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Things allowed (Hjälpmedel): a calculator.

Scores rating (Betygsgränser): 8-11 points giving rate 3; 11.5-14.5 points giving rate 4; 15-18 points giving rate 5.

## 1 (3 points)

Two events  $A$  and  $B$  are independent with  $P(A) = 0.3$  and  $P(B) = 0.5$ .

(1.1) (0.5p) Find the probability  $P(A \cap B)$ .

(1.2) (0.5p) Find the probability  $P(A \cup B)$ .

(1.3) (1p) Find the conditional probability  $P(A \mid A \cup B)$ .

(1.4) (1p) Find the probability  $P(A' \cap B)$ , where  $A'$  is the complement of  $A$ .

## 2 (3 points)

A PHEV (plug-in hybrid electric vehicle) car has two engines: a diesel engine and an electric engine. Let  $X$  be lifespan (in years) of the diesel engine, and  $Y$  lifespan (in years) of the electric engine. Assume that  $X \sim \text{Exp}(\frac{1}{10})$  and  $Y \sim \text{Exp}(\frac{1}{12})$  are independent and exponential random variables, namely  $X$  has mean 10 years and  $Y$  has mean 12 years.

(2.1) (1.5p) Find the probability that the electric engine fails before the diesel engine, namely,  $P(X > Y)$ .

(2.2) (1.5p) Find the probability that the total lifespan of the PHEV is at least 25 years, namely,  $P(X + Y \geq 25)$ .

## 3 (3 points)

Every week Emma goes to gym a few times. Let  $X$  be the distribution of numbers of times that Emma goes to gym in one week. Assume that  $X$  is a discrete random variable with the following probability mass function:

$X$	1	2	3
$p(x)$	0.2	0.5	0.3

For example,  $P(X = 1) = 0.2$  means that in one week Emma goes to gym only 1 time with probability 20%.

(3.1) (1p) Find the mean  $\mu = E(X)$  and variance  $\sigma^2 = V(X)$  of  $X$ .

(3.2) (2p) Assume that gym activities for Emma in different weeks are independent. Find the probability that Emma goes to gym at least 120 times in the next 52 weeks? (Hint: Central Limit Theorem (CLT))

## 4 (3 points)

(4.1) (1p) A population  $X$  is a continuous random variable with a probability density function

$$f(x) = \frac{1}{a - 10}, \quad \text{for } 10 \leq x \leq a,$$

where  $a > 10$  is unknown. A sample  $\{x_1, x_2, \dots, x_n\}$  is taken from this population. Use the method of moments to find a point estimate  $\hat{a}_{MM}$  of  $a$ .

(4.2) (2p) Another population  $Y$  is a continuous random variable with a probability density function

$$f(y) = \frac{1}{b - c}, \quad \text{for } c \leq y \leq b,$$

where  $b$  and  $c$  (with  $b > c$ ) are unknown. A sample  $\{y_1, y_2, \dots, y_n\}$  is taken from this population. Use the maximum-likelihood method to find point estimates  $\hat{b}_{ML}$  and  $\hat{c}_{ML}$  of  $b$  and  $c$ .

## 5 (3 points)

A new medicine is to be introduced. It is believed that the risk of side effects is affected by the dose size. To study this, 600 patients are randomly selected and divided into two groups with each group consisting of 300 patients. Two different dose sizes are given to these two groups and the results are:

Groups	Dose sizes	How many patients have side effects?
Group A: 300 patients	0.20	54
Group B: 300 patients	0.30	95

Let  $p_A$  and  $p_B$  denote the proportions of patients having side effects with the above mentioned two dose sizes. Does the above sample provide any evidence that the risk of side effects is affected by the dose size? Answer this using an appropriate two-sided 95% confidence interval. (Hint: Do NOT use the method of Hypotheses Test!).

## 6 (3 points)

A coffee supplier has plenty of 500 g coffee boxes. The coffee supplier claims that the distribution  $X$  of weights of all these coffee boxes is  $X \sim N(500, 2^2)$ . To check this claim, a randomly selected 100 coffee boxes give the following:

Weight of coffee boxes	< 498	498 – 502	> 502
Number of coffee boxes	39	45	16

The above results say that, among the randomly selected 100 coffee boxes: there are 39 coffee boxes whose weights are less than 498 g; there are 45 coffee boxes whose weights are between 498 g and 502 g, and there are 16 coffee boxes whose weights are more than 502 g. Use  $\chi^2$ -test with a significance level 5% to check whether or not the sample provides any evidence showing the coffee supplier's claim is wrong.

## 1. Basic probability

(1.1) Conditional probability  $P(A|B) = \frac{P(A \cap B)}{P(B)}$ .

(1.2) Total probability  $P(B) = \sum_{i=1}^k P(B|A_i)P(A_i)$  where  $\{A_i\}$  are disjoint and  $\cup_{i=1}^k A_i = S$ .

(1.3) Bayes' Theorem  $P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^k P(B|A_j)P(A_j)}$  where  $\{A_i\}$  are in (1.2).

## 2. Random variables (r.v.s)

(2.1) Discrete r.v.  $X$  has a pmf  $p(x) = P(X = x)$  satisfying  $p(x) \geq 0$  and  $\sum p(x_i) = 1$ ,

$$\begin{array}{c|ccccccc} X & x_1 & x_2 & \cdots & x_n & \cdots \\ \hline p(x) & p(x_1) & p(x_2) & \cdots & p(x_n) & \cdots \end{array}$$

Expectation (or *Expected value* or *mean*)  $\mu_X = E(X) = \sum x_i p(x_i)$ ;

Variance  $\sigma_X^2 = V(X) = E(X - \mu_X)^2 = E(X^2) - \mu_X^2 = \sum x_i^2 p(x_i) - (\sum x_i p(x_i))^2$ .

