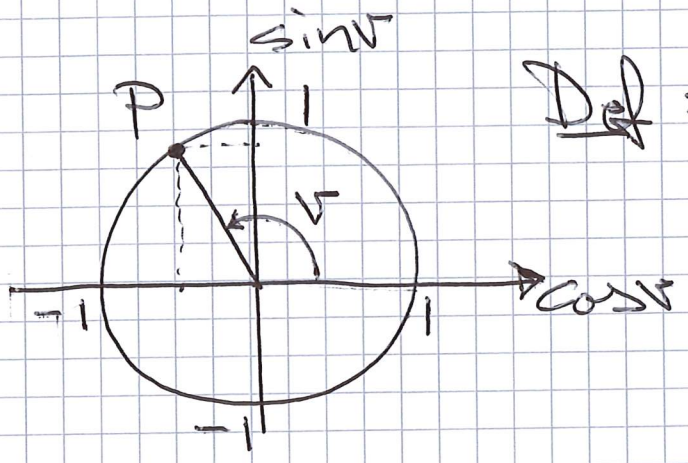
Fö 4 | Trigonometriska funktioner

läs 2.4. Se även Fö 3 nollep.

Rita enhetscirkeln.

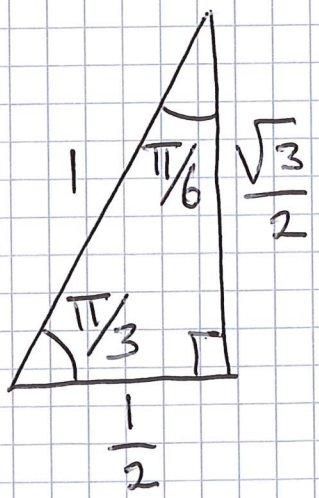


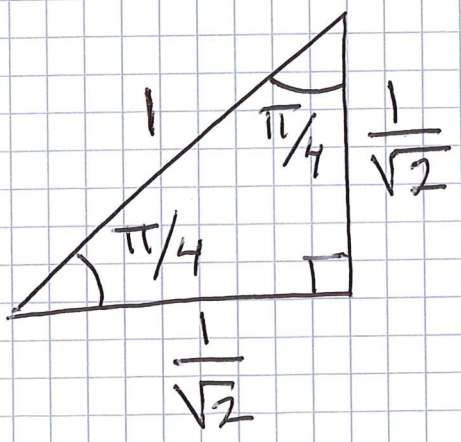
Def: $P = (\cos v, \sin v)$

t.ex

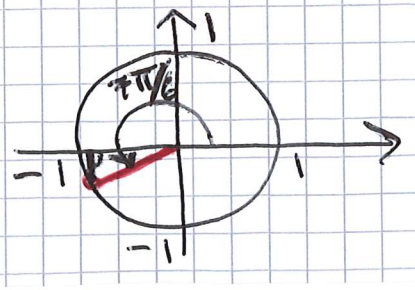
$\cos \pi = -1$

$\sin\left(-\frac{5\pi}{2}\right) = -1$



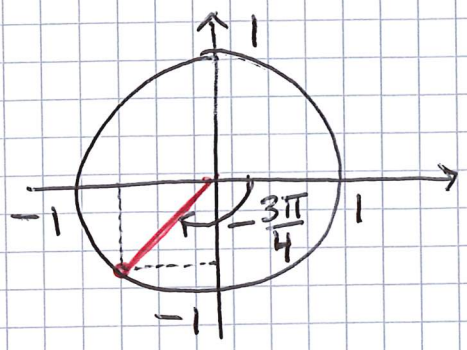


Ex. $\sin \frac{7\pi}{6} = -\frac{1}{2}$



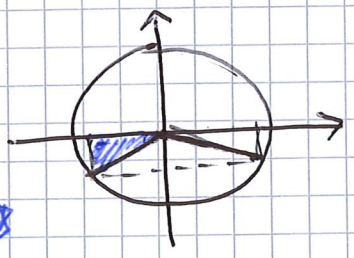
Rita.

