## TAMS38 Computer exercises 3

**Preparation:** Read about Two-Way ANOVA and corresponding non-parametric methods and the methods to construct the confidence intervals.

Bring collection of formulas and tables, and calculators to computer exercise.

## 1 – Complete Two-Way ANOVA

The following data material consists of life in hours for three types of aircraft wings subjected to two kinds of vibration, one kind of constant frequency with the second frequency which sweeps back and forth in a given band.

	Types of aircraft wings					
	1	2	3			
Vibr 1	1676	883	1212			
	1510	961	1090			
Vibr 2	1915	948	1091			
	1763	1062	1212			

We should analysis data with the complete two-factor model using Minitab.

a) How does this two-factor model look like?.

Open the new project in Minitab. Put the observations in column C1, levels of the factor A (vibration type) in C2 and levels of factor B (type of aircraft wings) in C3. It may be practical to enter the name in the name row in the data window. Call C1 by Y, C2 by A, C3 by B. Instead of A and B you can use the appropriate words to speak about what factors involved. It is wise to start by plotting the data in order to possibly be able to detect outliers. For example using comments

PLOT C1\*C2 PLOT C1\*C3 we get plots against each of the different factor levels. Command

TABLE C2 C3; MEAN C1.

give the cell averages.

Go to **Stat/ANOVA/Interactions plot** so you get a plots on which you can see how each type of the wings responds to the two vibration types. Do factors appear to be additive? Variance analysis can be done for example via **Stat/ANOVA/Balanced Anova**. Choose

Responses: Y Model: A|B Graphs: Normal plot of residuals Residuals versus fits Results: Display means corr... A|B

The line in A|B means that we require an analysis under the complete model with all possible interaction effects. Some comments on the plots? Answer the following questions

b) Does type of vibration have importance for lifetime? Perform appropriate test of main effects on the level 0.01.

c) Does the wing type have importance for lifetime? Perform appropriate test of main effects on the 0.01.

d) Is there interaction effect between vibratory and wing type? Perform appropriate test on the 0.05. Compare to interactions plots.

e) Despite the result in d) keep the complete two-factor model  $Y_{ijk} = \mu_{ij} + \varepsilon_{ijk}$ , where  $\varepsilon_{ijk}$  are independent and  $N(0, \sigma)$ . Do the pairwise comparisons between different types of wings for each vibration type using the appropriate condidence interval with the simultaneous confidence level at most 0.88. Is some wing type superior to the other?

## 2 – Additive two-factor model

Charging status for a car battery is given by the specific capacity of the electrolyte and for lead-acid battery the responds value is 1.260 when battery is completely charged. They want to examine three kinds of batteries ability to maintain its charge status. Three different cars were used for the experiment, and because cars could be assumed to have different characteristics we have ensured that each battery was used once for each vehicle. Vehicle number j had three test periods of for each battery type 1, 2, 3 in random order

(trials with randomized within each block = car). During the test periods cars were used in a similar way. It took three batteries of each type. After each test period battery charging status was measured. Results (decimal point is omitted):

		Car	
Type of battery	1	2	3
1	1250	1252	1244
2	1258	1258	1250
3	1248	1252	1244

Analyze the data using the additive two-factor analysis (model 1). Let A be a battery type and B be a car type. Open a new project in Minitab. Put observations Y in C1, A-levels in C2 and B-levels in C3 and call the columns accordingly. Do variance analysis for additive model and obtain row- and column-averages through **Stat/ANOVA/Balanced Anova** and

Responses: Y Model: A B Storage: Klicka för Residuals och Fits Results: Display means... A B

Here, A B in the model indicate that we have chosen additive model. Now, we have obtained residuals in C4 and estimated expectations (fits) in C5. For safety reasons we also make

NSCORES C4 C6 PLOT C6\*C4; SYMBOL C3.

which gives normal probability plot, where the observations for those different cars is marked (of course residual plots are more interesting for somewhat larger datasets).

Moreover, analyze the data according to a one-factor model (model 2) where we have sample for each battery type (this means that the cars, in the model, are considered to be equivalent). Do the normal probability plot for this model is the same way as above.

Answer the following questions, where a)-c) are about model 1 and d)-g) about model 2.

a) Write out the model 1, i.e., the additive two-factor model. How to estimate  $\sigma^2$ ? Degrees of freedom?

b) Determine using the test on the level 0.05 if there is difference between the battery types in the ability to maintain the charge status. Differences between the cars? Use level 0.05.

c) Make pairwise comparisons between battery types using Tukey method. Use table with  $\alpha = 0.05$ , (k = amount of battery types and f = degrees of freedom for  $SS_E$ ).

d) Write out the model 2. How to estimate  $\sigma^2$ ? Degrees of freedom? Compare to a).

e) Determine using the test on level 0.05 if there is difference between the battery types in the ability to maintain the charge status. Compare to result of b).

f) Look at the normal probability plot. Is there any systematic pattern?

g) Which of the two models would you prefer? Why?

## 3 – Friedmans test

" The following table shows the heart rate of nine patients with congestive heart failure before and shortly after administration of enalaprilat, an angiotensin-converting enzyme inhibitor. Measurements were taken before and at 30, 60 and 120 minutes after drug administration."

Table Short-term effect of enalaprilat on heart rate								
(beats per minute) (Maskin et al., 1985)								
Time (mins)								
Subject	0	30	60	120	Mean	$\operatorname{StDev}$		
1	96	92	86	92	91.50	4.1		
2	110	106	108	114	109.50	3.4		
3	89	86	85	83	85.75	2.5		
4	95	78	78	83	83.50	8.0		
5	128	124	118	118	122.00	4.9		
6	100	98	100	94	98.00	2.8		
7	72	68	67	71	69.50	2.4		
8	79	75	74	74	75.50	2.4		
9	100	106	104	102	103.00	2.6		
Mean	96.56	92.56	91.11	92.33	93.14			
StDev	16.4	17.8	17.2	16.5	16.4			

We want to examine how enaprilat affects the number of heartbeats per minute at different times. At first glance, the experiment quite similar to what we studied in exercise 2, but there is one crucial difference: This time there have been repeated measurements on **the same** patient. We should try to take into account in the model by considering each patient as a "block". If one can assume normal distribution we use

Model: Patient no. *i* and time point no. *j* give where  $\tau_i$  = effect of patient,  $\beta_j$  = effect of time and  $\varepsilon_{ij}$  describe random variations.

Clean window and go to **File/Open project** and open uppg3.MPJ. Look carefully for how data is entered.

a) Go to Stat/Nonparametrics/Friedman. Write

Response:Y Treatment: T Blocks: P

Give  $P = \dots$  Can we, with some probability, claim that there is difference between time points in regards to the amount of heartbeats per minute?

b) Do the ANOVA-analysis.

Can we, with some probability, claim that there is difference between time points in regards to the amount of heartbeats per minute? We obtain P=...

Compare results of a) and b).

c) Compare time point 1 (t = 0) with time point 2 (t = 30) using **Data-Unstack** so that the *y*-values for the different times comes in the different columns. Build differences between time point 1 and time point 2

let C.... = C.... - C....

and then use **Stat/Nonparametrics/Wilcoxon one-sample** to construct the 95% confidence interval for the systematic difference between time point 1 and time point 2.