Contents

1 Repetitious tasks

Ex. A Anna's doctors suspect that she was suffering from hypokalaemia, i.e., low levels of potassium in the blood. Repeated measurements of the potassium value of a person gives different results, partly because of individual variations from day to day, partly due to measurement error. It has been found that it is reasonable to assume that a measured potassium value of a person is normally distributed with parameters μ and σ, where μ is the characteristic potassium value of the person and $\sigma = 0.2$. A person classed as potassium hypokalaemic if the value is below 3.5. Assume that Anna has $\mu = 3.7$.

a) What is the probability that Anna is classified as hypokalaemic if you make a single potassium measurement?
b) What is the probability that Anna is classified as hypokalaemic if one makes four independent measurements at appropriate time intervals and the mean of these measurements are compared with 3.5?

Ex. B Some researchers compared the microbiological method and hydroxylamine method for analysis of ampicillin. In a series of experiments analyzed pair of equivalent tablets using both methods. In the table below are measured in units of ampicillin per cent of the claimed amount of ampicillin (this is only a subset of the material):

Experiment nr.	Mikrobiol. method	Hydroxylamin method
1	97.2	97.2
2	105.8	97.8
3	99.5	96.2
4	100.0	101.8
5	93.8	88.0
6	79.2	74.0
7	72.0	75.0
8	72.0	67.5
9	69.5	65.8
10	20.5	21.2

Can we, on the basis of this data, conclude that there is a systematic difference between the two methods? Answer the question by means of a suitable 95% confidence interval or test. Normal distribution may be assumed. Present your findings clearly.

Ex. C For certain types of mining to get the waste products that are weakly radioactive. In unfortunate circumstances, these via wastewater leak into the groundwater and reach any source of drinking water. For drinking water the recommended threshold of 5 picocurie per liter of water. From a city's drinking water took 24 water samples and investigated the radiation, resulting in the average value $\bar{x} = 4.61$. Assume that x_i are observations of r.v. $X_i = \mu + \epsilon_i$, $\epsilon_i \sim N(0, 0.87)$.

a) Test $H_0 : \mu = 5$ against $H_1 = \mu < 5$ at level 0.01.

b) Calculate the power of the test for $\mu = 4.5$.

c) Instead for the hypothesis testing in a) the water company would be able to try $H_0' : \mu = 5$ against $H_1' : \mu > 5$ at level 0.01. Which of tests you prefer? Justify your answer by describing how the level of significance can be interpreted in both cases.

Ex. D In one study has investigated the histamine levels in the sputum of nine allergic people and thirteen healthy (Thomas & Simmons 1969), measurement values x_i and y_i, respectively, are:

allergic	31.0	39.6	64.7	65.9	67.9	100.0	102.4	1112.0	1651.0
non allergic	4.7	5.2	6.6	18.9	27.3	29.1	32.4	34.3	35.4
	41.7	45.5	48.0	48.1					

It is quite obvious that variations in the levels are much higher for people with allergies than non-allergy sufferers. Therefore, studying instead of logarithmic values. For the transformed values $u_i = \ln x_i$ and $v_j = \ln y_j$ we have

$\bar{u} = 4.816 \quad s_u = 1.415$
$\bar{v} = 3.122 \quad s_v = 0.855$

Model: The r.v. $U_1, \ldots, U_9$ and $V_1, \ldots, V_{13}$ are independent, $U_i \sim N(\mu_1, \sigma)$ and $V_j \sim N(\mu_2, \sigma)$.

a) Can one with any certainty say that allergic individuals have elevated histamine values compared with healthy people? Justify your answer with an appropriate 95% confidence interval. One suspected already before the measurements that allergic individuals had higher values.

b) Estimate the probability of an allergic and healthy person, respectively, has a histamine value greater than 50.

Comments to the task Ex. D: In this task, it is not obvious that the transformation provides the same standard deviation for the two samples. In the book there is a method of case $\sigma_u \neq \sigma_v$.

Ex. E a) When measuring a quality variable, it is considered reasonable to assume that the measured value $X_i \sim N(\mu, 0.5)$. One wants to have $P(X_i \leq 1) < 0.01$. What is the corresponding condition for μ?
b) At 16 independent measurements we have received the following values:

2.14	2.57	2.01	1.73	2.63	3.14	2.82	2.54
1.42	2.95	2.59	2.51	2.19	2.26	3.49	2.96

Can one with any certainty claim that the condition in a) is fulfilled? Answer the question using an appropriate confidence intervals or test. Significance level 0.05.
c) Seems the assumption that $\sigma = 0.5$ reasonable? Justify your answer using a suitable two-sided confidence interval with confidence level 95%.

