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Things allowed (Hjälpmedel): a calculator.

Scores rating (Betygsgränser): 8-11 points giving rate 3; 11.5-14.5 points giving rate 4; 15-18 points giving rate 5.

Write down all necessary steps in solutions in order to receive as many points as possible.

1 (3 points)

A small town has only six families marked as F_1, F_2, F_3, F_4, F_5 and F_6 . Suppose that each family has bought 1 lottery ticket. The lottery company will randomly select 4 winning lottery tickets from the town.

(1.1) (1p) Find the probability that these 4 winning lottery tickets are from families F_1, F_2, F_3 and F_4 .

(1.2) (2p) Find the probability that the family F_1 has a winning lottery ticket.

Solution. (1.1) It is from combination that

$$P(F_1, F_2, F_3, F_4) = \frac{P(\text{how many ways to choose } F_1, F_2, F_3, F_4)}{P(\text{how many ways in total})} = \frac{1}{\binom{6}{4}} = \frac{1}{15} = 0.0667.$$

(1.2)

$$\begin{aligned} P(F_1) &= P(F_1 \text{ has a winning lottery}) = 1 - P(F_1 \text{ does not have a winning lottery}) \\ &= 1 - P(\text{all 4 winning lotteries are from } F_2, F_3, F_4, F_5, F_6) \\ &= 1 - \frac{\binom{5}{4}}{\binom{6}{4}} = 1 - \frac{1}{3} = \frac{2}{3} = 0.667. \end{aligned}$$

□

2 (3 points)

Let (X, Y) be a two dimensional continuous random variable with a joint probability density function $f(x, y)$:

$$f(x, y) = \begin{cases} c \cdot x \cdot y, & \text{if } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

(2.1) (0.5p) Find the value of c so that $f(x, y)$ is indeed a joint probability density function.

(2.2) (0.5p) Find the mean $E(X \cdot Y)$.

(2.3) (1p) Find the conditional probability $P(X + Y \leq 1)$.

(2.4) (1p) Find the conditional probability $P(X + Y \leq 1 \mid X \leq 1)$.

Solution. (2.1) A probability density function should satisfy

$$1 = \int_0^2 \left(\int_0^3 f(x, y) dy \right) dx = c \left(\int_0^2 x dx \right) \left(\int_0^3 y dy \right) = c \cdot 2 \cdot 9/2 = 9c \Rightarrow c = 1/9.$$

(2.2)

$$\begin{aligned} E(X \cdot Y) &= \int_0^2 \left(\int_0^3 x \cdot y \cdot f(x, y) dy \right) dx = c \cdot \int_0^2 x^2 dx \cdot \int_0^3 y^2 dy \\ &= c \cdot \frac{8}{3} \cdot 9 = 24c = \frac{8}{3} = 2.6667. \end{aligned}$$

(2.3)

$$P(X + Y \leq 1) = \int_0^1 \left(\int_0^{1-x} c \cdot x \cdot y dy \right) dx = c \int_0^1 x \cdot \frac{1}{2} (1-x)^2 dx = \frac{c}{2} \int_0^1 (x - 2x^2 + x^3) dx = \frac{c}{24} = \frac{1}{216} = 0.0046.$$

(2.4) The marginal probability density function $f(x)$ of X is

$$f(x) = \int_0^3 f(x, y) dy = 9cx/2.$$

Therefore

$$P(X \leq 1) = \int_0^1 f(x) dx = \int_0^1 9cx/2 dx = 9c/4.$$

Thus

$$P(X + Y \leq 1 \mid X \leq 1) = \frac{P(X + Y \leq 1, X \leq 1)}{P(X \leq 1)} = \frac{P(X + Y \leq 1)}{P(X \leq 1)} = \frac{c/24}{9c/4} = \frac{1}{54} = 0.0185.$$

□

3 (3 points)

Suppose that at LiU 25% students have Android phones.

(3.1) (1p) Among 10 randomly selected LiU students, what is the probability that there are at least 5 students who have Android phones?

(3.2) (2p) Among 100 randomly selected LiU students, what is the probability that there are at least 42 students who have Android phones?

Solution. (3.1) Let X be the number of students who have Android phones among these 10 randomly selected students, then $X \sim \text{Bin}(10, 25\%)$. Therefore

$$P(\text{at least } 5) = P(X \geq 5) = 1 - P(X \leq 4) = (\text{Binomial table}) = 1 - 0.9219 = 0.0781.$$

(3.2) Let Y be the number of students who have Android phones among these 100 randomly selected students, then $Y \sim \text{Bin}(100, 25\%)$. Normal approximation to Binomial shall be used

$$\begin{aligned} P(\text{at least } 42) &= P(Y \geq 42) = 1 - P(Y \leq 41) = 1 - P(\text{Bin}(100, 25\%) \leq 41) \\ &= 1 - P(N(100 \cdot 25\%, 100 \cdot 25\% \cdot (1 - 25\%)) \leq 41) \\ &= 1 - P(N(25, 18.75) \leq 41) = 1 - P(N(0, 1) \leq \frac{41 - 25}{\sqrt{18.75}}) \\ &= 1 - P(N(0, 1) \leq 3.70) = 1 - \Phi(3.70) = 1 - 0.9999 = 0.0001. \end{aligned}$$

□

4 (3 points)

A coin has two sides: Head (denoted as 0) and Tail (denoted as 1). We don't know whether or not the coin is fair, and let X denote the upper side in a flip of the coin. Suppose that

$$\begin{array}{c|cc} X & 0 & 1 \\ \hline p(x) & \theta & 1 - \theta \end{array}$$

where $0 < \theta < 1$ is unknown. To estimate θ , the coin has been flipped 5 times with results: $\{1, 0, 1, 1, 0\}$.

(4.1) (1p) Find a point estimate $\hat{\theta}_{MM}$ of θ based on the method of moments.

(4.2) (2p) Find a point estimate $\hat{\theta}_{ML}$ of θ based on the maximum-likelihood method.

Solution. (4.1) We first compute the mean $E(X)$ of X as follows

$$E(X) = 0 \cdot \theta + 1 \cdot (1 - \theta) = 1 - \theta.$$

Therefore, the equation $E(X) = \bar{x}$ gives (with $\bar{x} = \frac{1+0+1+1+0}{5} = 0.6$)

$$1 - \theta = \bar{x} \implies \hat{\theta}_{MM} = 1 - \bar{x} = 1 - 0.6 = 0.4.$$

(4.2) The likelihood function is

$$L(\theta) = p(x_1) \cdot p(x_2) \cdot p(x_3) \cdot p(x_4) \cdot p(x_5) = p(1) \cdot p(0) \cdot p(1) \cdot p(1) \cdot p(0) = \theta^2 \cdot (1 - \theta)^3.$$

The log likelihood function is $\ln L(\theta) = 2 \ln(\theta) + 3 \ln(1 - \theta)$. The derivative $(\ln L(\theta))' = 0$ gives

$$0 = (\ln L(\theta))' = \frac{2}{\theta} - \frac{3}{1 - \theta} \quad \implies \quad 2(1 - \theta) = 3\theta \quad \implies \quad \hat{\theta}_{ML} = \frac{2}{5} = 0.4.$$

□

5 (3 points)

A sample $\{2, 4, 6\}$ (with sample mean 4 and sample standard deviation 2) is taken from a population $X \sim N(\mu, \sigma^2)$.

