neq 2$

test: $f(2) = 5$ $f^{-1}(5) = 2$

$$f \circ f^{-1} = f(f^{-1}(x)) = \dots = x$$

$$f^{-1} \circ f = f^{-1}(f(x)) = \dots = x$$

Ex. Visa att $f(x) = x^3 + x$ har
invers.

Beräkna $f^{-1}(10)$

L: f strängt växande $\Rightarrow f$ injektiv.

$$x_1, x_2 \in D_f.$$

$$\begin{aligned} x_2 > x_1 &\Rightarrow f(x_2) - f(x_1) = x_2^3 + x_2 - (x_1^3 + x_1) = \\ &= \underbrace{x_2^3 - x_1^3}_{>0} + \underbrace{x_2 - x_1}_{>0} > 0 \end{aligned}$$

$$10 = x^3 + x \Rightarrow x = 2.$$

$$10 = f(2)$$

$$f^{-1}(10) = f^{-1}(f(2)) = 2$$

$$\text{Svar: } f^{-1}(10) = 2$$

$$\text{Allmänt: } e^{\ln x} = x, \quad x > 0$$

$$\ln e^x = x$$

log. lagar mm

Se även
Fö2 notep.

$$\ln(xy) = \ln x + \ln y$$

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

$$\ln x^\alpha = \alpha \ln x$$

$$e^x = \exp(x)$$

skrivsätt

$$e^0 = 1$$

$$\ln 1 = 0$$

$$\ln e = 1$$

Ex. Lös $4^x - 2^{x+1} = 8$

$$L: (2^x)^2 - 2 \cdot 2^x = 8$$

Sub. $t = 2^x$
 $t > 0$

$$t^2 - 2t = 8$$

$$t^2 - 2t - 8 = 0$$

$$\begin{cases} t_1 = -2 \\ t_2 = 4 \end{cases}$$

$$4 = 2^x$$

$$x = 2$$

Svar: $x = 2$

Ex. Lös ekv. $2^{x+1} = 3^x$

L: $2^{x+1} = 3^x$

$$\ln 2^{x+1} = \ln 3^x$$

$$(x+1)\ln 2 = x\ln 3$$

$$x\ln 2 + \ln 2 = x\ln 3$$

$$x\ln 2 - x\ln 3 = -\ln 2$$

$$x(\ln 2 - \ln 3) = -\ln 2$$

$$x = \frac{-\ln 2}{\ln 2 - \ln 3}$$

Svar: $x = \frac{\ln 2}{\ln 3 - \ln 2}$

Ex. Beräkna $\lg 500 + \lg 20$

L: $\lg 500 + \lg 20 = \lg 10000 = \lg 10^4 = 4$

$\log \cdot \log$

Svar: 4.

$$\ln(xy) = \ln x + \ln y$$

Ex. Lös ekv. $\ln x - \ln(x+3) = 1$

L: $\ln x - \ln(x+3) = 1$

KRAV.

$x > 0$.

$\Leftrightarrow \ln \frac{x}{x+3} = 1$

$\Leftrightarrow e^{\ln \frac{x}{x+3}} = e^1$

$\Leftrightarrow \frac{x}{x+3} = e$

$\Leftrightarrow x = xe + 3e$

$\Leftrightarrow x - xe = 3e$

$\Leftrightarrow x(1-e) = 3e$

$\Leftrightarrow x = \frac{3e}{1-e} < 0$

log. lag.

$\ln a - \ln b =$

$= \ln \frac{a}{b}$

Obs! exp.

ej termvis.

t.ex $1+0=1$

$e^1 + e^0 \neq e^1$

Svar: lösning saknas.

Ex. För vilka reella tal x gäller
olikheten $5 + 4e^x - e^{2x} > 0$?

$$L: 5 + 4e^x - (e^x)^2 > 0$$

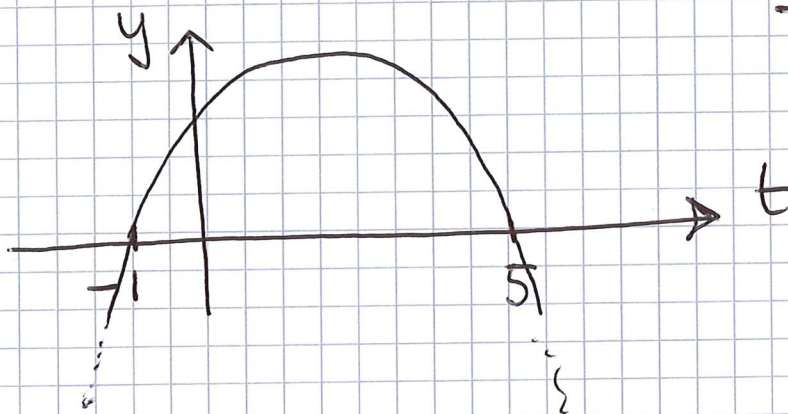
$$\text{Sub. } t = e^x, \quad t > 0.$$

$$5 + 4t - t^2 > 0$$

$$= g(t)$$

$$\Leftrightarrow$$

$$-1 < t < 5$$



$$t = e^x$$

$$-1 < e^x < 5$$

V.D. $-1 < e^x$ gäller för alla $x \in \mathbb{R}$.

$$\text{H.O. } e^x < 5 \Leftrightarrow x < \ln 5$$

(e^x strängt
väx.)

$$\text{Svar: } x < \ln 5.$$

Potenzfunktionen

$$f(x) = x^\alpha = e^{\ln x^\alpha} = e^{\alpha \ln x}$$

$$\alpha \in \mathbb{R}, \quad x > 0$$

$$\sqrt[n]{x} = x^{\frac{1}{n}}, \quad n \text{ heltal.}$$

$$\text{Ex. } \sqrt[3]{-64} = -4$$

$$\text{Ex. } \sqrt{-9} \text{ ej defineret.}$$

$$\text{Ex. } \sqrt{9} = 9^{\frac{1}{2}} = 3$$

$$\sqrt{x^2} = |x|$$
