

Fö 6

Gränsvärde

Läs 3.1-3.2

$$\frac{x+2}{x} = 1 + \frac{2}{x} \rightarrow 1$$

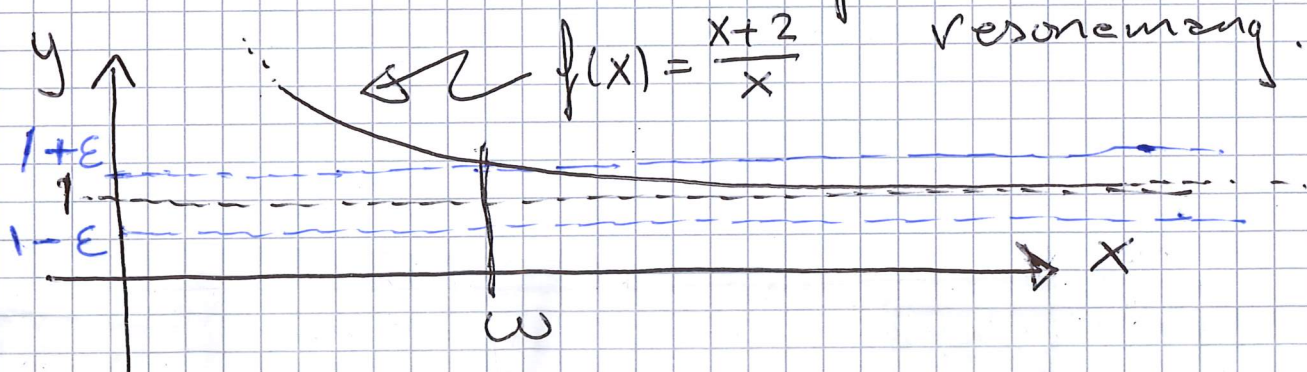
$\rightarrow 0$

då $x \rightarrow \infty$

$$\lim_{x \rightarrow \infty} \frac{x+2}{x} = 1$$

limit = gräns

Se boken
för ϵ, ω
resonemang.



Ta $\epsilon > 0$ godtyckligt.

$$\left| \frac{x+2}{x} - 1 \right| = \left| 1 + \frac{2}{x} - 1 \right| = \frac{2}{x} < \epsilon$$

$$\Leftrightarrow x > \frac{2}{\epsilon} = \omega$$

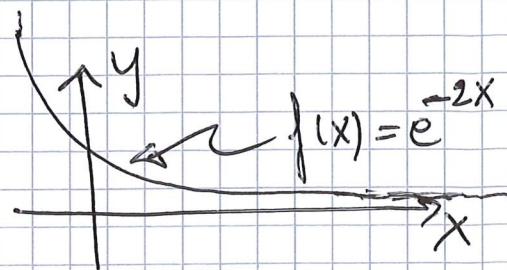
$x > 0$

ϵ = noggrannheten

Ex. $\lim_{x \rightarrow \infty} e^{-2x} = 0$

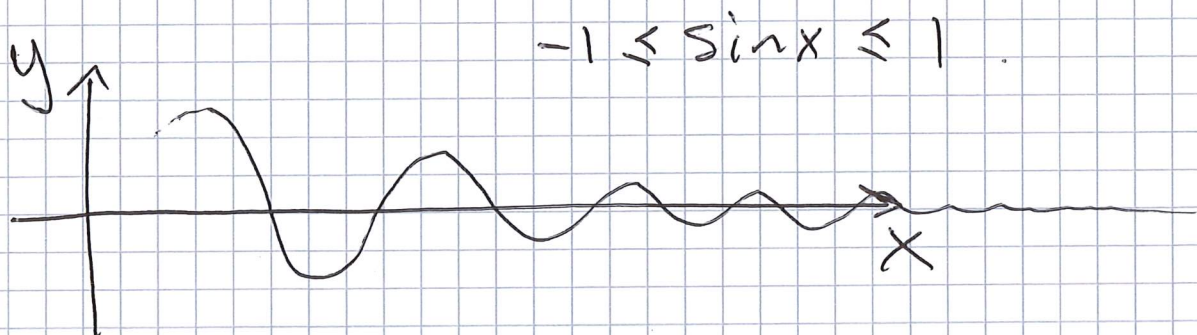
$$e^{-2x} = \frac{1}{e^{2x}}$$

Skissa grafer.