(5.1) (1.5p) Does the sample provide any evidence that $\mu < 7$? Answer this question using appropriate one-sided 95% confidence interval.

(5.2) (1.5p) Does the sample provide any evidence that $\sigma > 1$? Answer this question using appropriate one-sided 95% confidence interval.

Solution. (5.1) One-sided 95% confidence interval of μ is

$$I_\mu = (-\infty, \bar{x} + t_\alpha(n-1) \frac{s}{\sqrt{n}}) = (-\infty, 4 + t_{0.05}(2) \frac{2}{\sqrt{3}}) = (-\infty, 4 + 2.92 \frac{2}{\sqrt{3}}) = (-\infty, 7.37).$$

Since $7 \in I_\mu$, we can NOT say that $\mu < 7$. (Namely, no evidence!).

(5.2) One-sided 95% confidence interval of σ^2 is

$$I_{\sigma^2} = (\frac{(n-1)s^2}{\chi_\alpha^2(n-1)}, \infty) = (\frac{2 \cdot 2^2}{\chi_{0.05}^2(2)}, \infty) = (\frac{8}{5.99}, \infty) = (1.3356, \infty).$$

Since $1 \notin I_{\sigma^2}$, it indicates that $\sigma^2 > 1$ (Equivalently, $\sigma > 1$, that is, there is evidence!).

□

6 (3 points)

Extensive experience shows that if a certain disease is treated in the traditional way, then the probability that a patient recovers is only 0.6. Now there is a new medicine against the disease, and we want to study its recovery rate p . In a treatment experiment, there were 72 recovered out of 100 patients using this new medicine. Does this experiment provide any evidence that the new recovery rate $p > 0.6$? Answer this question by hypothesis testing with a significance level $\alpha = 0.05$.

(6.1). (1p) Write down H_0 and H_a .

(6.2). (1p) Is H_0 rejected? Why?

(6.3). (1p) For the test in (6.1), what is the probability of not concluding that $p > 0.6$ when the actual $p = 0.75$?

Solution. (6.1)

$$\begin{cases} H_0 : & p = 0.6 \\ H_a : & p > 0.6 \end{cases}$$

(6.2) It directly holds that the observed test statistic under H_0 is

$$TS = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{\frac{72}{100} - 0.6}{\sqrt{0.6(1 - 0.6)/100}} = 2.4495,$$

and the rejection region is

$$C = [z_\alpha, +\infty) = [z_{0.05}, +\infty) = [1.64, +\infty).$$

Since $TS \in C$, we reject H_0 (that is, the sample does provide any evidence that $p > 0.6$).

(6.3) This is actually type-II error

$$\begin{aligned}\beta(0.75) &= P(H_0 \text{ is not rejected when } p = 0.75) = P(TS \notin C \text{ when } p = 0.75) \\&= P\left(\frac{\hat{P} - p_0}{\sqrt{p_0(1-p_0)/n}} < 1.64 \text{ when } p = 0.75\right) = P(\hat{P} < p_0 + 1.64\sqrt{p_0(1-p_0)/n} \text{ when } p = 0.75) \\&= P(\hat{P} < 0.68 \text{ when } p = 0.75) \\&= P\left(\frac{\hat{P} - p}{\sqrt{p(1-p)/n}} < \frac{0.68 - p}{\sqrt{p(1-p)/n}} \text{ when } p = 0.75\right) \\&= P(N(0,1) < \frac{0.68 - 0.75}{\sqrt{0.75(1-0.75)/100}}) = P(N(0,1) < -1.62) = \Phi(-1.62) = 1 - \Phi(1.62) = 1 - 0.9474 = 0.0526.\end{aligned}$$

□

1. Basic probability

$$(1.1) \text{ Conditional probability } P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

$$(1.2) \text{ Total probability } P(B) = \sum_{i=1}^k P(B|A_i)P(A_i) \text{ where } \{A_i\} \text{ are disjoint and } \cup_{i=1}^k A_i = S.$$

$$(1.3) \text{ Bayes' Theorem } P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^k P(B|A_i)P(A_i)} \text{ where } \{A_i\} \text{ are in (1.2).}$$

2. Random variables (r.v.s)

$$(2.1) \text{ Discrete r.v. } X \text{ has a pmf } p(x) = P(X = x) \text{ satisfying } p(x) \geq 0 \text{ and } \sum p(x_i) = 1,$$

$$\begin{array}{c|cccc} X & x_1 & x_2 & \cdots & x_n & \cdots \\ \hline p(x) & p(x_1) & p(x_2) & \cdots & p(x_n) & \cdots \end{array}$$

Expectation (or *Expected value* or *mean*) $\mu_X = E(X) = \sum x_i p(x_i)$;

$$\text{Variance } \sigma_X^2 = V(X) = E(X - \mu_X)^2 = E(X^2) - \mu_X^2 = \sum x_i^2 p(x_i) - \left(\sum x_i p(x_i) \right)^2.$$

$$(2.2) \text{ Continuous r.v. } X \text{ has a pdf } f(x) \text{ satisfying } f(x) \geq 0 \text{ and } \int_{-\infty}^{\infty} f(x) dx = 1,$$

$$P(a < X < b) = \int_a^b f(x) dx.$$

Expectation (or *Expected value* or *mean*) $\mu_X = E(X) = \int_{-\infty}^{\infty} x f(x) dx$;

$$\text{Variance } \sigma_X^2 = V(X) = E(X - \mu_X)^2 = E(X^2) - \mu_X^2 = \int_{-\infty}^{\infty} x^2 f(x) dx - \left(\int_{-\infty}^{\infty} x f(x) dx \right)^2.$$

$$(2.3) \text{ Cumulative distribution function (cdf) of a r.v. } X \text{ is } F(x) = P(X \leq x).$$

$$(2.4) X \text{ and } Y \text{ are r.v.s, } a, b \text{ and } c \text{ are scalars, then}$$

$$E(aX + bY + c) = aE(X) + bE(Y) + c,$$

$$V(aX + bY + c) = a^2 V(X) + b^2 V(Y) + 2ab \operatorname{cov}(X, Y),$$

$$E(g(X, Y)) = \begin{cases} \sum_{i,j} g(x_i, y_j) \cdot p(x_i, y_j), & \text{for discrete } (X, Y), \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \cdot f(x, y) dx dy, & \text{for continuous } (X, Y). \end{cases}$$

$$(2.5) \bullet \text{ Discrete r.v. } (X, Y) \text{ has a joint pmf } p(x, y) \text{ satisfying } p(x, y) \geq 0 \text{ and } \sum x_i \sum y_j p(x_i, y_j) = 1.$$

The *marginal pmf* of X is $p_X(x) = \sum_y p(x, y)$;

The *marginal pmf* of Y is $p_Y(y) = \sum_x p(x, y)$;

X and Y are *independent* if $p(x, y) = p_X(x) \cdot p_Y(y)$.

$$\bullet \text{ Continuous r.v. } (X, Y) \text{ has a joint pdf } p(x, y) \text{ satisfying } f(x, y) \geq 0 \text{ and } \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1.$$

The *marginal pdf* of X is $f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy$;

The *marginal pdf* of Y is $f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx$;

X and Y are *independent* if $f(x, y) = f_X(x) \cdot f_Y(y)$.