Ex. F In connection with calibration of a measuring instrument one has made repeated measurements at different points within measuring range and received sample standard deviations

$$s_1 = 0.223 \quad s_2 = 0.260 \quad s_3 = 0.236$$
$$s_4 = 0.304 \quad s_5 = 0.181 \quad s_6 = 0.178$$

Because of a misunderstanding s_1, s_2, s_3 have been based on five measurements each and s_4, s_5, s_6 on four measurement each.
Model: A metric x is the observation of a stochastic variable $X = \mu + \epsilon$, where μ is the true value and $\epsilon \sim N(0, \sigma)$ is a measurement error. The measurement errors in the various measurements are independent.
Construct a 95% **bounded from above** confidence interval for σ based on all the data.

Ex. G One has made repeated measurements of the concentration of HCl in the solution by titration. Two different color indicators have been utilized to find the end point of the titration. Results:

Indicator	Mean	Sample standard deviation	Number of measurements
Methyl red	$\bar{x} = 0.08686$	$s_x = 0.00098$	16
Bromocresol green	$\bar{y} = 0.08641$	$s_y = 0.00113$	26

Model: We have two independent random samples from $N(\mu_1, \sigma)$ and $N(\mu_2, \sigma)$, respectively.
Seems that both indicators give equivalent results? Justify your answer using a suitable double-sided 95% confidence interval.

Ex. H The transmission of a digital image with a certain system takes an average of 3.45 seconds. By compressing the data (which need not lead to a worse picture of the recipient) one can cut down transmission time. A new algorithm that compresses the information, gives transit times that are $N(\mu, \sigma)$, where $\sigma = 0.32$ seconds. Fifteen independent image transfers gave the average transfer time $\bar{x} = 2.42$ seconds.
a) Test at level 0.05 $H_0 : \mu = 2.5$ against $H_1 : \mu < 2.5$.
b) Calculate the power of the test in a) if $\mu = 2.40$.

Ex. I At a wastewater treatment plant in the laboratory conducted a series of experiments to determine the phosphate reduction is y in percent because of the waste water pH-value x. Results:

Row	x	y
1	9.2	86.5
2	9.9	93.0
3	11.0	90.5
4	10.4	89.5
5	10.8	89.2
6	12.5	64.5
7	12.3	64.0
8	12.3	64.6
9	10.5	91.7
10	9.4	90.2
11	9.6	91.0
12	9.4	84.6
13	10.0	89.7
14	10.8	85.3
15	11.0	82.6
16	9.9	85.8
17	9.1	79.4
18	9.7	84.2
19	9.9	91.9
20	10.0	93.6

The data were analyzed in Minitab according to two different models

$$Y = \tilde{\beta}_0 + \tilde{\beta}_1 x + \tilde{\epsilon}, \quad (1)$$

$$Y = \beta_0 + \beta_1 x + \beta_2 x^2 + \epsilon, \quad (2)$$

where ϵ-variables are assumed to be independent and $N(0, \sigma)$ distributed. Minitab output and plots are given below.

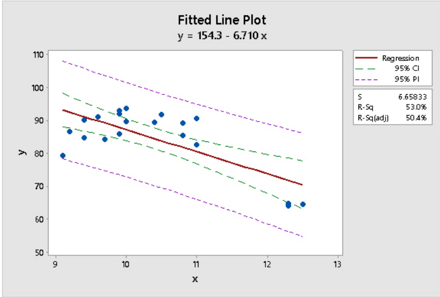
a) Explain briefly on the basis of plots why model 2 describes the data better than the model 1.

b) How it appears from the analysis that term x^2 is essential to the model 2. Motivate your answer with help of appropriate 95% confidence interval.

c) Which value of pH is optimal according to the model 2. Motivate your answer using the appropriate calculations.

d) Estimate difference $m_{10} - m_{11}$ between $\mathbb{E}(Y)$ for $x = 10$ and $\mathbb{E}(Y)$ for $x = 11$ in model 2.