(2.2) Continuous r.v.  $X$  has a pdf  $f(x)$  satisfying  $f(x) \geq 0$  and  $\int_{-\infty}^{\infty} f(x) dx = 1$ ,

$$P(a < X < b) = \int_a^b f(x) dx.$$

Expectation (or *Expected value* or *mean*)  $\mu_X = E(X) = \int_{-\infty}^{\infty} x f(x) dx$ ;

Variance  $\sigma_X^2 = V(X) = E(X - \mu_X)^2 = E(X^2) - \mu_X^2 = \int_{-\infty}^{\infty} x^2 f(x) dx - (\int_{-\infty}^{\infty} x f(x) dx)^2$ .

(2.3) Cumulative distribution function (cdf) of a r.v.  $X$  is  $F(x) = P(X \leq x)$ .

(2.4)  $X$  and  $Y$  are r.v.s,  $a, b$  and  $c$  are scalars, then

$$E(aX + bY + c) = aE(X) + bE(Y) + c,$$

$$V(aX + bY + c) = a^2V(X) + b^2V(Y) + 2ab \operatorname{cov}(X, Y),$$

$$E(g(X, Y)) = \begin{cases} \sum_{i,j} g(x_i, y_j) \cdot p(x_i, y_j), & \text{for discrete } (X, Y), \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \cdot f(x, y) dx dy, & \text{for continuous } (X, Y). \end{cases}$$

(2.5) • Discrete r.v.  $(X, Y)$  has a joint pmf  $p(x, y)$  satisfying  $p(x, y) \geq 0$  and  $\sum_{x_i} \sum_{y_j} p(x_i, y_j) = 1$ .

The *marginal pmf* of  $X$  is  $p_X(x) = \sum_y p(x, y)$ ;

The *marginal pmf* of  $Y$  is  $p_Y(y) = \sum_x p(x, y)$ ;

$X$  and  $Y$  are *independent* if  $p(x, y) = p_X(x) \cdot p_Y(y)$ .

• Continuous r.v.  $(X, Y)$  has a joint pdf  $p(x, y)$  satisfying  $f(x, y) \geq 0$  and  $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$ .

The *marginal pdf* of  $X$  is  $f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy$ ;

The *marginal pdf* of  $Y$  is  $f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx$ ;

$X$  and  $Y$  are *independent* if  $f(x, y) = f_X(x) \cdot f_Y(y)$ .

## 3. Several special r.v.s

(3.1)  $X \sim \operatorname{Bin}(n, p)$  has a pmf  $p(x) = P(X = x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$ ,  $x = 0, 1, 2, \dots, n$ .

$$E(X) = n \cdot p, \quad V(X) = n \cdot p \cdot (1-p).$$

(3.2)  $X \sim \operatorname{Po}(\lambda)$  has a pmf  $p(x) = P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$ ,  $x = 0, 1, 2, \dots$   
 $E(X) = \lambda, \quad V(X) = \lambda$ .

(3.3)  $X \sim \operatorname{Hypergeometric}$  has a pmf  $p(x) = P(X = x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}$ .

(3.4)  $X \sim \operatorname{Exp}(\lambda)$  has a pdf

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(3.5)  $X \sim N(\mu, \sigma^2)$  has a pdf

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty.$$

(3.6)  $X \sim U(a, b)$  has a pdf

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise.} \end{cases}$$

$$E(X) = \frac{a+b}{2}, \quad V(X) = \frac{(b-a)^2}{12}.$$

## 4. Central Limit Theorem (CLT)

Suppose that a population has mean  $= \mu$  and variance  $= \sigma^2$ . A random sample  $\{X_1, X_2, \dots, X_n\}$  from this population is given. Then for large  $n \geq 30$ ,

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1). \tag{1}$$

• If the population is normal, then (1) holds for any  $n$ .

• Note that  $\mu = E(\bar{X})$  and  $(\sigma/\sqrt{n})^2 = V(\bar{X})$ .

## 5. Several notations in statistics

(5.1) Sample mean:  $\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n} = \sum \frac{X_i}{n}$ ;  $\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \sum \frac{x_i}{n}$ .

(5.2) Sample variance:

$$S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \frac{1}{n-1} \left( \sum X_i^2 - \frac{(\sum X_i)^2}{n} \right); \quad s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{1}{n-1} \left( \sum x_i^2 - \frac{(\sum x_i)^2}{n} \right).$$

• Capital letters  $\bar{X}$  and  $S^2$  refer to the objects based on random sample (therefore they are in general r.v.s), while small letters  $\bar{x}$  and  $s^2$  are the objects based on observations (so they are scalars).

(5.3) A point estimator of  $\theta$  obtained by Methods of Moments is denoted as  $\hat{\theta}_{MM}$ .

(5.4) A point estimator of  $\theta$  obtained by Maximum Likelihood method is denoted as  $\hat{\theta}_{ML}$ .

## 6. Confidence Interval (CI)

In this course, three types of confidence intervals are studied depending on the unknown population parameter(s): CI-1 (confidence intervals for population mean(s)), CI-2 (confidence intervals for population variance(s)), and CI-3 (confidence intervals for population proportion(s)).

**CI-1: (1 - α) CI of a population mean μ**

**case 1.1 (any n)** If population  $X \sim N(\mu, \sigma^2)$  and  $\sigma^2$  is known, then  $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0, 1)$  and

$$I_\mu = (\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}) := \bar{x} \mp z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}.$$

**case 1.2 (n ≥ 30)** For any population X, it holds that  $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0, 1)$  and

$$I_\mu = \bar{x} \mp z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \text{ or } I_\mu = \bar{x} \mp z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}}.$$

**case 1.3 (any n)** If population  $X \sim N(\mu, \sigma^2)$  and  $\sigma^2$  is unknown, then  $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim T(n-1)$  and

$$I_\mu = \bar{x} \mp t_{\alpha/2}(n-1) \cdot \frac{s}{\sqrt{n}}.$$

**CI-1': (1 - α) CI of the difference of two population means  $\mu_X - \mu_Y$**

**case 1.1' (any  $n_1, n_2$ )** If independent populations  $X \sim N(\mu_X, \sigma_X^2), Y \sim N(\mu_Y, \sigma_Y^2)$ , and  $\sigma_X^2, \sigma_Y^2$  are known, then  $\frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}} \sim N(0, 1)$ , and  $I_{\mu_X-\mu_Y} = (\bar{x}-\bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}$ .

