

Lesson 10

- Ex. 2.7.1** a) $n \approx 120$
 b) $n \approx 100$

- Ex. 2.6.1** a) Curvature is examined with $v_{PQ} = 0.0056 \approx 0.01 \ll 39.86$. No tendency to curvature.

$$SS_{PQ} = \frac{(\bar{y}_F - \bar{y}_C)^2}{\frac{1}{4} + \frac{1}{2}} = 0.02083$$

$SS_E = (2 - 1) \cdot s_C^2 = 3.7538$, where s_C^2 = sample standard deviation for measurements from center point.

- b) Since we did not find any tendency to curvature, it is not likely that (i) will be successful. We follow (ii) and move from $x_1 = 0, x_2 = 0$ in direction $(6.33, 3.10)$.

Ex. 2.7.2 $0.05 = 1 - \Phi\left(\frac{K - \frac{n}{2}}{\sqrt{n}/2}\right)$

$$0.8 = 1 - \Phi\left(\frac{K - 0.7n}{\sqrt{0.21n}}\right)$$

gives $n = 37$. Calculation using binomial distribution without approximation provide $n = 37$: $K = 24, \alpha = 0.049, \text{power} = 0.807$.

- Ex. I** a) The observed points follow the curved curve much better. The straight line in the first plot seems to be systematically wrong in relation to the observed values.

b) $I_{\beta_2} = (\hat{\beta}_2 \pm t \cdot s \cdot \sqrt{h_{22}}) = (-7.11; -4.11)$. We see that $0 \notin I_{\beta_2}$. Hence, x^2 is useful as an explanatory variable.

c) For the estimated regression relationship $x = 10.21$ is the value that gives highest reduction of the phosphate. This is only an estimate of the optimum x -value.

d) $\hat{m}_{10} - \hat{m}_{11} = -\hat{\beta}_1 - 21\hat{\beta}_2 = 3.226$. Hence, pH=10 seems to be better than pH=11.

- Ex. 2.6.2** a) $v_{PQ} = 125.8 > 18.51$. There is, with high probability, curvature of the response surface, which means that there is an optimum point in that particular area.

b) New measurements should be taken in $(-\sqrt{2}, 0), (\sqrt{2}, 0), (0, -\sqrt{2}), (0, \sqrt{2})$. In addition, you should make additional measurements in the center point to get safer σ^2 -estimator.

- Ex. 2.6.3** Starting from zero $(0,0)$ one should move in direction $(2.1, -3.5)$, for example make new measurements of y -value in points $(0.6, -1), (1.2, -2), (1.8, -3), \dots$ and continue so long the value y is increasing and both x_1 and x_2 remains within the acceptable range.

- Ex. 2.7.3** at least 16 people.

- Ex. 2.7.4** a) Significance level $\alpha \approx 0.07$ (using normal approximation)
 b) Power ≈ 0.993 (using Poisson approximation)