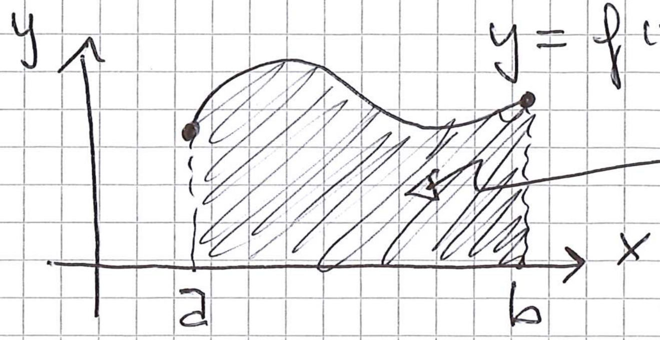
Fö3 | Läs 6.1–6.4.

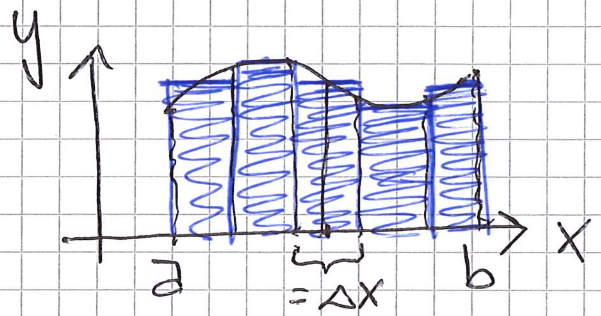
Integraler

derivatan

$y = f(x)$, $a \leq x \leq b$.



Arean söks
$= I$.



Dela intervallet $[a,b]$ i n st bitar

Δx = bredden.

$f(x_i)$ = höjden.

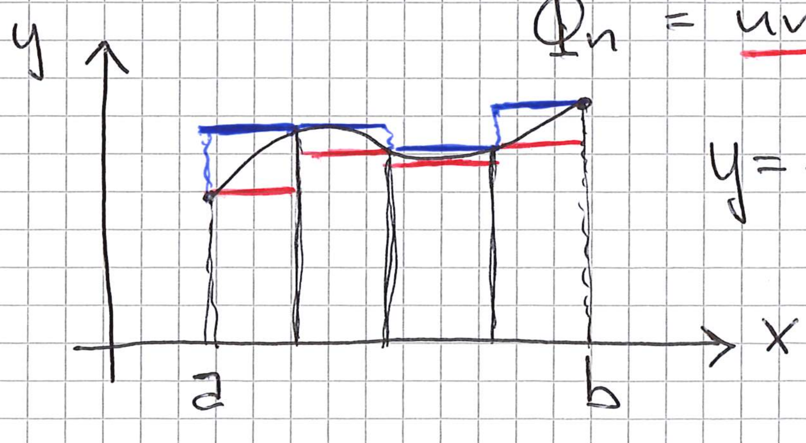
$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = I = \int_a^b f(x)\,dx$$

läs $\int$ som ett $\sum$-tecken "summering"

Se boken.

Ψ_n = övertrappa

Φ_n = undertrappa.



$y = f(x)$, $a \leq x \leq b$ kontinuerlig.

$$I(\Phi_n) \leq I \leq I(\Psi_n)$$

låt $n \to \infty$. Se sid 277-278.

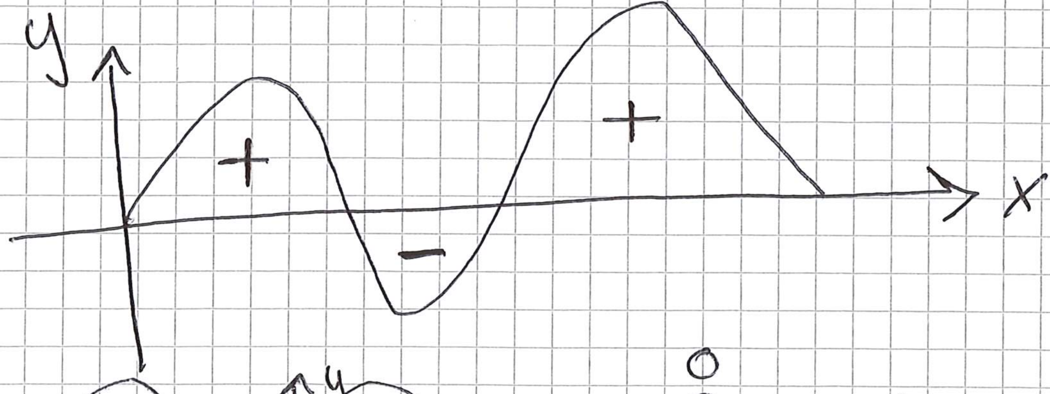
Sats 6.2. Räknelagar.