MODEL 1...............................




```
MTB > Fitline 'y' 'x';
SUBC> Confidence 95.0;
SUBC> CI;
SUBC> PI.
```

Regression Analysis: y versus x

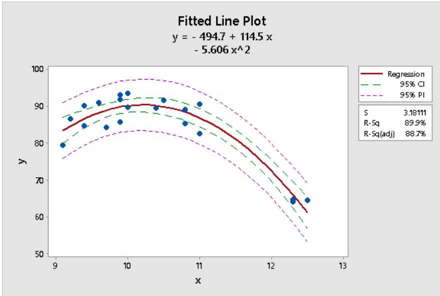
The regression equation is
y = 154.3 - 6.710 x

S = 6.65833 R-Sq = 53.0% R-Sq(adj) = 50.4%

Analysis of Variance

Source	DF	SS	MS
Regression	1	900.68	900.677
Error	18	798.00	44.333
Total	19	1698.68	

MODEL 2...............................




```
MTB > Fitline 'y' 'x';
SUBC> Poly 2;
SUBC> Confidence 95.0;
SUBC> CI;
SUBC> PI.
```

$$(X^T X)^{-1} = \begin{pmatrix} 678.164 & -126.211 & 5.8116 \\ -126.211 & 23.5343 & -1.08585 \\ 5.8116 & -1.08585 & 0.050207 \end{pmatrix}$$

Polynomial Regression Analysis: y versus x

The regression equation is
y = - 494.7 + 114.5 x - 5.606 x^2

S = 3.18111 R-Sq = 89.9% R-Sq(adj) = 88.7%

Analysis of Variance

Source	DF	SS	MS
Regression	2	1526.65	763.323
Error	17	172.03	10.119
Total	19	1698.68	

Sequential Analysis of Variance

Source	DF	SS	F	P
Linear	1	900.677	20.32	0.000
Quadratic	1	625.970	61.86	0.000

2 Exercises

2.1 One-way analysis of variance and variance component model

Ex. 2.1.1 In the manufacture of roof trusses four different splicing methods were examined. At test load one obtained the following buckling strengths (unit: 10^4N):

Method 1:	1.32	1.64	1.11	1.72
Method 2:	2.08	1.86	1.79	2.11
Method 3:	1.76	1.58	1.89	1.87
Method 4:	1.39	1.43	1.58	1.27

According to analysis given below the following results were obtained:

Group	$\bar{y}_{i\cdot}$	s_i
1	1.4475	0.2837
2	1.96	0.1590
3	1.775	0.1420
4	1.4175	0.1279

VARIANCE ANALYSIS

	Sum of squares	Df
Between groups	0.82715	3
Within group	0.42685	12

a) Have splicing methods influence on strength? Answer the question with the help of a suitable test. Significance level 1%.
b) Makes pairwise comparisons between the methods by calculating confidence intervals for the various differences $\mu_i - \mu_j$ with simultaneous confidence level exactly 99%.
The common one-way factor model is presumed in both a) and b).

Ex. 2.1.2 The following data set shows the yields of soybeans (unit: bushels/acre) sown with plant spacing 2 inches on equivalent areas and line spacing 20, 24, 28 32 inches respectively:

line spacing							$\bar{y}_{i\cdot}$	s_i
20	23.1	22.8	23.2	23.4	23.6	21.7	22.967	0.677
24	21.7	23.0	22.4	21.1	21.9	23.4	22.250	0.855
28	21.9	21.3	21.6	20.2	21.6	23.8	21.733	1.172
32	19.8	20.4	19.3	18.5	19.1	21.9	19.833	1.199

a) If one is to analyze the data according to the common one-way factor model, one must assume that the standard deviations are equal. Show that this assumption is reasonable using appropriate tests each at significance level 0.05. It is fine to do one of the tests and specify the simultaneous confidence level.
b) Now consider the data as four random samples from $N(\mu_i, \sigma)$. Examine

with an appropriate confidence interval or test at significance level 0.05 if it is possible that

$$\mu_1 - \mu_3 = 2(\mu_3 - \mu_4) \qquad \Leftrightarrow \qquad \mu_1 - 3\mu_3 + 2\mu_4 = 0.$$

Ex. 2.1.3 A plastic factory receives raw materials that are manufactured in different manufacturing batches (melts). One randomly chose five samples from some manufacturing batches and observed their tensile strengths. Results (MEAN and STDEV from Minitab):

Batch	Tensile strength					MEAN	STDEV
A_1	8032	7982	8065	8020	8040	8027.8	30.4
A_2	8238	8201	8306	8302	8322	8273.8	51.8
A_3	8239	8376	8320	8305	8256	8299.2	54.4

Model: Let y_{ij} be observation nr j for batch nr i, where $Y_{ij} = \mu + \tau_i + \epsilon_{ij}$ and where $\tau_1, \ldots, \tau_3$ is $N(0, \sigma_\tau)$ and $\epsilon_{11}, \ldots, \epsilon_{35}$ is $N(0, \sigma)$.
a) Examine using a suitable test of the level 0.01

$$H_0 : \sigma_\tau^2 = 0 \;\; against \;\; H_1 : \sigma_\tau^2 \neq 0.$$

b) Construct a 95% confidence interval for μ.