**case 1.2' ( $n_1, n_2 \geq 30$ )** For any independent populations X and Y, it holds that

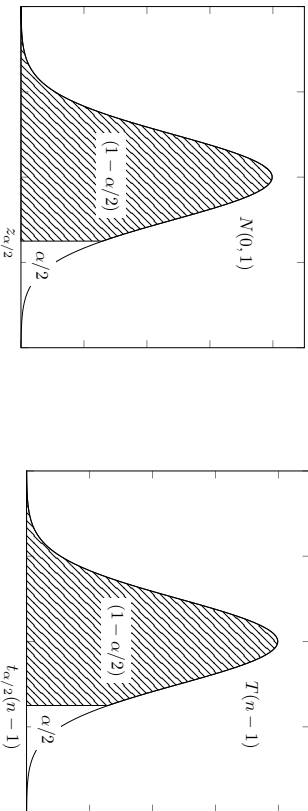
$$\frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}} \sim N(0, 1) \text{ and}$$

$$I_{\mu_X-\mu_Y} = (\bar{x}-\bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}} \text{ or } I_{\mu_X-\mu_Y} = (\bar{x}-\bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{\sigma}_X^2}{n_1} + \frac{\hat{\sigma}_Y^2}{n_2}}.$$

**case 1.3' (any  $n_1, n_2$ )** If independent populations  $X \sim N(\mu_X, \sigma_X^2), Y \sim N(\mu_Y, \sigma_Y^2)$ , where  $\sigma_X^2, \sigma_Y^2$  are unknown but  $\sigma_X^2 = \sigma_Y^2$ , then

$$\frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim T(n_1+n_2-2), \text{ where } S^2 = \frac{(n_1-1)S_X^2 + (n_2-1)S_Y^2}{n_1+n_2-2}, \text{ and}$$

$$I_{\mu_X-\mu_Y} = (\bar{x}-\bar{y}) \mp t_{\alpha/2}(n_1+n_2-2) \cdot s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}.$$



**CI-2: (1 - α) CI of population variance(s)  $\sigma^2$**

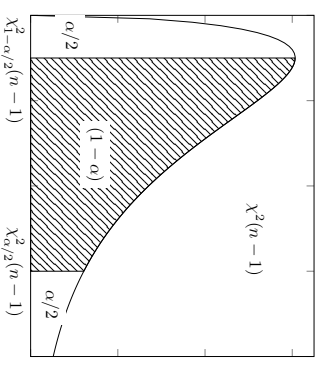
• If a population  $X \sim N(\mu, \sigma^2)$  and  $\sigma^2$  is unknown, then  $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$ , and

$$I_{\sigma^2} = \left( \frac{(n-1)s^2}{\chi_{\alpha/2}^2(n-1)}, \frac{(n-1)s^2}{\chi_{1-\alpha/2}^2(n-1)} \right).$$

• If two independent populations  $X \sim N(\mu_X, \sigma^2)$  and  $Y \sim N(\mu_Y, \sigma^2)$ , and  $\sigma^2$  is unknown, then  $\frac{(n_1+n_2-2)S^2}{\sigma^2} \sim \chi^2(n_1+n_2-2)$ , and

$$I_{\sigma^2} = \left( \frac{(n_1+n_2-2)s^2}{\chi_{\alpha/2}^2(n_1+n_2-2)}, \frac{(n_1+n_2-2)s^2}{\chi_{1-\alpha/2}^2(n_1+n_2-2)} \right),$$

where  $S^2 = \frac{(n_1-1)S_X^2 + (n_2-1)S_Y^2}{n_1+n_2-2}$ .



**CI-3: (1 - α) CI of population proportion(s)**

- If a (large) population has an unknown proportion p, then  $\frac{\hat{p}-p}{\sqrt{p(1-p)/n}} \sim N(0, 1)$  if  $n\hat{p} \geq 10$  and  $n(1-\hat{p}) \geq 10$  with  $\hat{p} = x/n$ , and  $I_p = \hat{p} \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ .
- If two independent (large) populations have unknown proportions  $p_1$  and  $p_2$ , then

$$\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}} \sim N(0, 1)$$

if  $n_i\hat{p}_i \geq 10$  and  $n_i(1-\hat{p}_i) \geq 10$  for  $i = 1, 2$ , and  $I_{p_1-p_2} = (\hat{p}_1-\hat{p}_2) \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ .

**7. Hypothesis Test (HT)**

	$H_0$ is true	$H_0$ is false and $\theta = \theta_1$
reject $H_0$	(type I error or significance level) $\alpha$	(power) $h(\theta_1)$
don't reject $H_0$	$1 - \alpha$	(type II error) $\beta(\theta_1) = 1 - h(\theta_1)$

reject  $H_0 \Leftrightarrow TS \in C \Leftrightarrow p\text{-value} < \alpha$

**$\chi^2$  tests for populations (non-parametric tests)**

Suppose that for a random sample of a population X, the n elements of it are classified into k disjoint groups  $A_i, 1 \leq i \leq k$ . For each group  $A_i, 1 \leq i \leq k$ , suppose that there are  $N_i, 1 \leq i \leq k$  elements inside. Let  $p_i = P(A_i)$  assuming a given distribution of X. Note that  $p_1 + p_2 + \dots + p_k = 1$  and  $N_1 + N_2 + \dots + N_k = n$ . One wants to test the hypotheses

$$H_0 : P(A_i) = p_i, \quad 1 \leq i \leq k, \quad H_a : P(A_i) \neq p_i \text{ for some } 1 \leq i \leq k.$$

If  $n$  is large in the sense that  $np_i \geq 5$  for all  $1 \leq i \leq k$ , then the test statistic is

$$\sum_{i=1}^k \frac{(N_i - np_i)^2}{np_i} \approx \chi^2(k-1).$$

Therefore the observation of the test statistic is

$$TS = \sum_{i=1}^k \frac{(n_i - np_i)^2}{np_i}, \text{ where } n_i \text{ is the observation of } N_i, 1 \leq i \leq k.$$

For the critical region  $C$ , one can take (note that if  $H_0$  is true, then  $TS$  should be close to zero)

$$C = (\chi^2_{\alpha}(k-1), \infty).$$

The conclusion would be  $TS \in C \iff H_0$  is rejected.