Ex. $\tan\left(-\frac{3\pi}{4}\right) = \frac{\sin\left(-\frac{3\pi}{4}\right)}{\cos\left(-\frac{3\pi}{4}\right)} = \frac{-\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = 1$

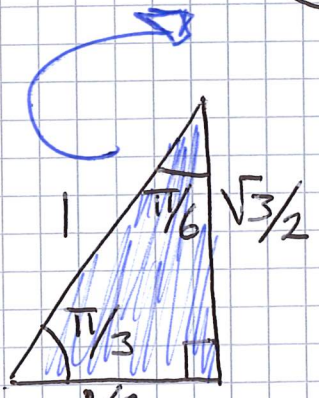


Ex. Lös ekv. $\sin x = -\frac{1}{2}$

L:



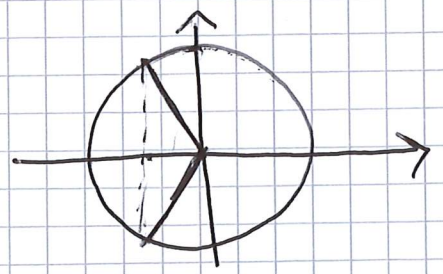
$$\sin x = -\frac{1}{2} \iff x = \begin{cases} \frac{7\pi}{6} + n2\pi \\ -\frac{\pi}{6} + n2\pi \end{cases}, n \in \mathbb{Z}.$$



Ex. Lös ekv. $\cos 3x = -\frac{1}{2}$

obs! Direkt

L. $\cos 3x = -\frac{1}{2} \iff 3x = \pm\frac{2\pi}{3} + n2\pi$



$$\iff x = \pm\frac{2\pi}{9} + \frac{n2\pi}{3}, n \in \mathbb{Z}.$$

Trig. formler

Se nollep. Fö 3.

$$\sin^2 x + \cos^2 x = 1 \quad \text{"trig. ettan"}$$

obs!

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin 2x = 2 \sin x \cos x \qquad \text{mfl.}$$

Ex. Lös ekv. $\cos 2x + \cos x + 1 = 0$

L: $2\cos^2 x - 1 + \cos x + 1 = 0$

$2\cos^2 x + \cos x = 0$

$\cos x (2\cos x + 1) = 0$

Bryt ut gemensam faktor.

fall 1. $\cos x = 0$

$x = \pm \frac{\pi}{2} + n 2\pi$

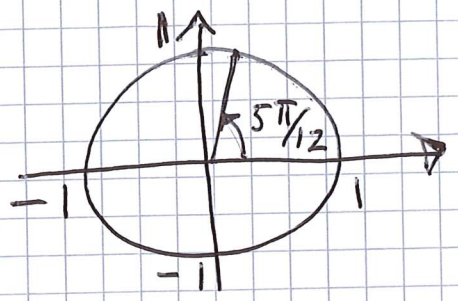
fall 2. $\cos x = -\frac{1}{2}$

$x = \pm \frac{2\pi}{3} + n 2\pi$

$n \in \mathbb{Z}$.

Ex. Beräkna $\sin\frac{5\pi}{12}$

L: $\sin\frac{5\pi}{12} = \sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right) =$



$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6} =$

$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

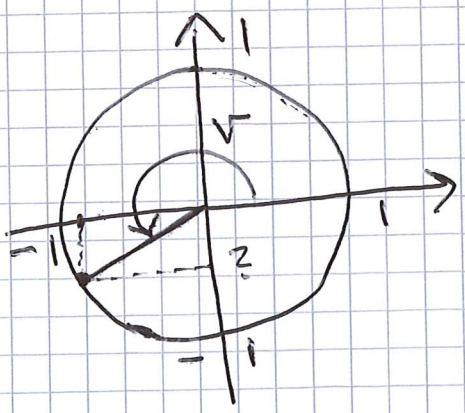
Ex. Räkna ut $\cos(v+\frac{\pi}{6})$

då $\cos v = -\frac{2}{3}$ och $\pi < v < 2\pi$

L: $\cos(v+\frac{\pi}{6}) = \cos v\cos\frac{\pi}{6} - \sin v\sin\frac{\pi}{6}$

$= -\frac{2}{3}\cdot\frac{\sqrt{3}}{2} - \sin v\cdot\frac{1}{2} = -\frac{\sqrt{3}}{3} + \frac{\sqrt{5}}{3}\cdot\frac{1}{2} =$