3. Several special r.v.s

$$(3.1) X \sim \operatorname{Bin}(n, p) \text{ has a pmf } p(x) = P(X = x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n.$$

$$E(X) = n \cdot p, \quad V(X) = n \cdot p \cdot (1-p).$$

$$(3.2) X \sim \operatorname{Po}(\lambda) \text{ has a pmf } p(x) = P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, \dots$$

$$E(X) = \lambda, \quad V(X) = \lambda.$$

$$(3.3) X \sim \operatorname{Hypergeometric} \text{ has a pmf } p(x) = P(X = x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}.$$

$$(3.4) X \sim \operatorname{Exp}(\lambda) \text{ has a pdf}$$

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$E(X) = \frac{1}{\lambda}, \quad V(X) = \left(\frac{1}{\lambda}\right)^2.$$

$$(3.5) X \sim N(\mu, \sigma^2) \text{ has a pdf}$$

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty.$$

$$E(X) = \mu, \quad V(X) = \sigma^2.$$

$$(3.6) X \sim U(a, b) \text{ has a pdf}$$

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise.} \end{cases}$$

$$E(X) = \frac{a+b}{2}, \quad V(X) = \frac{(b-a)^2}{12}.$$

4. Central Limit Theorem (CLT)

Suppose that a population has mean $= \mu$ and variance $= \sigma^2$. A random sample $\{X_1, X_2, \dots, X_n\}$ from this population is given. Then for large $n \geq 30$,

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1). \quad (1)$$

$$\bullet \text{ If the population is normal, then (1) holds for any } n.$$

$$\bullet \text{ Note that } \mu = E(\bar{X}) \text{ and } (\sigma / \sqrt{n})^2 = V(\bar{X}).$$

5. Several notations in statistics

$$(5.1) \text{ Sample mean: } \bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n} = \sum \frac{X_i}{n}; \quad \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \sum \frac{x_i}{n}.$$

$$(5.2) \text{ Sample variance:}$$

$$S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \frac{1}{n-1} \left(\sum X_i^2 - \frac{(\sum X_i)^2}{n} \right); \quad s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{1}{n-1} \left(\sum x_i^2 - \frac{(\sum x_i)^2}{n} \right).$$

$\bullet$ Capital letters $\bar{X}$ and S^2 refer to the objects based on random sample (therefore they are in general r.v.s), while small letters $\bar{x}$ and s^2 are the objects based on observations (so they are scalars).

$$(5.3) \text{ A point estimator of } \theta \text{ obtained by Methods of Moments is denoted as } \hat{\theta}_{MM}.$$

$$(5.4) \text{ A point estimator of } \theta \text{ obtained by Maximum Likelihood method is denoted as } \hat{\theta}_{ML}.$$

6. Confidence Interval (CI)

In this course, three types of confidence intervals are studied depending on the unknown population parameter(s): CI-1 (confidence intervals for population mean(s)), CI-2 (confidence intervals for population variance(s)), and CI-3 (confidence intervals for population proportion(s)).

CI-1: $(1 - \alpha)$ CI of a population mean μ

case 1.1 (any n) If population $X \sim N(\mu, \sigma^2)$ and σ^2 is known, then $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$ and

$$I_\mu = \left(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right) := \bar{x} \mp z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}.$$

case 1.2 ($n \geq 30$) For any population X , it holds that $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$ and

$$I_\mu = \bar{x} \mp z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \text{ or } I_\mu = \bar{x} \mp z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}}.$$

case 1.3 (any n) If population $X \sim N(\mu, \sigma^2)$ and σ^2 is unknown, then $\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim T(n-1)$ and

$$I_\mu = \bar{x} \mp t_{\alpha/2}(n-1) \cdot \frac{s}{\sqrt{n}}.$$

CI-1': $(1 - \alpha)$ CI of the difference of two population means $\mu_X - \mu_Y$

case 1.1' (any n_1, n_2) If independent populations $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$, and σ_X^2, σ_Y^2 are known, then

$$\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}} \sim N(0, 1), \text{ and } I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}.$$

case 1.2' ($n_1, n_2 \geq 30$) For any independent populations X and Y , it holds that

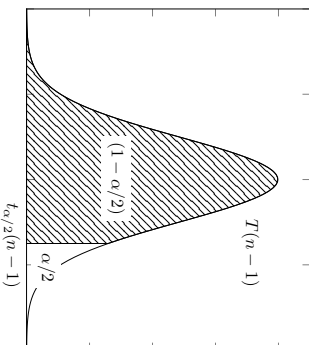
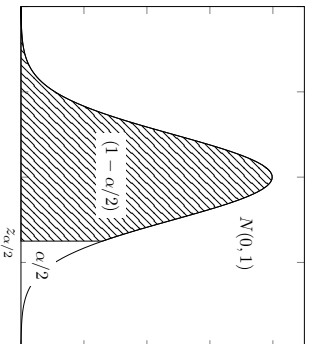
$$\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}} \sim N(0, 1) \text{ and}$$

$$I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}} \text{ or } I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{\sigma}_X^2}{n_1} + \frac{\hat{\sigma}_Y^2}{n_2}}.$$

case 1.3' (any n_1, n_2) If independent populations $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$, where σ_X^2, σ_Y^2 are unknown but $\sigma_X^2 = \sigma_Y^2$, then

$$\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim T(n_1 + n_2 - 2), \text{ where } S^2 = \frac{(n_1 - 1)S_X^2 + (n_2 - 1)S_Y^2}{n_1 + n_2 - 2}, \text{ and}$$

$$I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp t_{\alpha/2}(n_1 + n_2 - 2) \cdot s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}.$$



CI-2: $(1 - \alpha)$ CI of population variance(s) σ^2

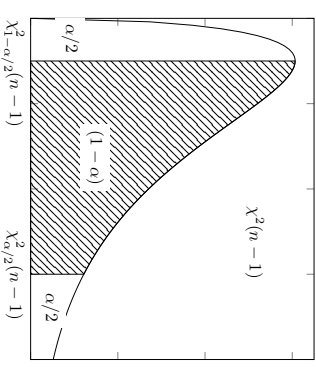
• If a population $X \sim N(\mu, \sigma^2)$ and σ^2 is unknown, then $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, and

$$I_{\sigma^2} = \left(\frac{(n-1)s^2}{\chi_{\alpha/2}^2(n-1)}, \frac{(n-1)s^2}{\chi_{1-\alpha/2}^2(n-1)} \right).$$