## 8. Linear and logistic regression

**(Multiple) linear regression:**  $Y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + \varepsilon$ ,  $\varepsilon \sim N(0, \sigma^2)$ .

- $Y$  : response variable (which is normal r.v.),  $\{x_1, \dots, x_k\}$  : predictors (which are scalars).
- sample:  $\{(x_{11}, \dots, x_{1k}; y_1), (x_{21}, \dots, x_{2k}; y_2), \dots, (x_{n1}, \dots, x_{nk}; y_n)\}$ .
- how to estimate  $\beta_j \approx \hat{\beta}_j$  : least square method, that is, to minimize  $\sum_{i=1}^n (y_i - \hat{y}_i)^2$ , where the estimated (multiple) linear regression line  $\hat{y}$  is

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k.$$

- $\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \sim T(n-k-1)$ , this helps determine whether or not the real  $\beta_j = 0$ ?
- $\sigma^2 \approx \frac{SSE}{n-k-1}$ , this gives an estimation of the size of the error.
- $R^2 = \frac{SSR}{SSY}$ , this gives how well the model is (if  $R^2 \approx 1$ , then the model fits the sample very well).
- How to test  $\beta_1 = \dots = \beta_k = 0$  ? Use the random variable  $\frac{SSR/k}{SSE/(n-k-1)} \sim F(k, n-k-1)$ .

**Logistic regression:** Let  $Y$  can only take 0 or 1 with  $P(Y=1) = p$  and  $P(Y=0) = 1-p$ .

$$E(Y) = p(x_1, \dots, x_k) = \frac{e^{\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k}}{1 + e^{\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k}}.$$

- $Y$  : response variable (which is Bernoulli r.v.  $P(Y=1) = p$  and  $P(Y=0) = 1-p$ , so  $E(Y) = p$ ),  $\{x_1, \dots, x_k\}$  : predictors (which are scalars).
- sample:  $\{(x_{11}, \dots, x_{1k}; y_1), (x_{21}, \dots, x_{2k}; y_2), \dots, (x_{n1}, \dots, x_{nk}; y_n)\}$ .
- how to estimate  $\beta_j \approx \hat{\beta}_j$  : maximal likelihood method (maximize  $\prod_{i=1}^n p(x_{i1}, \dots, x_{ik})^{y_i} (1 - p(x_{i1}, \dots, x_{ik}))^{1-y_i}$ ).
- $\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \approx N(0, 1)$  for large  $n \geq 30$ , this helps determine whether or not the real  $\beta_j = 0$ ?
- Classification of a new object  $Y(x_1, \dots, x_k)$  as 1 or 0 according

$$Y(x_1, \dots, x_k) = \begin{cases} 1, & \text{if } \hat{p}(x_1, \dots, x_k) \geq 0.5, \\ 0, & \text{if } \hat{p}(x_1, \dots, x_k) < 0.5, \end{cases}$$

where the estimated logit function  $\hat{p}(x_1, \dots, x_k)$  is

$$\hat{p}(x_1, \dots, x_k) = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k}}{1 + e^{\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k}}.$$

## 9. Tables

(9.1) Table for  $N(0, 1)$  standard normal random variable  $\Phi(x) = P(N(0, 1) \leq x)$ ,  $x \geq 0$ .  
There is an important relation  $\Phi(-x) = 1 - \Phi(x)$ ,  $x \geq 0$ .

x	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9564	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9993	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

(9.2) Table for  $T(f)$  random variable  $F(x) = P(T(f) \leq x)$ ,  
where  $f$  is a parameter called 'degrees of freedom'.

f	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.9995
1	1.00	3.08	6.31	12.71	31.82	63.66	127.32	636.62
2	0.82	1.89	2.92	4.30	6.96	9.92	14.09	31.60
3	0.76	1.64	2.35	3.18	4.54	5.84	7.45	12.92
4	0.74	1.53	2.13	2.78	3.75	4.60	5.60	8.61
5	0.73	1.48	2.02	2.57	3.36	4.03	4.77	6.87
6	0.72	1.44	1.94	2.45	3.14	3.71	4.32	5.96
7	0.71	1.41	1.89	2.36	3.00	3.50	4.03	5.41
8	0.71	1.40	1.86	2.31	2.90	3.36	3.83	5.04
9	0.70	1.38	1.83	2.26	2.82	3.25	3.69	4.78
10	0.70	1.37	1.81	2.23	2.76	3.17	3.58	4.59
11	0.70	1.36	1.80	2.20	2.72	3.11	3.50	4.44
12	0.70	1.36	1.78	2.18	2.68	3.05	3.43	4.32
13	0.69	1.35	1.77	2.16	2.65	3.01	3.37	4.22
14	0.69	1.35	1.76	2.14	2.62	2.98	3.33	4.14
15	0.69	1.34	1.75	2.13	2.60	2.95	3.29	4.07
16	0.69	1.34	1.75	2.12	2.58	2.92	3.25	4.01
17	0.69	1.33	1.74	2.11	2.57	2.90	3.22	3.97
18	0.69	1.33	1.73	2.10	2.55	2.88	3.20	3.92
19	0.69	1.33	1.73	2.09	2.54	2.86	3.17	3.88
20	0.69	1.33	1.72	2.09	2.53	2.85	3.15	3.85
21	0.69	1.32	1.72	2.08	2.52	2.83	3.14	3.82
22	0.69	1.32	1.72	2.07	2.51	2.82	3.12	3.79
23	0.69	1.32	1.71	2.07	2.50	2.81	3.10	3.77
24	0.68	1.32	1.71	2.06	2.49	2.80	3.09	3.75
25	0.68	1.32	1.71	2.06	2.49	2.79	3.08	3.73
26	0.68	1.31	1.71	2.06	2.48	2.78	3.07	3.71
27	0.68	1.31	1.70	2.05	2.47	2.77	3.06	3.69
28	0.68	1.31	1.70	2.05	2.47	2.76	3.05	3.67
29	0.68	1.31	1.70	2.05	2.46	2.76	3.04	3.66
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03	3.65
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97	3.55
50	0.68	1.30	1.68	2.01	2.40	2.68	2.94	3.50
60	0.68	1.30	1.67	2.00	2.39	2.66	2.91	3.46
100	0.68	1.29	1.66	1.98	2.36	2.63	2.87	3.39
$\infty$	0.67	1.28	1.65	1.96	2.33	2.58	2.81	3.29