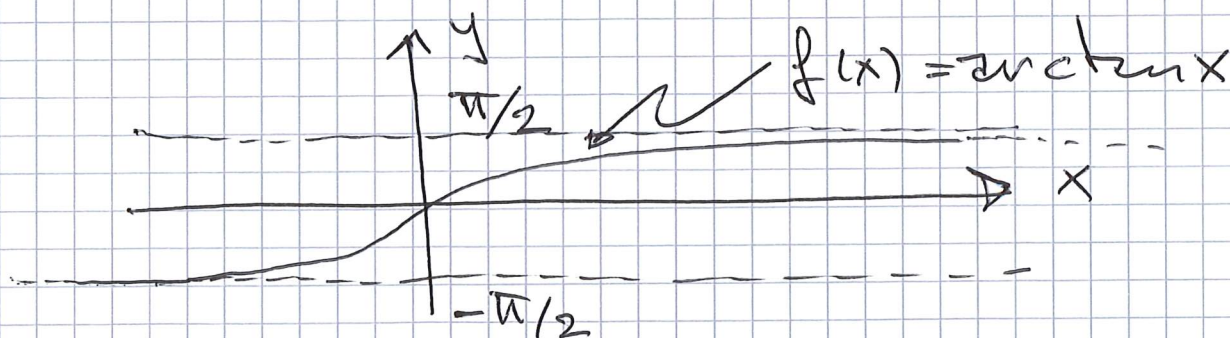


Ex. $\lim_{x \rightarrow \infty} \sin x$ exist. ej.

Ex. $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$



Ex. $\lim_{x \rightarrow -\infty} \arctan x = -\frac{\pi}{2}$



Räkna ut ...

Ex. $\lim_{x \rightarrow \infty} \frac{x^2 - 3x + 100}{5x^2 + 2x + 7} = \frac{\text{Bryt ut}}{\text{dominerande}} \frac{\text{faktor}}{\text{faktor}}$

$$= \lim_{x \rightarrow \infty} \frac{\cancel{x^2} \left(1 - \overset{\rightarrow 0}{\frac{3}{x}} + \overset{\rightarrow 0}{\frac{100}{x^2}} \right)}{\cancel{x^2} \left(5 + \overset{\rightarrow 0}{\frac{2}{x}} + \overset{\rightarrow 0}{\frac{7}{x^2}} \right)} = \frac{1}{5}$$

Svar:

$$\lim_{x \rightarrow \infty} \frac{x^2 - 3x + 100}{5x^2 + 2x + 7} = \frac{1}{5}$$

obs!

Grensvärde

Alt. skrivsätt.

$$\frac{x^2 - 3x + 100}{5x^2 + 2x + 7} = \frac{\cancel{x^2} \left(1 - \overset{\rightarrow 0}{\frac{3}{x}} + \overset{\rightarrow 0}{\frac{100}{x^2}} \right)}{\cancel{x^2} \left(5 + \overset{\rightarrow 0}{\frac{2}{x}} + \overset{\rightarrow 0}{\frac{7}{x^2}} \right)} \rightarrow \frac{1}{5}$$

da $x \rightarrow \infty$.

/2s $\rightarrow$ som
går emot.

Viktigast. Bryt ut dominerande faktor.

men.

dominant.

Ex. $x^3 + 2x^2 - 5x = x^3 \left(1 + \frac{2}{x} - \frac{5}{x^2}\right)$
då $x \rightarrow \pm \infty$.

dominant.
 $x^3 + 2x^2 - 5x = x(x^2 + 2x - 5)$
då $x \rightarrow 0$.

Ex. Beräkna $\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 + 1}}{5x - 1}$

$$L = \frac{\sqrt{3x^2 + 1}}{5x - 1} = \frac{\sqrt{x^2 \left(3 + \frac{1}{x^2}\right)}}{x \left(5 - \frac{1}{x}\right)} =$$

$$= \frac{\sqrt{x^2} \sqrt{3 + \frac{1}{x^2}}}{x \left(5 - \frac{1}{x}\right)} = \frac{x \sqrt{3 + \frac{1}{x^2}}}{x \left(5 - \frac{1}{x}\right)} \rightarrow \frac{\sqrt{3}}{5}$$

då $x \rightarrow \infty$

$$\sqrt{x^2} = |x| = \begin{cases} x & \text{om } x \geq 0 \\ -x & \text{om } x < 0 \end{cases}$$

Svar: $\frac{\sqrt{3}}{5}$

obs!

$$\text{Ex. } \lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2+1}}{5x-1} = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2} \sqrt{3+\frac{1}{x^2}}}{x(5-\frac{1}{x})} =$$

$$\stackrel{=}{=} \lim_{x \rightarrow -\infty} \frac{-x \sqrt{3+\frac{1}{x^2} \downarrow 0}}{x(5-\frac{1}{x} \rightarrow 0)} = -\frac{\sqrt{3}}{5}$$

$$\sqrt{x^2} = -x \text{ da } x < 0.$$

Vänligt jobb med konjugatet.