• If two independent populations $X \sim N(\mu_X, \sigma^2)$ and $Y \sim N(\mu_Y, \sigma^2)$, and σ^2 is unknown, then $\frac{(n_1+n_2-2)S^2}{\sigma^2} \sim \chi^2(n_1+n_2-2)$, and

$$I_{\sigma^2} = \left(\frac{(n_1+n_2-2)s^2}{\chi_{\alpha/2}^2(n_1+n_2-2)}, \frac{(n_1+n_2-2)s^2}{\chi_{1-\alpha/2}^2(n_1+n_2-2)} \right),$$

where $S^2 = \frac{(n_1-1)S_X^2 + (n_2-1)S_Y^2}{n_1+n_2-2}$.



CI-3: $(1 - \alpha)$ CI of population proportion(s)

- If a (large) population has an unknown proportion p , then $\frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \sim N(0, 1)$ if $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ with $\hat{p} = x/n$, and $I_p = \hat{p} \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.
- If two independent (large) populations have unknown proportions p_1 and p_2 , then

$$\frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} \sim N(0, 1)$$

if $n_i \hat{p}_i \geq 10$ and $n_i(1 - \hat{p}_i) \geq 10$ for $i = 1, 2$, and $I_{p_1 - p_2} = (\hat{p}_1 - \hat{p}_2) \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$.

7. Hypothesis Test (HT)

	H_0 is true	H_0 is false and $\theta = \theta_1$
reject H_0	(type I error or significance level) α	(power) $h(\theta_1)$
don't reject H_0	$1 - \alpha$	(type II error) $\beta(\theta_1) = 1 - h(\theta_1)$

reject $H_0 \Leftrightarrow TS \in C \Leftrightarrow p\text{-value} < \alpha$

χ^2 tests for populations (non-parametric tests)

Suppose that for a random sample of a population X , the n elements of it are classified into k disjoint groups A_i , $1 \leq i \leq k$. For each group A_i , $1 \leq i \leq k$, suppose that there are N_i , $1 \leq i \leq k$ elements inside. Let $p_i = P(A_i)$ assuming a given distribution of X . Note that $p_1 + p_2 + \dots + p_k = 1$ and $N_1 + N_2 + \dots + N_k = n$. One wants to test the hypotheses

$$H_0 : P(A_i) = p_i, \quad 1 \leq i \leq k, \quad H_a : P(A_i) \neq p_i \text{ for some } 1 \leq i \leq k.$$

If n is large in the sense that $np_i \geq 5$ for all $1 \leq i \leq k$, then the test statistic is

$$\sum_{i=1}^k \frac{(N_i - np_i)^2}{np_i} \approx \chi^2(k-1).$$

Therefore the observation of the test statistic is

$$TS = \sum_{i=1}^k \frac{(n_i - np_i)^2}{np_i}, \text{ where } n_i \text{ is the observation of } N_i, 1 \leq i \leq k.$$

For the critical region C , one can take (note that if H_0 is true, then TS should be close to zero)

$$C = [\chi_\alpha^2(k-1), \infty).$$

The conclusion would be $TS \in C \iff H_0$ is rejected.

8. Linear and logistic regression

(Multiple) linear regression: $Y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + \varepsilon$, $\varepsilon \sim N(0, \sigma^2)$.

- Y : response variable (which is normal r.v.), $\{x_1, \dots, x_k\}$: predictors (which are scalars).
- sample: $\{(x_{11}, \dots, x_{1k}; y_1), (x_{21}, \dots, x_{2k}; y_2), \dots, (x_{n1}, \dots, x_{nk}; y_n)\}$.
- how to estimate $\beta_j \approx \hat{\beta}_j$: least square method, that is, to minimize $\sum_{i=1}^n (\hat{y}_i - y_i)^2$, where the estimated (multiple) linear regression line $\hat{y}$ is

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k.$$

- $\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \sim T(n-k-1)$, this helps determine whether or not the real $\beta_j = 0$?
- $\sigma^2 \approx \frac{SSE}{n-k-1}$, this gives an estimation of the size of the error.
- $R^2 = \frac{SSR}{SS_T}$, this gives how well the model is (if $R^2 \approx 1$, then the model fits the sample very well).
- How to test $\beta_1 = \dots = \beta_k = 0$? Use the random variable $\frac{SSR/k}{SSE/(n-k-1)} \sim F(k, n-k-1)$.

Logistic regression: Let Y can only take 0 or 1 with $P(Y=1) = p$ and $P(Y=0) = 1-p$.

$$E(Y) = p(x_1, \dots, x_k) = \frac{e^{\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k}}{1 + e^{\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k}}.$$

- Y : response variable (which is Bernoulli r.v. $P(Y=1) = p$ and $P(Y=0) = 1-p$, so $E(Y) = p$), $\{x_1, \dots, x_k\}$: predictors (which are scalars).
- sample: $\{(x_{11}, \dots, x_{1k}; y_1), (x_{21}, \dots, x_{2k}; y_2), \dots, (x_{n1}, \dots, x_{nk}; y_n)\}$.
- how to estimate $\beta_j \approx \hat{\beta}_j$: maximal likelihood method (maximize $\prod_{i=1}^n p(x_{i1}, \dots, x_{ik})^{y_i} (1 - p(x_{i1}, \dots, x_{ik}))^{1-y_i}$).
- $\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \approx N(0, 1)$ for large $n \geq 30$, this helps determine whether or not the real $\beta_j = 0$?
- Classification of a new object $Y(x_1, \dots, x_k)$ as 1 or 0 according

$$Y(x_1, \dots, x_k) = \begin{cases} 1, & \text{if } \hat{p}(x_1, \dots, x_k) \geq 0.5, \\ 0, & \text{if } \hat{p}(x_1, \dots, x_k) < 0.5, \end{cases}$$

where the estimated logit function $\hat{p}(x_1, \dots, x_k)$ is

$$\hat{p}(x_1, \dots, x_k) = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k}}{1 + e^{\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k}}.$$

9. Tables

(9.1) Table for $N(0, 1)$ standard normal random variable $\Phi(x) = P(N(0, 1) \leq x)$, $x \geq 0$.
There is an important relation $\Phi(-x) = 1 - \Phi(x)$, $x \geq 0$.

x	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9935
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

(9.2) Table for $T(f)$ random variable $F(x) = P(T(f) \leq x)$,
where f is a parameter called 'degrees of freedom'.