(9.3) Table for  $\chi^2(f)$  random variable  $F(x) = P(\chi^2(f) \leq x)$ , where  $f$  is a parameter.

$f$	0.0005	0.001	0.005	0.01	0.025	0.05	0.10	0.20	0.30	0.40	0.50
1	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.15	0.27	0.45
2	0.00	0.00	0.01	0.02	0.05	0.10	0.21	0.45	0.71	1.02	1.39
3	0.02	0.02	0.07	0.11	0.22	0.35	0.58	1.01	1.42	1.87	2.37
4	0.06	0.09	0.21	0.30	0.48	0.71	1.06	1.65	2.19	2.75	3.36
5	0.16	0.21	0.41	0.55	0.83	1.15	1.61	2.34	3.00	3.66	4.35
6	0.30	0.38	0.68	0.87	1.24	1.64	2.20	3.07	3.83	4.57	5.35
7	0.48	0.60	0.99	1.24	1.69	2.17	2.73	3.62	4.67	5.49	6.35
8	0.71	0.86	1.34	1.65	2.18	2.73	3.49	4.59	5.53	6.42	7.34
9	0.97	1.15	1.73	2.09	2.70	3.33	4.17	5.38	6.39	7.36	8.34
10	1.26	1.48	2.16	2.56	3.25	3.94	4.87	6.18	7.27	8.30	9.34
11	1.59	1.83	2.60	3.05	3.82	4.57	5.58	6.99	8.15	9.24	10.34
12	1.93	2.21	3.07	3.57	4.40	5.23	6.30	7.81	9.03	10.18	11.34
13	2.31	2.62	3.57	4.11	5.01	5.89	7.04	8.63	9.93	11.13	12.34
14	2.70	3.04	4.07	4.66	5.63	6.57	7.79	9.47	10.82	12.08	13.34
15	3.11	3.48	4.60	5.23	6.26	7.26	8.55	10.31	11.72	13.03	14.34
16	3.54	3.94	5.14	5.81	6.91	7.96	9.31	11.15	12.62	13.98	15.34
17	3.98	4.42	5.70	6.41	7.56	8.67	10.09	12.00	13.53	14.94	16.34
18	4.44	4.90	6.26	7.01	8.23	9.39	10.86	12.86	14.44	15.89	17.34
19	4.91	5.41	6.84	7.63	8.91	10.12	11.65	13.72	15.35	16.85	18.34
20	5.40	5.92	7.43	8.26	9.59	10.85	12.44	14.58	16.27	17.81	19.34
21	5.90	6.45	8.03	8.90	10.28	11.59	13.24	15.44	17.18	18.77	20.34
22	6.40	6.98	8.64	9.54	10.98	12.34	14.04	16.31	18.10	19.73	21.34
23	6.92	7.53	9.26	10.20	11.69	13.09	14.85	17.19	19.02	20.69	22.34
24	7.45	8.08	9.89	10.86	12.40	13.85	15.66	18.06	19.94	21.65	23.34
25	7.99	8.65	10.52	11.52	13.12	14.61	16.47	18.94	20.87	22.62	24.34
26	8.54	9.22	11.16	12.20	13.84	15.38	17.29	19.82	21.79	23.58	25.34
27	9.09	9.80	11.81	12.88	14.57	16.15	18.11	20.70	22.72	24.54	26.34
28	9.66	10.39	12.46	13.56	15.31	16.93	18.94	21.59	23.65	25.51	27.34
29	10.23	10.99	13.12	14.26	16.05	17.71	19.77	22.48	24.58	26.48	28.34
30	10.80	11.59	13.79	14.95	16.79	18.49	20.60	23.36	25.51	27.44	29.34
40	16.91	17.92	20.71	22.16	24.43	26.51	29.05	32.34	34.87	37.13	39.34
50	23.46	24.67	27.99	29.71	32.36	34.76	37.69	41.45	46.86	49.33	51.33
60	30.34	31.74	35.53	37.48	40.48	43.19	46.46	50.64	53.81	56.62	59.33
100	59.90	61.92	67.33	70.06	74.22	77.93	82.36	87.95	92.13	95.81	99.33

Table for  $\chi^2(f)$  random variable  $F(x) = P(\chi^2(f) \leq x)$ , where  $f$  is a parameter.