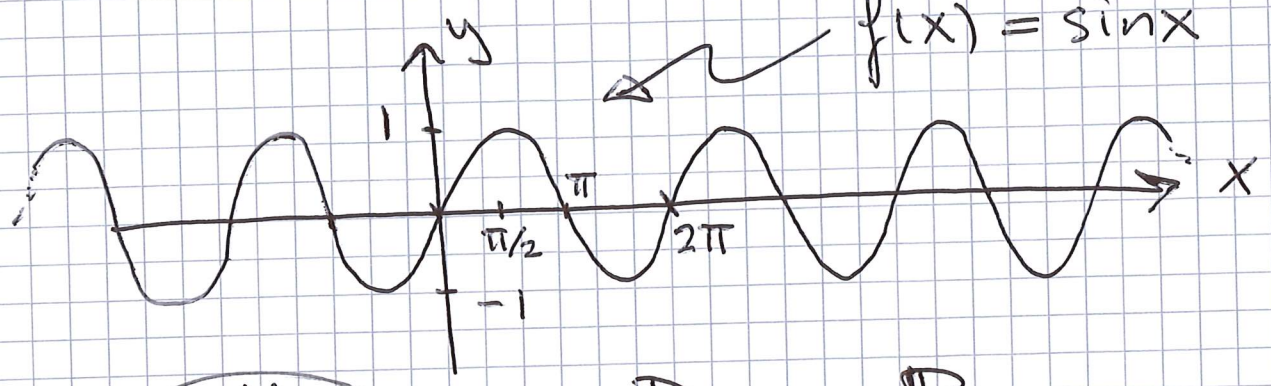
$= \frac{\sqrt{5}-2\sqrt{3}}{6}$.



$\sin^2 v + \left(-\frac{2}{3}\right)^2 = 1^2$

$\sin v = \pm\frac{\sqrt{5}}{3}$

De trigonometriska funktionerna



udda.

period = 2π.

$D_{\sin x} = \mathbb{R}$.

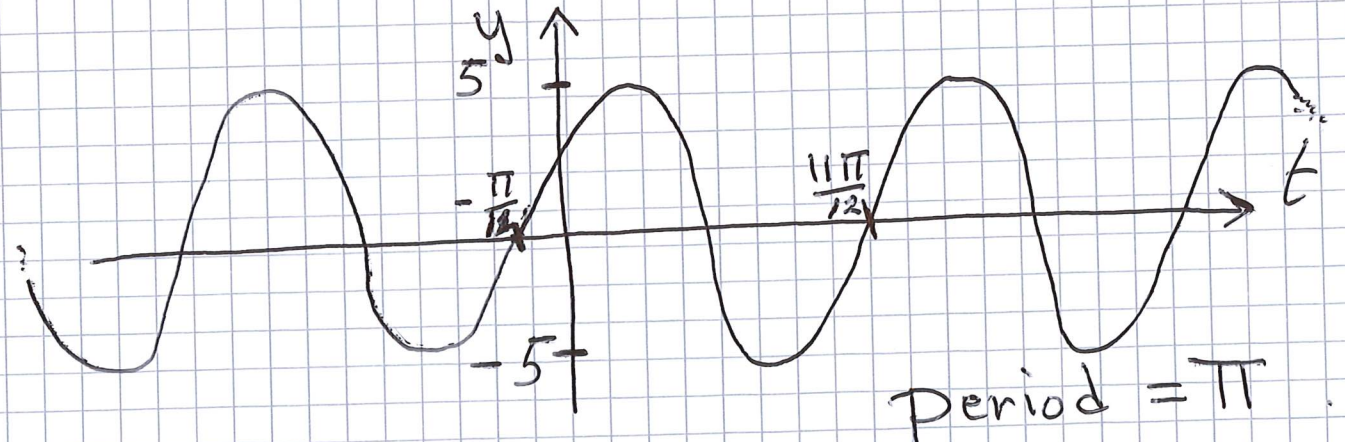
$V_{\sin x} = [-1, 1]$

Övn: Skissa $f(x) = \cos x$.