f	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.9995
1	1.00	3.08	6.31	12.71	31.82	63.66	127.32	636.62
2	0.82	1.89	2.92	4.30	6.96	9.92	14.09	31.60
3	0.76	1.64	2.35	3.18	4.54	5.84	7.45	12.92
4	0.74	1.53	2.13	2.78	3.75	4.60	5.60	8.61
5	0.73	1.48	2.02	2.57	3.36	4.03	4.77	6.87
6	0.72	1.44	1.94	2.45	3.14	3.71	4.32	5.96
7	0.71	1.41	1.89	2.36	3.00	3.50	4.03	5.41
8	0.71	1.40	1.86	2.31	2.90	3.36	3.83	5.04
9	0.70	1.38	1.83	2.26	2.82	3.25	3.69	4.78
10	0.70	1.37	1.81	2.23	2.76	3.17	3.58	4.59
11	0.70	1.36	1.80	2.20	2.72	3.11	3.50	4.44
12	0.70	1.36	1.78	2.18	2.68	3.05	3.43	4.32
13	0.69	1.35	1.77	2.16	2.65	3.01	3.37	4.22
14	0.69	1.35	1.76	2.14	2.62	2.98	3.33	4.14
15	0.69	1.34	1.75	2.13	2.60	2.95	3.29	4.07
16	0.69	1.34	1.75	2.12	2.58	2.92	3.25	4.01
17	0.69	1.33	1.74	2.11	2.57	2.90	3.22	3.97
18	0.69	1.33	1.73	2.10	2.55	2.88	3.20	3.92
19	0.69	1.33	1.73	2.09	2.54	2.86	3.17	3.88
20	0.69	1.33	1.72	2.09	2.53	2.85	3.15	3.85
21	0.69	1.32	1.72	2.08	2.52	2.83	3.14	3.82
22	0.69	1.32	1.72	2.07	2.51	2.82	3.12	3.79
23	0.69	1.32	1.71	2.07	2.50	2.81	3.10	3.77
24	0.68	1.32	1.71	2.06	2.49	2.80	3.09	3.75
25	0.68	1.32	1.71	2.06	2.49	2.79	3.08	3.73
26	0.68	1.31	1.71	2.06	2.48	2.78	3.07	3.71
27	0.68	1.31	1.70	2.05	2.47	2.77	3.06	3.69
28	0.68	1.31	1.70	2.05	2.47	2.76	3.05	3.67
29	0.68	1.31	1.70	2.05	2.46	2.76	3.04	3.66
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03	3.65
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97	3.55
50	0.68	1.30	1.68	2.01	2.40	2.68	2.94	3.50
60	0.68	1.30	1.67	2.00	2.39	2.66	2.91	3.46
100	0.68	1.29	1.66	1.98	2.36	2.63	2.87	3.39
∞	0.67	1.28	1.65	1.96	2.33	2.58	2.81	3.29

(9.3) Table for $\chi^2(f)$ random variable $F(x) = P(\chi^2(f) \leq x)$, where f is a parameter.

f	$F(x)$														
	0.0005	0.001	0.005	0.01	0.025	0.05	0.10	0.20	0.30	0.40	0.50				
1	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.15	0.27	0.45				
2	0.00	0.00	0.01	0.02	0.05	0.10	0.21	0.45	0.71	1.02	1.39				
3	0.02	0.02	0.07	0.11	0.22	0.35	0.58	1.01	1.42	1.87	2.37				
4	0.06	0.09	0.21	0.30	0.48	0.71	1.06	1.65	2.19	2.75	3.36				
5	0.16	0.21	0.41	0.55	0.83	1.15	1.61	2.34	3.00	3.66	4.35				
6	0.30	0.38	0.68	0.87	1.24	1.64	2.20	3.07	3.83	4.57	5.35				
7	0.48	0.60	0.99	1.24	1.69	2.17	2.83	3.82	4.67	5.49	6.35				
8	0.71	0.86	1.34	1.65	2.18	2.73	3.49	4.59	5.53	6.42	7.34				
9	0.97	1.15	1.73	2.09	2.70	3.33	4.17	5.38	6.39	7.36	8.34				
10	1.26	1.48	2.16	2.56	3.25	3.94	4.87	6.18	7.27	8.30	9.34				
11	1.59	1.83	2.60	3.05	3.82	4.57	5.58	6.99	8.15	9.24	10.34				
12	1.93	2.21	3.07	3.57	4.40	5.23	6.30	7.81	9.03	10.18	11.34				
13	2.31	2.62	3.57	4.11	5.01	5.89	7.04	8.63	9.93	11.13	12.34				
14	2.70	3.04	4.07	4.66	5.63	6.57	7.79	9.47	10.82	12.08	13.34				
15	3.11	3.48	4.60	5.23	6.26	7.26	8.55	10.31	11.72	13.03	14.34				
16	3.54	3.94	5.14	5.81	6.91	7.96	9.31	11.15	12.62	13.98	15.34				
17	3.98	4.42	5.70	6.41	7.56	8.67	10.09	12.00	13.53	14.94	16.34				
18	4.44	4.90	6.26	7.01	8.23	9.39	10.86	12.86	14.44	15.89	17.34				
19	4.91	5.41	6.84	7.63	8.91	10.12	11.65	13.72	15.35	16.85	18.34				
20	5.40	5.92	7.43	8.26	9.59	10.85	12.44	14.58	16.27	17.81	19.34				
21	5.90	6.45	8.03	8.90	10.28	11.59	13.24	15.44	17.18	18.77	20.34				
22	6.40	6.98	8.64	9.54	10.98	12.34	14.04	16.31	18.10	19.73	21.34				
23	6.92	7.53	9.26	10.20	11.69	13.09	14.85	17.19	19.02	20.69	22.34				
24	7.45	8.08	9.89	10.86	12.40	13.85	15.66	18.06	19.94	21.65	23.34				
25	7.99	8.65	10.52	11.52	13.12	14.61	16.47	18.94	20.87	22.62	24.34				
26	8.54	9.22	11.16	12.20	13.84	15.38	17.29	19.82	21.79	23.58	25.34				
27	9.09	9.80	11.81	12.88	14.57	16.15	18.11	20.70	22.72	24.54	26.34				
28	9.66	10.39	12.46	13.56	15.31	16.93	18.94	21.59	23.65	25.51	27.34				
29	10.23	10.99	13.12	14.26	16.05	17.71	19.77	22.48	24.58	26.48	28.34				
30	10.80	11.59	13.79	14.95	16.79	18.49	20.60	23.36	25.51	27.44	29.34				
40	16.91	17.92	20.71	22.16	24.43	26.51	29.05	32.34	34.87	37.13	39.34				
50	23.46	24.67	27.99	29.71	32.36	34.76	37.69	41.45	44.31	46.86	49.33				
60	30.34	31.74	35.53	37.48	40.48	43.19	46.46	50.64	53.81	56.62	59.33				
100	59.90	61.92	67.33	70.06	74.22	77.93	82.36	87.95	92.13	95.81	99.33				

Table for $\chi^2(f)$ random variable $F(x) = P(\chi^2(f) \leq x)$, where f is a parameter.