$f$	0.60	0.70	0.80	0.90	0.95	0.975	0.99	0.995	0.999	0.9995
1	0.71	1.07	1.64	2.71	3.84	5.02	6.63	7.88	10.83	12.12
2	1.83	2.41	3.22	4.61	5.99	7.38	9.21	10.60	13.82	15.20
3	2.95	3.66	4.64	6.25	7.81	9.35	11.34	12.84	16.27	17.73
4	4.04	4.88	5.99	7.78	9.49	11.14	13.28	14.86	18.47	20.00
5	5.13	6.06	7.29	9.24	11.07	12.83	15.09	16.75	20.52	22.11
6	6.21	7.23	8.56	10.64	12.59	14.45	16.81	18.55	22.46	24.10
7	7.28	8.38	9.80	12.02	14.07	16.01	18.48	20.28	24.32	26.02
8	8.35	9.52	11.03	13.36	15.51	17.53	20.09	21.95	26.12	27.87
9	9.41	10.66	12.24	14.68	16.92	19.02	21.67	23.59	27.88	29.67
10	10.47	11.78	13.44	15.99	18.31	20.48	23.21	25.19	29.59	31.42
11	11.53	12.90	14.63	17.28	19.68	21.92	24.72	26.76	31.26	33.14
12	12.58	14.01	15.81	18.55	21.03	23.34	26.22	28.30	32.91	34.82
13	13.64	15.12	16.98	19.81	22.36	24.74	27.69	29.82	34.53	36.48
14	14.69	16.22	18.15	21.06	23.68	26.12	29.14	31.32	36.12	38.11
15	15.73	17.32	19.31	22.31	25.00	27.49	30.58	32.80	37.70	39.72
16	16.78	18.42	20.47	23.54	26.30	28.85	32.00	34.27	39.25	41.31
17	17.82	19.51	21.61	24.77	27.59	30.19	33.41	35.72	40.79	42.88
18	18.87	20.60	22.76	25.99	28.87	31.53	34.81	37.16	42.31	44.43
19	19.91	21.69	23.90	27.20	30.14	32.85	36.19	38.58	43.82	45.97
20	20.95	22.77	25.04	28.41	31.41	34.17	37.57	40.00	45.31	47.50
21	21.99	23.86	26.17	29.62	32.67	35.48	38.93	41.40	46.80	49.01
22	23.03	24.94	27.30	30.81	33.92	36.78	40.29	42.80	48.27	50.51
23	24.07	26.02	28.43	32.01	35.17	38.08	41.64	44.18	49.73	52.00
24	25.11	27.10	29.55	33.20	36.42	39.36	42.98	45.56	51.18	53.48
25	26.14	28.17	30.68	34.38	37.65	40.65	44.31	46.93	52.62	54.95
26	27.18	29.25	31.79	35.56	38.89	41.92	45.64	48.29	54.05	56.41
27	28.21	30.32	32.91	36.74	40.11	43.19	46.96	49.64	55.48	57.86
28	29.25	31.39	34.03	37.92	41.34	44.46	48.28	50.99	56.89	59.30
29	30.28	32.46	35.14	39.09	42.56	45.72	49.59	52.34	58.30	60.73
30	31.32	33.53	36.25	40.26	43.77	46.98	50.89	53.67	59.70	62.16
40	41.62	44.16	47.27	51.81	55.76	59.34	63.69	66.77	73.40	76.09
50	51.89	54.72	58.16	63.17	67.50	71.42	79.15	79.49	86.66	89.56
60	62.13	65.23	68.97	74.40	79.08	83.30	88.38	91.95	99.61	102.69
100	102.95	106.91	111.67	118.50	124.34	129.56	135.81	140.17	149.45	153.17

(9.4) Table for Binomial random variable  $P(Bin(n, p) \leq k)$  if  $p \leq 0.5$ .  
 If  $p > 0.5$ , then  $P(Bin(n, p) \leq k) = P(Bin(n, 1 - p) \geq n - k)$ .

$n$	$k$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
	1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500
3	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250
	1	0.9928	0.9720	0.9392	0.8960	0.8438	0.7840	0.7183	0.6480	0.5747	0.5000
4	0	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1256	0.0915	0.0625
	1	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4735	0.3910	0.3125
5	0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313
	1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875
6	0	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
	1	0.9672	0.8847	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094
7	0	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
	1	0.9566	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625
8	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
	1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352
9	0	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
	1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195

Table for Binomial random variable  $P(Bin(n, p) \leq k)$  if  $p \leq 0.5$ .  
 If  $p > 0.5$ , then  $P(Bin(n, p) \leq k) = P(Bin(n, 1 - p) \geq n - k)$ .

$n$	$k$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
10	0	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107
11	0	0.5688	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005
	1	0.8981	0.6974	0.4922	0.3221	0.1971	0.1130	0.0606	0.0302	0.0139	0.0059
12	0	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002
	1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032



Table for Binomial random variable  $P(\text{Bin}(n, p) \leq k)$  if  $p \leq 0.5$ .  
 If  $p > 0.5$ , then  $P(\text{Bin}(n, p) \leq k) = P(\text{Bin}(n, 1 - p) \geq n - k)$ .