Amplitud - fasvinkelform

$$f(t) = A\sin(\omega t + \alpha)$$

Ex. Skissa $f(t) = 5\sin(2t + \frac{\pi}{6})$

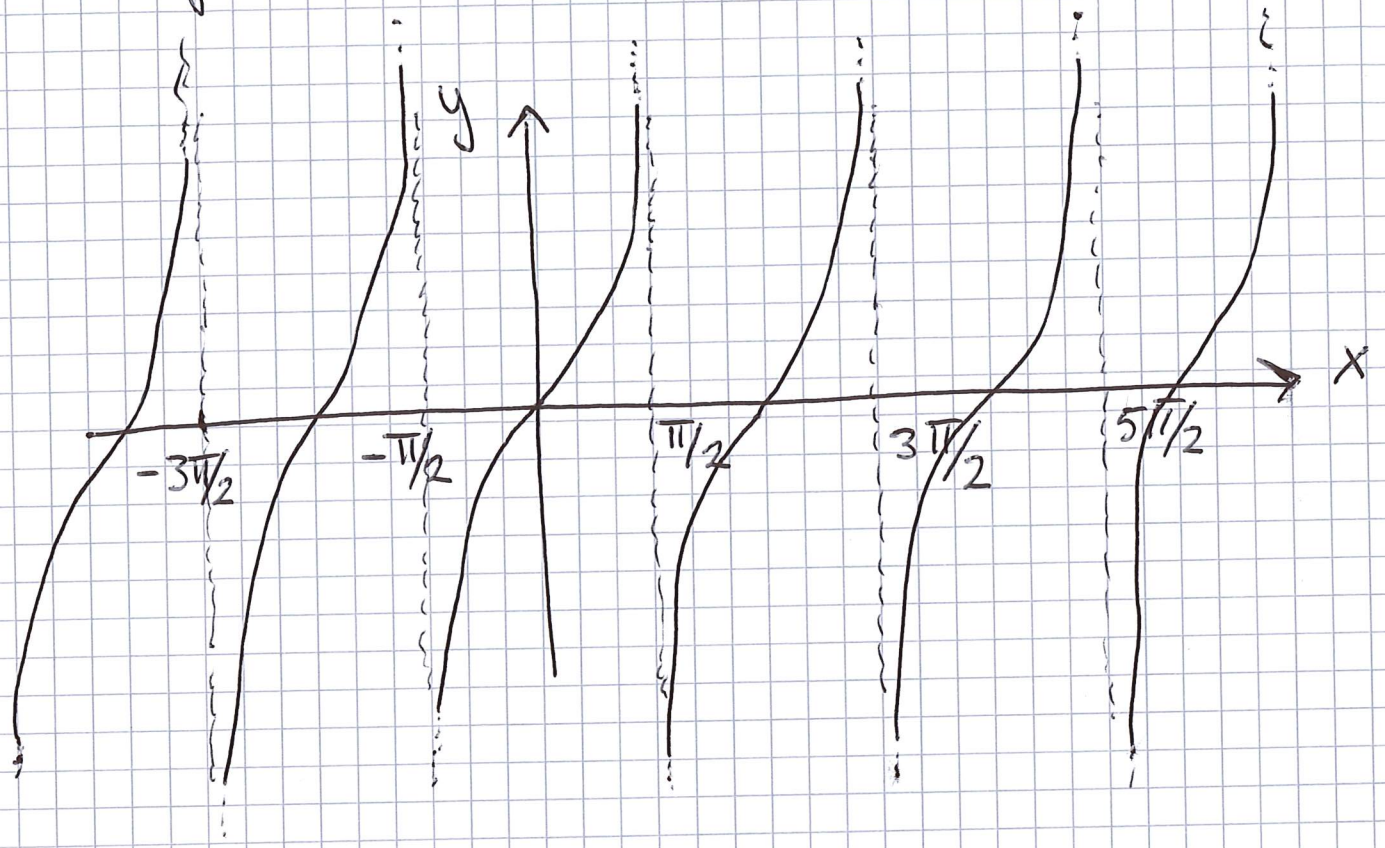


period = π.

Funktionerna tan och cot

$n \in \mathbb{Z}$.

$$f(x) = \tan x = \frac{\sin x}{\cos x}, \quad x \neq \frac{\pi}{2} + n\pi$$



$$\text{period} = \pi.$$

$$y = \tan x, \quad x \neq \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}$$

$$V_{\tan x} =]-\infty, \infty[$$

Övn: Rita $f(x) = \cot x$

$$\cot x = \frac{\cos x}{\sin x}, \quad x \neq n\pi, \quad n \in \mathbb{Z}.$$

Hjälpvinkelmetoden

Skriv om $a\sin\omega x + b\cos\omega x$ på formen $A\sin(\omega x + \alpha)$.

Kom ihåg! $\boxed{\sin(x+y) = \sin x\cos y + \cos x\sin y}$

$$A\sin(\omega x+\alpha) = A(\sin\omega x\cos\alpha + \cos\omega x\sin\alpha) =$$

$$= \underbrace{A\cos\alpha}_{=a}\sin\omega x + \underbrace{A\sin\alpha}_{=b}\cos\omega x =$$

$$= a\sin\omega x + b\cos\omega x$$

ger

$$\begin{cases} A\cos\alpha = a \\ A\sin\alpha = b \end{cases} \iff \begin{matrix} A = \sqrt{a^2+b^2} \\ \tan\alpha = \frac{b}{a} \end{matrix} \quad (A>0)$$

Ex. Skriv $\sqrt{3}\sin x + \cos x$ på formen $A\sin(\omega x+\alpha)$.