	$F(x)$													
f	0.60	0.70	0.80	0.90	0.95	0.975	0.99	0.995	0.999	0.9995				
1	0.71	1.07	1.64	2.71	3.84	5.02	6.63	7.88	10.83	12.12				
2	1.83	2.41	3.22	4.61	5.99	7.38	9.21	10.60	13.82	15.20				
3	2.95	3.66	4.64	6.25	7.81	9.35	11.34	12.84	16.27	17.73				
4	4.04	4.88	5.99	7.78	9.49	11.14	13.28	14.86	18.47	20.00				
5	5.13	6.06	7.29	9.24	11.07	12.83	15.09	16.75	20.52	22.11				
6	6.21	7.23	8.56	10.64	12.59	14.45	16.81	18.55	22.46	24.10				
7	7.28	8.38	9.80	12.02	14.07	16.01	18.48	20.28	24.32	26.02				
8	8.35	9.52	11.03	13.36	15.51	17.53	20.09	21.95	26.12	27.87				
9	9.41	10.66	12.24	14.68	16.92	19.02	21.67	23.59	27.88	29.67				
10	10.47	11.78	13.44	15.99	18.31	20.48	23.21	25.19	29.59	31.42				
11	11.53	12.90	14.63	17.28	19.68	21.92	24.72	26.76	31.26	33.14				
12	12.58	14.01	15.81	18.55	21.03	23.34	26.22	28.30	32.91	34.82				
13	13.64	15.12	16.98	19.81	22.36	24.74	27.69	29.82	34.53	36.48				
14	14.69	16.22	18.15	21.06	23.68	26.12	29.14	31.32	36.12	38.11				
15	15.73	17.32	19.31	22.31	25.00	27.49	30.58	32.80	37.70	39.72				
16	16.78	18.42	20.47	23.54	26.30	28.85	32.00	34.27	39.25	41.31				
17	17.82	19.51	21.61	24.77	27.59	30.19	33.41	35.72	40.79	42.88				
18	18.87	20.60	22.76	25.99	28.87	31.53	34.81	37.16	42.31	44.43				
19	19.91	21.69	23.90	27.20	30.14	32.85	36.19	38.58	43.82	45.97				
20	20.95	22.77	25.04	28.41	31.41	34.17	37.57	40.00	45.31	47.50				
21	21.99	23.86	26.17	29.62	32.67	35.48	38.93	41.40	46.80	49.01				
22	23.03	24.94	27.30	30.81	33.92	36.78	40.29	42.80	48.27	50.51				
23	24.07	26.02	28.43	32.01	35.17	38.08	41.64	44.18	49.73	52.00				
24	25.11	27.10	29.55	33.20	36.42	39.36	42.98	45.56	51.18	53.48				
25	26.14	28.17	30.68	34.38	37.65	40.65	44.31	46.93	52.62	54.95				
26	27.18	29.25	31.79	35.56	38.89	41.92	45.64	48.29	54.05	56.41				
27	28.21	30.32	32.91	36.74	40.11	43.19	46.96	49.64	55.48	57.86				
28	29.25	31.39	34.03	37.92	41.34	44.46	48.28	50.99	56.89	59.30				
29	30.28	32.46	35.14	39.09	42.56	45.72	49.59	52.34	58.30	60.73				
30	31.32	33.53	36.25	40.26	43.77	46.98	50.89	53.67	59.70	62.16				
40	41.62	44.16	47.27	51.81	55.76	59.34	63.69	66.77	73.40	76.09				
50	51.89	54.72	58.16	63.17	67.50	71.42	76.15	79.49	86.66	89.56				
60	62.13	65.23	68.97	74.40	79.08	83.30	88.38	91.95	99.61	102.69				
100	102.95	106.91	111.67	118.50	124.34	129.56	135.81	140.17	149.45	153.17				

(9.4) Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1 - p) \geq n - k)$.

n	k	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
	1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500
3	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250
	1	0.9928	0.9720	0.9392	0.8960	0.8438	0.7840	0.7183	0.6480	0.5747	0.5000
4	0	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750
	1	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625
5	0	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125
	1	0.9974	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875
6	0	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000
	1	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125
7	0	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688
	1	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
8	0	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094
	1	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438
9	0	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563
	1	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906
10	0	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844
	1	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
11	0	0.9356	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625
	1	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266
12	0	0.9998	0.9973	0.9879	0.9673	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000
	1	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734
13	0	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375
	1	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9987	0.9964	0.9915	0.9648
14	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
	1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352
15	0	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445
	1	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633
16	0	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367
	1	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555
17	0	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648
	1	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961
18	0	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
	1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
19	0	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2218	0.1495	0.0898
	1	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
20	0	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
	1	1.0000	0.9999	0.9994	0.9999	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
21	0	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102
	1	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805
22	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980

Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1 - p) \geq n - k)$.

n	k	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
10	0	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107
11	0	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547
	1	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719
12	0	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770
	1	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230
13	0	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281
	1	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453
14	0	0.5688	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005
	1	0.8981	0.6974	0.4922	0.3221	0.1971	0.1130	0.0606	0.0302	0.0139	0.0059
15	0	0.9848	0.9104	0.7788	0.6174	0.4552	0.3127	0.2001	0.1189	0.0652	0.0327
	1	0.9984	0.9815	0.9306	0.8389	0.7133	0.5696	0.4256	0.2963	0.1911	0.1133
16	0	0.9999	0.9972	0.9841	0.9496	0.8854	0.7897	0.6683	0.5328	0.3971	0.2744
	1	1.0000	0.9997	0.9973	0.9883	0.9657	0.9218	0.8513	0.7535	0.6331	0.5000
17	0	1.0000	1.0000	0.9997	0.9980	0.9924	0.9784	0.9499	0.9006	0.8262	0.7256
	1	1.0000	1.0000	1.0000	0.9998	0.9988	0.9957	0.9878	0.9707	0.9390	0.8867
18	0	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9980	0.9941	0.9852	0.9673
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9978	0.9941
19	0	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0003
	1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032
20	0	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193
	1	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730
21	0	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938
	1	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872
22	0	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.9154	0.8418	0.7393	0.6128
	1	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062
23	0	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270
	1	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9972	0.9921	0.9807
24	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998

Table for Binomial random variable $P(\text{Bin}(n, p) \leq k)$ if $p \leq 0.5$.
If $p > 0.5$, then $P(\text{Bin}(n, p) \leq k) = P(\text{Bin}(n, 1 - p) \geq n - k)$.