$n$	$k$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
14	0	0.4877	0.2288	0.1028	0.0440	0.0178	0.0068	0.0024	0.0008	0.0002	0.0001
	1	0.8470	0.5846	0.3567	0.2075	0.1010	0.0475	0.0205	0.0081	0.0029	0.0009
	2	0.9699	0.8416	0.6479	0.4481	0.2811	0.1608	0.0839	0.0398	0.0170	0.0065
	3	0.9958	0.9559	0.8535	0.6982	0.5213	0.3552	0.2205	0.1243	0.0632	0.0287
	4	0.9996	0.9908	0.9533	0.8702	0.7415	0.5842	0.4227	0.2793	0.1672	0.0898
	5	1.0000	0.9985	0.9885	0.9561	0.8883	0.7805	0.6405	0.4859	0.3373	0.2120
	6	1.0000	0.9998	0.9978	0.9884	0.9617	0.9067	0.8164	0.6925	0.5461	0.3953
	7	1.0000	1.0000	0.9997	0.9976	0.9897	0.9685	0.9247	0.8499	0.7414	0.6047
	8	1.0000	1.0000	1.0000	0.9996	0.9978	0.9978	0.9917	0.9417	0.8811	0.7880
	9	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9983	0.9940	0.9825	0.9102
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9998	0.9989	0.9961	0.9886
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9978	0.9935
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
15	0	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000
	1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005
	2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037
	3	0.9945	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176
	4	0.9994	0.9873	0.9383	0.8338	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592
	5	0.9999	0.9978	0.9832	0.9389	0.8516	0.7216	0.5816	0.4303	0.2608	0.1509
	6	1.0000	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036
	7	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000
	8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964
	9	1.0000	1.0000	1.0000	0.9999	0.9999	0.9992	0.9923	0.9662	0.9231	0.8491
	10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9981	0.9937	0.9824
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9963
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9995
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	16	0	0.4401	0.1853	0.0743	0.0281	0.0100	0.0033	0.0010	0.0003	0.0001
1		0.8108	0.5147	0.2839	0.1407	0.0635	0.0261	0.0098	0.0033	0.0010	0.0003
2		0.9571	0.7892	0.5614	0.3518	0.1971	0.0994	0.0451	0.0183	0.0066	0.0021
3		0.9930	0.9316	0.7899	0.5981	0.4050	0.2459	0.1339	0.0651	0.0281	0.0106
4		0.9991	0.9830	0.9209	0.7982	0.6302	0.4499	0.2892	0.1666	0.0853	0.0384
5		0.9999	0.9967	0.9765	0.9183	0.8103	0.6598	0.4900	0.3288	0.1976	0.1051
6		1.0000	0.9995	0.9944	0.9733	0.9204	0.8247	0.6881	0.5272	0.3660	0.2272
7		1.0000	0.9999	0.9989	0.9930	0.9729	0.9236	0.8406	0.7161	0.5629	0.4018
8		1.0000	1.0000	0.9998	0.9985	0.9925	0.9743	0.9329	0.8577	0.7441	0.5982
9		1.0000	1.0000	1.0000	0.9998	0.9988	0.9929	0.9711	0.9417	0.8759	0.7728
10		1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9938	0.9809	0.9551	0.8949
11		1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9851	0.9616
12		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9991	0.9965	0.9894
13		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9994	0.9979
14		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997
15		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table for Binomial random variable  $P(\text{Bin}(n, p) \leq k)$  if  $p \leq 0.5$ .  
 If  $p > 0.5$ , then  $P(\text{Bin}(n, p) \leq k) = P(\text{Bin}(n, 1 - p) \geq n - k)$ .

$n$	$k$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
17	0	0.4181	0.1668	0.0631	0.0225	0.0075	0.0023	0.0007	0.0002	0.0000	0.0000
	1	0.7922	0.4818	0.2525	0.1182	0.0501	0.0193	0.0067	0.0021	0.0006	0.0001
	2	0.9497	0.7618	0.5198	0.3096	0.1637	0.0774	0.0327	0.0123	0.0041	0.0012
	3	0.9912	0.9174	0.7556	0.5489	0.3530	0.2019	0.1028	0.0464	0.0184	0.0064
	4	0.9988	0.9779	0.9013	0.7582	0.5739	0.3887	0.2348	0.1260	0.0596	0.0245
	5	0.9999	0.9953	0.9681	0.8943	0.7653	0.5668	0.4197	0.2639	0.1471	0.0717
	6	1.0000	0.9992	0.9922	0.9623	0.8929	0.7752	0.6188	0.4478	0.2902	0.1672
	7	1.0000	0.9999	0.9983	0.9891	0.9598	0.8954	0.8054	0.7872	0.6405	0.4743
	8	1.0000	1.0000	0.9997	0.9974	0.9876	0.9597	0.9006	0.8011	0.6626	0.5000
	9	1.0000	1.0000	1.0000	0.9995	0.9969	0.9969	0.9873	0.9617	0.9081	0.8166
	10	1.0000	1.0000	1.0000	0.9999	0.9994	0.9994	0.9980	0.9852	0.9174	0.8338
	11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9993	0.9970	0.9894	0.9699
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9914	0.9755
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9981	0.9936
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9988
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	18	0	0.3972	0.1501	0.0536	0.0180	0.0056	0.0016	0.0004	0.0001	0.0000
1		0.7735	0.4503	0.2241	0.0991	0.0395	0.0142	0.0046	0.0013	0.0003	0.0001
2		0.9419	0.7338	0.4797	0.2713	0.1353	0.0600	0.0236	0.0082	0.0025	0.0008
3		0.9891	0.9018	0.7202	0.5010	0.3057	0.1646	0.0783	0.0328	0.0120	0.0038
4		0.9985	0.9718	0.8794	0.7164	0.5187	0.3327	0.1886	0.0942	0.0411	0.0154
5		0.9998	0.9936	0.9351	0.8671	0.7175	0.5344	0.3550	0.2088	0.1077	0.0481
6		1.0000	0.9988	0.9882	0.9487	0.8610	0.7217	0.5491	0.3258	0.1189	0.0403
7		1.0000	0.9998	0.9973	0.9837	0.9431	0.8593	0.7283	0.5634	0.3915	0.2403
8		1.0000	1.0000	0.9995	0.9957	0.9807	0.9404	0.8609	0.7368	0.5778	0.4073
9		1.0000	1.0000	0.9999	0.9991	0.9946	0.9790	0.9403	0.8653	0.7473	0.5927
10		1.0000	1.0000	1.0000	0.9998	0.9988	0.9939	0.9788	0.9259	0.8720	0.7597
11		1.0000	1.0000	1.0000	0.9998	0.9998	0.9986	0.9937	0.9797	0.9463	0.8811
12		1.0000	1.0000	1.0000	1.0000	0.9997	0.9997	0.9986	0.9942	0.9817	0.9519
13		1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9987	0.9951	0.9846
14		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9998	0.9990	0.9962
15		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9993
16		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
19	0	0.3774	0.1351	0.0456	0.0144	0.0042	0.0011	0.0003	0.0001	0.0000	0.0000
	1	0.7547	0.4203	0.1985	0.0829	0.0310	0.0104	0.0031	0.0008	0.0002	0.0000
	2	0.9335	0.7054	0.4413	0.2369	0.1113	0.0462	0.0170	0.0055	0.0015	0.0002
	3	0.9868	0.8850	0.6841	0.4551	0.2631	0.1332	0.0591	0.0230	0.0077	0.0024
	4	0.9980	0.9648	0.8556	0.6733	0.4654	0.2822	0.1500	0.0696	0.0280	0.0096
	5	0.9998	0.9914	0.9463	0.8369	0.6678	0.4739	0.2968	0.1629	0.0777	0.0318
	6	1.0000	0.9983	0.9837	0.9324	0.8251	0.6655	0.4812	0.3081	0.1727	0.0835
	7	1.0000	0.9997	0.9959	0.9767	0.9225	0.8180	0.6656	0.4878	0.3169	0.1796
	8	1.0000	1.0000	0.9992	0.9933	0.9713	0.9161	0.8145	0.6675	0.4940	0.3238

(9.5) Table for Poisson random variable  $P(Po(\mu) \leq k)$ .