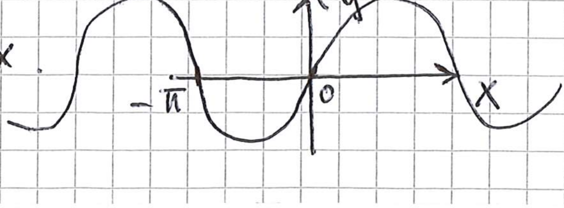
$$\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx \quad , \quad c = \text{konstant}.$$

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx \; .$$

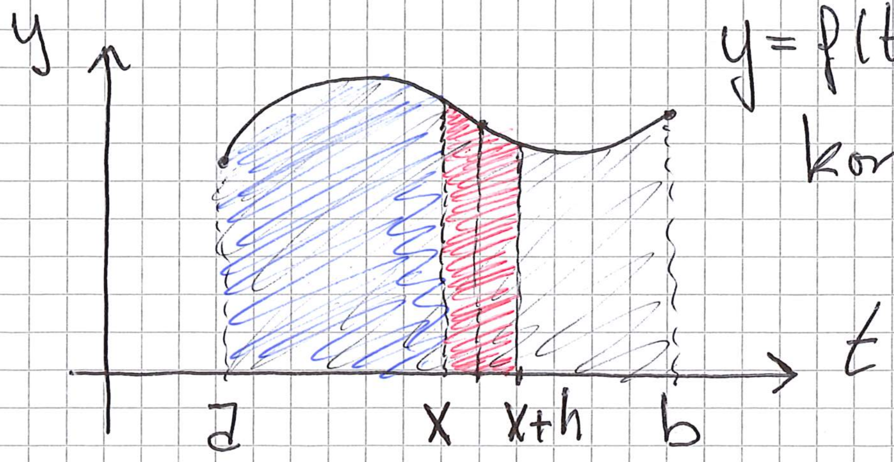
Kom ihåg: $\int$ är summering.



Ex. 

$$\int_{-\pi}^{0} \sin x\,dx < 0$$

Sambandet mellan integraler och derivator.



$y = f(t)$, $a \leq t \leq b$ kontinuerlig.

Bilda $S(x) = \int_a^x f(t)\,dt$

Medelvärdessatsen för integraler.

$$S(x+h) - S(x) = f(\xi)(x+h-x)$$

där $x \leq \xi \leq x+h$

$$\lim_{h \to 0} \frac{S(x+h) - S(x)}{h} = \lim_{\xi \to x} f(\xi)$$

$$S'(x) = f(x)$$

$$S(x) = F(x) + C$$

$x = a$ ger

$0 = F(a) + C$

$\Leftrightarrow$

$C = -F(a)$

$x = b$ ger $S(b) = F(b) - F(a)$

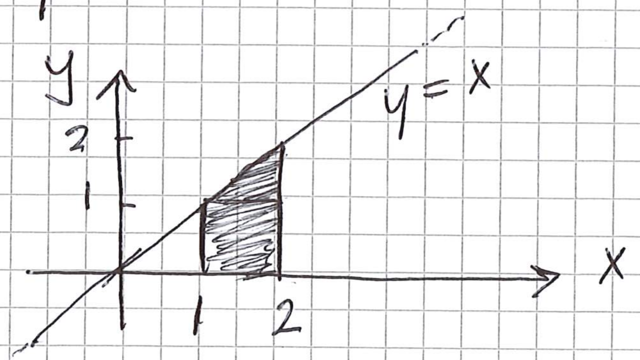
$$\int_a^b f(t)\,dt = F(b) - F(a)$$

Skrivsätt:

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$

Ex.

$$\int_1^2 x\,dx = \left[\frac{x^2}{2}\right]_1^2 = \frac{2^2}{2} - \frac{1^2}{2} = \frac{3}{2}$$



Ex.

$$\int_{-\pi}^0 \sin x\,dx = \Big[-\cos x\Big]_{-\pi}^0 =$$

$$= -\cos 0 - (-\cos(-\pi)) = -1 - 1 = -2$$

Ex. $$\int_0^1 \frac{x+3}{x^2+3x+2}\,dx = \int_0^1 \frac{x+3}{(x+1)(x+2)}\,dx =$$

$$\underset{\text{PBU}}{=} \int_0^1 \left(\frac{A}{x+1} + \frac{B}{x+2}\right) dx = \left/ \begin{matrix} \text{H.P. ger} \\ A=2 \\ B=-1 \end{matrix} \right/ =$$

$$= \int_0^1 \left(\frac{2}{x+1} - \frac{1}{x+2}\right) dx = \Big[2\ln|x+1| - \ln|x+2|\Big]_0^1 =$$

$$= 2\ln 2 - \ln 3 - \big(2\ln 1 - \ln 2\big) =$$

Viktig parentes.

$$= 2\ln 2 - \ln 3 + \ln 2 = 3\ln 2 - \ln 3 =$$

$$= \ln \frac{8}{3}$$

Ex. $$\int_0^{\pi/2} x \cos x\,dx \underset{\text{p.i}}{=} \Big[x\sin x\Big]_0^{\pi/2} - \int_0^{\pi/2} \sin x\,dx =$$