$$\text{Ex. Beräkna } \lim_{x \rightarrow \infty} (\sqrt{x^2+6x} - x)$$

$$L: \sqrt{x^2+6x} - x = \frac{(\sqrt{x^2+6x} - x)(\sqrt{x^2+6x} + x)}{\sqrt{x^2+6x} + x} =$$

$$= \frac{x^2+6x-x^2}{\sqrt{x^2(1+\frac{6}{x})} + x} = \frac{6x}{\sqrt{x^2} \sqrt{1+\frac{6}{x}} + x} =$$

$$= \frac{6x}{x(\sqrt{1+\frac{6}{x} \rightarrow 0} + 1)} \rightarrow \frac{6}{\sqrt{1}+1} = 3 \text{ da } x \rightarrow \infty$$

Svar: Gr.v = 3

Gränsvärden då $x \rightarrow a$.

Ex. Beräkna $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 + x - 2}$

L: $\frac{x^2 - 3x + 2}{x^2 + x - 2} = \frac{(x-1)(x-2)}{(x-1)(x+2)} =$

Faktorsatsen

ofta.
ofta.

$= \frac{x-2}{x+2} \rightarrow \frac{-1}{3} \text{ då } x \rightarrow 1$

obs! går emot $\rightarrow$

Svar: $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 + x - 2} = -\frac{1}{3}$

obs! likhet här.

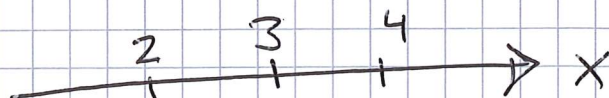
Se upp med "typ"

$\left[\frac{0}{0}\right], \left[\frac{\infty}{\infty}\right], [0 \cdot \infty], [\infty - \infty]$

Svårt.

Höger och Vänstergränsvärde.

Ex. $\lim_{x \rightarrow 3^+} \frac{x+2}{x-3} = \infty$



$$\lim_{x \rightarrow 3^-} \frac{x+2}{x-3} = -\infty$$

$\lim_{x \rightarrow a} f(x)$ exist. $\Leftrightarrow$ Både $\lim_{x \rightarrow a^+} f(x)$ och $\lim_{x \rightarrow a^-} f(x)$ existerar och är lika.

Ex. $\lim_{x \rightarrow 0^-} \frac{x+3}{x^2-2x} = \lim_{x \rightarrow 0^-} \frac{x+3}{x(x-2)} = \infty$

$$\text{Ex. } \lim_{x \rightarrow \infty} \frac{\ln x^4}{\ln 3x} = 4$$

$$\text{L: } \frac{\ln x^4}{\ln 3x} = \frac{4 \ln x}{\ln 3 + \ln x} = \frac{4 \cancel{\ln x}}{\ln x \left(\frac{\ln 3}{\ln x} + 1 \right)} \rightarrow 4$$

$\xrightarrow{0} \text{dix} \rightarrow \infty$

$$\text{Ex. } \lim_{x \rightarrow 1} \frac{x^5 - 1}{x^3 - 1} = \frac{5}{3}$$

$$\text{L } \frac{x^5 - 1}{x^3 - 1} = \frac{(x-1)(x^4 + x^3 + x^2 + x + 1)}{(x-1)(x^2 + x + 1)} \rightarrow \frac{5}{3}$$

$\xrightarrow{0} \text{dix} \rightarrow 1$

$$\text{Ex. } \lim_{x \rightarrow 2} \frac{\sqrt{x-1} - 1}{x-2} = \frac{1}{2}$$

$$\text{L: } \frac{(\sqrt{x-1} - 1)(\sqrt{x-1} + 1)}{(x-2)(\sqrt{x-1} + 1)} =$$

$$= \frac{x-1-1}{(x-2)(\sqrt{x-1} + 1)} = \frac{\cancel{x-2}}{(x-2)(\sqrt{x-1} + 1)} \rightarrow \frac{1}{2}$$

$\xrightarrow{0} \text{dix} \rightarrow 2$

Ex. $\lim_{x \rightarrow \infty} \frac{\ln(x + x^5)}{\ln(x^2 + x^3)}$

L: Tänk dominerande faktor.
men vad dominerar?

$$\lim_{x \rightarrow \infty} \frac{\ln(x^5(\frac{1}{x^4} + 1))}{\ln(x^3(\frac{1}{x} + 1))} = \dots = \frac{5}{3}$$

Ex. $\lim_{x \rightarrow 0^+} \frac{\ln(x^2 + x^5)}{\ln(x^3 + x^4)}$

L: Vad dominerar?

$$\lim_{x \rightarrow 0^+} \frac{\ln(x^2(1 + x^3))}{\ln(x^3(1 + x))} = \dots = \frac{2}{3}$$