$k$	$\mu$									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9536	0.9371	0.9197
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$k$	$\mu$									
	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
1	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337	0.4060
2	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037	0.6767
3	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747	0.8571
4	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559	0.9473
5	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868	0.9834
6	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966	0.9955
7	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992	0.9989
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$k$	$\mu$									
	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550	0.0498
1	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146	0.1991
2	0.6496	0.6227	0.5960	0.5697	0.5438	0.5184	0.4936	0.4695	0.4460	0.4232
3	0.8386	0.8194	0.7993	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696	0.6472
4	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8629	0.8477	0.8318	0.8153
5	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258	0.9161
6	0.9941	0.9925	0.9906	0.9884	0.9858	0.9828	0.9794	0.9756	0.9713	0.9665
7	0.9985	0.9980	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901	0.9881
8	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969	0.9962
9	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9993	0.9991	0.9989
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table for Poisson random variable  $P(Po(\mu) \leq k)$ .

$k$	$\mu$									
	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0
0	0.0408	0.0334	0.0273	0.0224	0.0183	0.0150	0.0123	0.0101	0.0082	0.0067
1	0.1712	0.1468	0.1257	0.1074	0.0916	0.0780	0.0663	0.0563	0.0477	0.0404
2	0.3799	0.3397	0.3027	0.2689	0.2381	0.2127	0.1851	0.1626	0.1425	0.1247
3	0.6025	0.5584	0.5152	0.4735	0.4335	0.3954	0.3594	0.3257	0.2942	0.2650
4	0.7806	0.7442	0.7064	0.6678	0.6288	0.5898	0.5512	0.5132	0.4763	0.4405
5	0.8946	0.8705	0.8441	0.8156	0.7851	0.7531	0.7199	0.6858	0.6510	0.6160
6	0.9534	0.9421	0.9267	0.9091	0.8893	0.8675	0.8436	0.8180	0.7908	0.7622
7	0.9832	0.9769	0.9682	0.9599	0.9509	0.9419	0.9314	0.9204	0.9087	0.8966
8	0.9943	0.9917	0.9883	0.9840	0.9786	0.9721	0.9642	0.9549	0.9442	0.9319
9	0.9982	0.9973	0.9960	0.9942	0.9919	0.9889	0.9851	0.9805	0.9749	0.9682
10	0.9995	0.9992	0.9987	0.9981	0.9972	0.9959	0.9943	0.9922	0.9896	0.9863
11	0.9999	0.9998	0.9996	0.9994	0.9991	0.9986	0.9980	0.9971	0.9960	0.9945
12	1.0000	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9993	0.9990	0.9986
13	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9996	0.9993	0.9990	0.9986
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
$k$	$\mu$									
	5.2	5.4	5.6	5.8	6.0	6.5	7.0	7.5	8.0	8.5
0	0.0055	0.0045	0.0037	0.0030	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002
1	0.0342	0.0289	0.0244	0.0206	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019
2	0.1088	0.0948	0.0824	0.0715	0.0620	0.0430	0.0286	0.0203	0.0138	0.0093
3	0.2381	0.2133	0.1906	0.1700	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301
4	0.4061	0.3733	0.3422	0.3127	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744
5	0.5809	0.5461	0.5119	0.4783	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496
6	0.7324	0.7017	0.6703	0.6384	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562
7	0.8449	0.8217	0.7970	0.7710	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856
8	0.9181	0.9027	0.8857	0.8672	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231
9	0.9603	0.9512	0.9409	0.9292	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530
10	0.9823	0.9775	0.9718	0.9651	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634
11	0.9927	0.9904	0.9875	0.9841	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487
12	0.9972	0.9962	0.9949	0.9932	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091
13	0.9990	0.9986	0.9980	0.9973	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486
14	0.9999	0.9995	0.9993	0.9990	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726
15	0.9999	0.9998	0.9998	0.9996	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862
16	1.0000	0.9999	0.9999	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934
17	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970
18	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table for Poisson random variable  $P(Po(\mu) \leq k)$ .

$k$	$\mu$																
	9.0	9.5	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0							
0	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000							
1	0.0012	0.0008	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000							
2	0.0062	0.0042	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000							
3	0.0212	0.0149	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000							
4	0.0550	0.0403	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002							
5	0.1157	0.0885	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007							
6	0.2068	0.1649	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021							
7	0.3239	0.2687	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054							
8	0.4557	0.3918	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126							
9	0.5874	0.5218	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261							
10	0.7060	0.6453	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491							
11	0.8030	0.7520	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847							
12	0.8758	0.8364	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350							
13	0.9261	0.8981	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009							
14	0.9585	0.9400	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808							
15	0.9780	0.9665	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715							
16	0.9889	0.9823	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677							
17	0.9947	0.9911	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640							
18	0.9976	0.9957	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550							
19	0.9989	0.9980	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363							
20	0.9996	0.9991	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055							
21	0.9998	0.9996	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615							
22	0.9999	0.9999	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047							
23	1.0000	0.9999	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367							
24	1.0000	1.0000	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594							
25	1.0000	1.0000	1.0000	1.0000	0.9999	0.9990	0.9974	0.9938	0.9869	0.9748							
26	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9987	0.9967	0.9925	0.9848							
27	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9983	0.9959	0.9912							
28	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9978	0.9950							
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9994	0.9986							
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993							
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9996							
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9996							
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999							
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999							
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000							