n	k	p														
		0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50					
14	0	0.4877	0.2288	0.1028	0.0440	0.0178	0.0068	0.0024	0.0008	0.0002	0.0001					
	1	0.8470	0.5846	0.3567	0.1979	0.1010	0.0475	0.0205	0.0081	0.0029	0.0009					
	2	0.9699	0.8416	0.6479	0.4481	0.2811	0.1608	0.0839	0.0398	0.0170	0.0065					
	3	0.9958	0.9559	0.8535	0.6982	0.5213	0.3552	0.2205	0.1243	0.0632	0.0287					
	4	0.9996	0.9908	0.9533	0.8702	0.7415	0.5842	0.4227	0.2793	0.1672	0.0898					
	5	1.0000	0.9985	0.9885	0.9561	0.8883	0.7805	0.6405	0.4859	0.3373	0.2120					
	6	1.0000	0.9998	0.9978	0.9884	0.9617	0.9067	0.8164	0.6925	0.5461	0.3953					
	7	1.0000	1.0000	0.9997	0.9976	0.9897	0.9685	0.9247	0.8499	0.7414	0.6047					
	8	1.0000	1.0000	1.0000	0.9996	0.9978	0.9917	0.9757	0.9417	0.8811	0.7880					
	9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9983	0.9940	0.9825	0.9574	0.9102					
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9989	0.9961	0.9886	0.9713					
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9978	0.9935					
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991					
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999					
15	0	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000					
	1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005					
	2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037					
	3	0.9945	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176					
	4	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592					
	5	0.9999	0.9832	0.9389	0.8389	0.6816	0.5216	0.3608	0.2193	0.1204	0.0592					
	6	1.0000	0.9997	0.9664	0.8819	0.7434	0.6098	0.4522	0.3036	0.1766	0.0937					
	7	1.0000	1.0000	0.9994	0.9819	0.8627	0.7500	0.6378	0.5272	0.4222	0.3036					
	8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964					
	9	1.0000	1.0000	1.0000	0.9999	0.9999	0.9993	0.9972	0.9931	0.9745	0.9408					
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9974	0.9963					
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999					
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000					
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000					
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000					
16	0	0.4401	0.1853	0.0743	0.0281	0.0100	0.0033	0.0010	0.0003	0.0001	0.0000					
	1	0.8108	0.5147	0.2839	0.1407	0.0635	0.0261	0.0098	0.0033	0.0010	0.0003					
	2	0.9571	0.7892	0.5614	0.3518	0.1971	0.0994	0.0451	0.0183	0.0066	0.0021					
	3	0.9930	0.9316	0.7899	0.5981	0.4050	0.2459	0.1339	0.0651	0.0281	0.0106					
	4	0.9991	0.9830	0.9209	0.7982	0.6302	0.4499	0.2892	0.1666	0.0853	0.0384					
	5	0.9999	0.9967	0.9765	0.9183	0.8103	0.6598	0.4900	0.3288	0.1976	0.1051					
	6	1.0000	0.9995	0.9949	0.9733	0.9204	0.8247	0.6881	0.5272	0.3660	0.2272					
	7	1.0000	0.9999	0.9989	0.9930	0.9729	0.9256	0.8406	0.7161	0.5629	0.4018					
	8	1.0000	1.0000	0.9998	0.9985	0.9925	0.9743	0.9271	0.8577	0.7441	0.5982					
	9	1.0000	1.0000	1.0000	0.9998	0.9984	0.9929	0.9711	0.9417	0.8759	0.7728					
	10	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9938	0.9809	0.9514	0.8949					
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9851	0.9616					
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9991	0.9965	0.9849					
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9994	0.9979					
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997					
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000					

Table for Binomial random variable $P(\text{Bin}(n, p) \leq k)$ if $p \leq 0.5$.
If $p > 0.5$, then $P(\text{Bin}(n, p) \leq k) = P(\text{Bin}(n, 1 - p) \geq n - k)$.

n	k	p														
		0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50					
17	0	0.4181	0.1668	0.0631	0.0225	0.0075	0.0023	0.0007	0.0002	0.0000	0.0000					
	1	0.7922	0.4818	0.2525	0.1182	0.0501	0.0193	0.0067	0.0021	0.0006	0.0001					
	2	0.9497	0.7618	0.5198	0.3096	0.1637	0.0774	0.0327	0.0123	0.0041	0.0012					
	3	0.9912	0.9174	0.7556	0.5489	0.3530	0.2019	0.1028	0.0464	0.0184	0.0064					
	4	0.9988	0.9779	0.9013	0.7582	0.5739	0.3887	0.2348	0.1260	0.0596	0.0245					
	5	0.9999	0.9953	0.9681	0.8943	0.7653	0.5968	0.4197	0.2639	0.1471	0.0717					
	6	1.0000	0.9992	0.9917	0.9623	0.8929	0.7752	0.6188	0.4478	0.2902	0.1662					
	7	1.0000	0.9999	0.9983	0.9891	0.9588	0.8954	0.7872	0.6405	0.4743	0.3145					
	8	1.0000	1.0000	0.9997	0.9974	0.9876	0.9597	0.9006	0.8011	0.6626	0.5000					
	9	1.0000	1.0000	1.0000	0.9995	0.9969	0.9873	0.9617	0.9081	0.8166	0.6855					
	10	1.0000	1.0000	1.0000	0.9999	0.9994	0.9968	0.9880	0.9652	0.9174	0.8338					
	11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9970	0.9894	0.9699	0.9283					
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9975	0.9914	0.9755					
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9981	0.9936					
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9988					
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999					
18	0	0.3972	0.1501	0.0536	0.0180	0.0056	0.0016	0.0004	0.0001	0.0000	0.0000					
	1	0.7735	0.4503	0.2241	0.0991	0.0395	0.0142	0.0046	0.0013	0.0003	0.0001					
	2	0.9419	0.7338	0.4797	0.2713	0.1353	0.0600	0.0236	0.0082	0.0025	0.0007					
	3	0.9891	0.9018	0.7202	0.5010	0.3057	0.1646	0.0783	0.0328	0.0120	0.0038					
	4	0.9985	0.9718	0.8794	0.7164	0.5187	0.3327	0.1866	0.0942	0.0411	0.0154					
	5	0.9998	0.9936	0.9581	0.8671	0.7175	0.5344	0.3550	0.2088	0.1077	0.0481					
	6	1.0000	0.9988	0.9882	0.9487	0.8610	0.7217	0.5491	0.3783	0.2258	0.1189					
	7	1.0000	0.9998	0.9973	0.9837	0.9431	0.8593	0.7283	0.5634	0.3915	0.2403					
	8	1.0000	1.0000	0.9995	0.9957	0.9807	0.9404	0.8609	0.7368	0.5778	0.4073					
	9	1.0000	1.0000	0.9999	0.9991	0.9946	0.9790	0.9403	0.8653	0.7473	0.5927					
	10	1.0000	1.0000	1.0000	0.9998	0.9988	0.9988	0.9939	0.9788	0.9424	0.8720					
	11	1.0000	1.0000	1.0000	1.0000	0.9998	0.9996	0.9938	0.9638	0.9797	0.9463					
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9986	0.9942	0.9817	0.9519					
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9846					
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9998	0.9990	0.9962					
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993					
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999						
19	0	0.3774	0.1351	0.0456	0.0144	0.0042	0.0011	0.0003	0.0001	0.0000	0.0000					
	1	0.7547	0.4203	0.1985	0.0829	0.0310	0.0104	0.0031	0.0008	0.0002	0.0004					
	2	0.9335	0.7054	0.4413	0.2369	0.1113	0.0462	0.0170	0.0055	0.0015	0.0004					
	3	0.9868	0.8550	0.6841	0.4551	0.2631	0.1332	0.0591	0.0230	0.0077	0.0022					
	4	0.9980	0.9648	0.8556	0.6733	0.4654	0.2822	0.1500	0.0696	0.0280	0.0096					
	5	0.9998	0.9914	0.9463	0.8369	0.6678	0.4739	0.2968	0.1629	0.0777	0.0318					
	6	1.0000	0.9983	0.9837	0.9324	0.8251	0.6655	0.4812	0.3081	0.1727	0.0835					
	7	1.0000	0.9997	0.9959	0.9767	0.9225	0.8180	0.6656	0.4587	0.3169	0.1796					
	8	1.0000	1.0000	0.9992	0.9933	0.9713	0.9161	0.8145	0.6675	0.4940	0.3238					
	9	1.0000	1.0000	0.9999	0.9984	0.9914	0.9674	0.9125	0.8139	0.6710	0.5000					
	10	1.0000	1.0000	1.0000	0.9997	0.9977	0.9895	0.9895	0.9863	0.9115	0.8159					
	11	1.0000	1.0000	1.0000	1.0000	0.9995	0.9972	0.9886	0.9868	0.9648	0.9129					
	12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9969	0.9884	0.9658	0.9165					
	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9969	0.9891	0.9682					
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9974	0.9904					
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9995	0.9978					
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996						