L: $\sqrt{3}\sin x + \cos x = 2\left(\underbrace{\frac{\sqrt{3}}{2}}_{=\cos\alpha}\sin x + \underbrace{\frac{1}{2}}_{=\sin\alpha}\cos x\right)$

$A = \sqrt{(\sqrt{3})^2+1^2}$

$\alpha = \frac{\pi}{6}$

Svar: $2\sin\left(x+\frac{\pi}{6}\right)$

Ex. Skriv $\sqrt{3}\sin 2x - 3\cos 2x$ på formen $A\sin(\omega x + \alpha)$.

L: $\omega = 2$

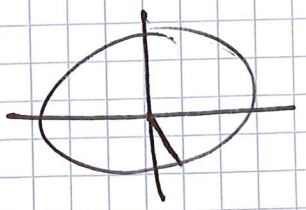
$$A = \sqrt{(\sqrt{3})^2 + (-3)^2} = \sqrt{12} = 2\sqrt{3}$$

$$\sqrt{3}\sin 2x - 3\cos 2x = 2\sqrt{3}\left(\frac{1}{2}\sin 2x - \frac{\sqrt{3}}{2}\cos 2x\right) =$$

↑ Bryt alltid först ut A.

$$= 2\sqrt{3}\left(\cos\alpha \sin 2x + \sin\alpha \cos 2x\right)$$

$$\begin{cases} \cos\alpha = \frac{1}{2} \\ \sin\alpha = -\frac{\sqrt{3}}{2} \end{cases}$$



$$\alpha = \frac{5\pi}{3}$$

en räcker.

Svar: $\sqrt{3}\sin 2x - 3\cos 2x = 2\sqrt{3}\sin\left(2x + \frac{5\pi}{3}\right)$

Ex. Lös ekv. $\sqrt{3}\sin x - \cos x = 1$

L: Först hjälpvinkelmetoden.

$$\sqrt{3}\sin x - \cos x = 2\left(\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x\right) =$$

$\frac{\sqrt{3}}{2} = \cos\alpha$ $\quad$ $\frac{1}{2} = \sin\alpha$

$\alpha = -\frac{\pi}{6}$.

↑ bryt ut

$$A = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$$

$$= 2\sin\left(x - \frac{\pi}{6}\right)$$

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forts. $\sqrt{3}\sin x - \cos x = 1 \iff$

$$2\sin\left(x - \frac{\pi}{6}\right) = 1 \iff$$

$$\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$$

$$\iff x - \frac{\pi}{6} = \begin{cases} \frac{\pi}{6} + n2\pi \\ \frac{5\pi}{6} + n2\pi \end{cases}, \quad n \in \mathbb{Z}.$$

Svar:

$$\iff x = \begin{cases} \frac{\pi}{3} + n2\pi \\ \pi + n2\pi \end{cases}, \quad n \in \mathbb{Z}.$$

Obs! Kontrollera.

Ex. För vilka $x \in [0, 2\pi]$ gäller att $\sqrt{3}\cos x + \sin x > \sqrt{3}$?

L: $\sqrt{3}\cos x + \sin x > \sqrt{3}$

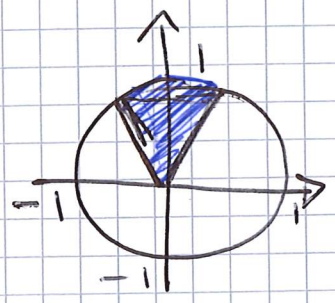
$$2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right) > \sqrt{3}$$

$\frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$ $\quad$ $\frac{1}{2} = \cos\frac{\pi}{3}$

$$2\sin\left(x + \frac{\pi}{3}\right) > \sqrt{3}$$

$$\sin\left(x + \frac{\pi}{3}\right) > \frac{\sqrt{3}}{2}$$

$\Leftrightarrow$



$$\frac{\pi}{3} + n2\pi < x + \frac{\pi}{3} < \frac{2\pi}{3} + n2\pi$$

$$n2\pi < x < \frac{\pi}{3} + n2\pi$$

där $n = 0$

Svar: $0 < x < \frac{\pi}{3}$