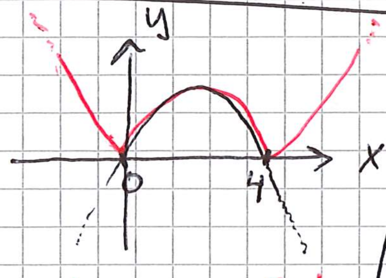
$$= \frac{\pi}{2} - 0 + \Big[\cos x\Big]_0^{\pi/2} = \frac{\pi}{2} - 1$$

Ex. $\displaystyle\int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx = \left/ \begin{array}{l} t = x^2+9 \\ \dfrac{dt}{dx} = 2x \\ \dfrac{dt}{2} = x\,dx \\ x=0 \Rightarrow t=9 \\ x=4 \Rightarrow t=25 \end{array} \right/ =$

$$= \int_9^{25} \frac{\frac{dt}{2}}{\sqrt{t}} = \int_9^{25} \frac{1}{2\sqrt{t}}\,dt = \Big[\sqrt{t}\Big]_9^{25} =$$

$$= 5 - 3 = 2$$

Ex. $\displaystyle\int_{-1}^{7} |4x - x^2|\,dx = \Big/$



$y = |4x - x^2|$ $\Big/ =$

$$= \int_{-1}^{0} -(4x-x^2)\,dx + \int_0^4 (4x-x^2)\,dx + \int_4^7 -(4x-x^2)\,dx$$

$$= \int_{-1}^{0} (x^2-4x)\,dx + \int_0^4 (4x-x^2)\,dx + \int_4^7 (x^2-4x)\,dx =$$

= ——— osv.

Exempel 6.11. Sid 292.

$$\int_0^3 |x-1|e^{-x}\,dx = \int_0^1 -(x-1)e^{-x}\,dx + \int_1^3 (x-1)e^{-x}\,dx =$$

$= \ldots\ldots$

Ex. $$\int_1^e \ln x\,dx = \int_1^e 1\cdot \ln x\,dx \underset{p.i}{=}$$

$$= \Big[x\ln x\Big]_1^e - \int_1^e x\cdot\frac{1}{x}\,dx =$$

$$= e\underbrace{\ln e}_{=1} - \underbrace{\ln 1}_{=0} - \Big[x\Big]_1^e = e-e+1 = 1$$

Ex. $$\int_0^{\pi/2} \sin 2x\, e^{\cos x}\,dx =$$

$$= \int_0^{\pi/2} 2\sin x\cos x\, e^{\cos x}\,dx = \left/ \begin{aligned} t &= \cos x \\ \frac{dt}{dx} &= -\sin x \\ dt &= -\sin x\,dx \\ (-1)dt &= \sin x\,dx \end{aligned} \right/ =$$

Byt även gränser.

$$= \int_1^0 2te^t(-1)\,dt = \int_0^1 \underset{\downarrow}{2t}\,\underset{\uparrow}{e^t}\,dt \underset{\text{p.i}}{=}$$

$$= \left[e^t 2t\right]_0^1 - \int_0^1 e^t 2\,dt =$$

$$= 2e - 0 - \left[2e^t\right]_0^1 = 2e - 2e + 2 = 2$$

Ex. $$\int_0^\pi \frac{\sin x}{3+\sin^2 x}\,dx = \int_0^\pi \frac{\sin x}{3+1-\cos^2 x}\,dx =$$

$$= \int_0^\pi \frac{\sin x}{4-\cos^2 x}\,dx = \left/ \begin{array}{l} t = \cos x \\ \frac{dt}{dx} = -\sin x \\ -dt = \sin x\,dx \end{array} \right| =$$

$$= \int_1^{-1} \frac{-dt}{4-t^2} = \int_{-1}^1 \frac{1}{4-t^2}\,dt =$$

$$= \int_{-1}^1 \frac{1}{(2+t)(2-t)}\,dt =$$

$$= \int_{-1}^{1} \left(\frac{\frac{1}{4}}{2+t} + \frac{\frac{1}{4}}{2-t} \right) dt =$$

$$= \frac{1}{4} \Big[\ln|2+t| + \underbrace{\ln|2-t|}_{-1} \Big]_{-1}^{1} =$$

obs!
lätt att missa.

$$= \frac{1}{4} \Big(\ln 3 - (-\ln 3) \Big) = \frac{\ln 3}{2}$$

Ex. $$\int_0^1 \cos^2 \pi x \, dx = \int_0^1 \frac{1 + \cos 2\pi x}{2} \, dx =$$

$$= \frac{1}{2} \left[x + \frac{\sin 2\pi x}{2\pi} \right]_0^1 =$$

$$= \frac{1}{2} \Big(1 + 0 - (0 + 0) \Big) = \frac{1}{2}$$