(9.5) Table for Poisson random variable $P(Po(\mu) \leq k)$.

k	μ									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9536	0.9371	0.9197
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
k	μ									
	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
1	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337	0.4060
2	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037	0.6767
3	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747	0.8571
4	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559	0.9473
5	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868	0.9834
6	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966	0.9955
7	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992	0.9989
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9998
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
k	μ									
	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550	0.0498
1	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146	0.1991
2	0.6496	0.6227	0.5960	0.5697	0.5438	0.5184	0.4936	0.4695	0.4460	0.4232
3	0.8386	0.8194	0.7993	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696	0.6472
4	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8629	0.8477	0.8318	0.8153
5	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258	0.9161
6	0.9941	0.9925	0.9906	0.9884	0.9858	0.9828	0.9794	0.9756	0.9713	0.9665
7	0.9985	0.9980	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901	0.9881
8	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969	0.9962
9	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9993	0.9991	0.9989
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table for Poisson random variable $P(Po(\mu) \leq k)$.

k	μ									
	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0
0	0.0408	0.0334	0.0273	0.0224	0.0183	0.0150	0.0123	0.0101	0.0082	0.0067
1	0.1712	0.1468	0.1257	0.1074	0.0916	0.0780	0.0663	0.0563	0.0477	0.0404
2	0.3799	0.3397	0.3027	0.2689	0.2381	0.2102	0.1851	0.1626	0.1425	0.1247
3	0.6025	0.5584	0.5152	0.4735	0.4335	0.3954	0.3594	0.3257	0.2942	0.2650
4	0.7806	0.7442	0.7064	0.6678	0.6288	0.5898	0.5512	0.5132	0.4763	0.4405
5	0.8946	0.8705	0.8441	0.8156	0.7851	0.7531	0.7199	0.6858	0.6510	0.6160
6	0.9654	0.9421	0.9267	0.9091	0.8893	0.8675	0.8436	0.8180	0.7908	0.7622
7	0.9832	0.9769	0.9682	0.9599	0.9509	0.9409	0.9361	0.9214	0.9049	0.8867
8	0.9943	0.9917	0.9883	0.9840	0.9786	0.9721	0.9642	0.9549	0.9442	0.9319
9	0.9982	0.9973	0.9960	0.9942	0.9919	0.9889	0.9851	0.9805	0.9749	0.9682
10	0.9995	0.9992	0.9987	0.9981	0.9972	0.9959	0.9943	0.9922	0.9896	0.9863
11	0.9999	0.9998	0.9996	0.9994	0.9991	0.9986	0.9980	0.9971	0.9960	0.9945
12	1.0000	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9993	0.9990	0.9986
13	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9998	0.9997	0.9995	0.9993
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
k	μ									
	5.2	5.4	5.6	5.8	6.0	6.5	7.0	7.5	8.0	8.5
0	0.0055	0.0045	0.0037	0.0030	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002
1	0.0342	0.0289	0.0244	0.0206	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019
2	0.1088	0.0948	0.0824	0.0715	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093
3	0.2381	0.2133	0.1906	0.1700	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301
4	0.4061	0.3733	0.3422	0.3127	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744
5	0.5589	0.5461	0.5119	0.4783	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496
6	0.7324	0.7017	0.6703	0.6384	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562
7	0.8449	0.8217	0.7970	0.7710	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856
8	0.9181	0.9027	0.8857	0.8672	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231
9	0.9603	0.9512	0.9409	0.9292	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530
10	0.9823	0.9775	0.9718	0.9651	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634
11	0.9927	0.9904	0.9875	0.9841	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487
12	0.9972	0.9962	0.9949	0.9932	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091
13	0.9990	0.9986	0.9980	0.9973	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486
14	0.9997	0.9995	0.9993	0.9990	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726
15	0.9999	0.9998	0.9998	0.9996	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862
16	1.0000	0.9999	0.9999	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934
17	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970
18	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table for Poisson random variable $P(Po(\mu) \leq k)$.

k	μ										
	9.0	9.5	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0	
0	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
1	0.0012	0.0008	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	
2	0.0062	0.0042	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	
3	0.0212	0.0149	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	
4	0.0550	0.0403	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	
5	0.1157	0.0885	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	
6	0.2068	0.1649	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	
7	0.3239	0.2687	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	
8	0.4557	0.3918	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	
9	0.5874	0.5218	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	
10	0.7060	0.6453	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	
11	0.8030	0.7520	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	
12	0.8758	0.8364	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	
13	0.9261	0.8981	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	
14	0.9585	0.9400	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	
15	0.9780	0.9665	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	
16	0.9889	0.9823	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677	
17	0.9947	0.9911	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640	
18	0.9976	0.9957	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550	
19	0.9989	0.9980	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363	
20	0.9996	0.9991	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055	
21	0.9998	0.9996	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615	
22	0.9999	0.9999	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047	
23	1.0000	0.9999	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367	
24	1.0000	1.0000	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594	
25	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9974	0.9938	0.9869	0.9748	
26	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9987	0.9967	0.9925	0.9848	
27	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9983	0.9959	0.9912	
28	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9978	0.9950	
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9986	
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9996	
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	