Ex. 2.1.4 In the production of a certain kind of robots one had a problem getting the plastic material in nosecone to withstand the high temperatures that arise during the flight. One decided to try five different additives to improve the plastic material and then measured the weight loss of the nose cone (unit: %) at three separate trials for each addition. Results:

Group	$\bar{y}_{i\cdot}$	s_i
1	7.03333	1.77858
2	8.73333	1.27017
3	5.93333	0.450925
4	5.06667	0.929157
5	5.66667	0.152753

VARIANCE ANALYSIS

	Sum of squeres	Df
Between groups	25.024	4
Within group	11.733	10

Model: We assume (with some hesitation) that samples come from $N(\mu_i, \sigma)$.
a) At what level is the difference in expected weight between the materials significant?
b) Estimate all pairwise differences between the expected values using t-, Tukey- and Scheffé-intervals. Let in all cases the simultaneous confidence level be at least 95%.
c) Compare lengths of confidence intervals given in b). Was it a coincidence that Tukey intervals became shorter and Scheffé intervals longest?
d) What is wrong with the following argument: The results of experiment show that the measured difference between materials 2 and 4 are larger

than the other differences. If we only want to determine if the difference between the five plastic materials is significant, therefore, we can be satisfied with a test the hypothesis $\mu_2 = \mu_4$ with a standard t-test. It finds that the difference between μ_2 and μ_4 is significant at the level 0.5%. Thus, the difference between the five plastic materials is also significant at the level 0.5%.

Ex. 2.1.5 A sensor indicates when the wavelength of a light source exceeds 7000 angstroms, meaning transition to the infrared zone. From a very large batch of sensors have been randomly selected 5 pieces and each of them has been tested 3 times, the man has determined the lowest wavelength at which the sensor indicated that the threshold exceeded 7000 angstroms. The aim is to draw conclusions for the whole party.
Results of the measurements:

Sensor nr i	Observations, y_{ij}			$\bar{y}_{i\cdot}$	s_i
1	7010	7016	7013	7013.00	3.000
2	6991	6984	6990	6988.33	3.786
3	6985	6989	6990	6988.00	2.646
4	7016	7010	7020	7015.33	5.033
5	7017	7020	7018	7018.33	1.528

a) Set up a variance component model and estimate all parameters in the model. Motivate shortly why a variance component model should be used.
b) Examine with a test on the significance level 0.01 if there are variations between sensors in respect of the wavelength at which the indication is given.

Ex. 2.1.6 In some company acid is being concentrated. Some parts of the equipment corrode and broke eventually. Three different suppliers A, B and C manufacture apparatus of required kind. The volume of production measured in hundreds of tons between installation and fault detection has been registered. Results:

	Production	$\bar{y}_i$	s_i
A	85 60 40 47 34 46	51.83	17.98
B	67 92 95 40 98 60 59 108 86 117	82.20	24.59
C	46 93 100 92 92	84.60	21.84

Model: We have three independent, random samples from $N(\mu_i, \sigma)$.

a) Estimate the expected value μ_A, μ_B and μ_C using intervals so that the simultaneous confidence level will be $\geq 94\%$.
b) At what level is the difference in expected production significant?
c) Estimate all pairwise differences between the expected values using t-intervals so that the simultaneous confidence level will be $\geq 94\%$.
d) Estimate all pairwise differences between the expected values using Scheffe's intervals so that the simultaneous confidence level will be $\geq 95\%$.

Ex. 2.1.7 A pharmaceutical manufacturer wants to investigate the bioactivity of a new drug. A completely randomized single-factor experiment wa conducted with three dosage levels, and the following results were obtained.

Dosage	Observations			
20g	24	28	37	30
30g	37	44	39	35
40g	42	47	52	38

a) Is there evidence that the dosage level affects bioactivity? Use $\alpha = 0.05$.
b) If it is appropriate to do so, make comparisons between the pairs of means. What conclusions can you draw?

Ex. 2.1.8 Three groups with equally big pigs were injected sedatives and for each pig time in minutes between injection and onset of sleep was measured. The pigs in the three groups were given 0.5 mg, 1.0 mg and 1.5 mg av sedatives. Results:

Dose		$\bar{y}_i$	s_i
0.5 mg:	21 23 19 24	21.75	2.22
1.0 mg:	19 21 20 18 22 20	20.00	1.41
1.5 mg:	15 10 13 14 11 15	13.00	2.10

Model: For dose nr i and pig nr j in the group the time y_{ij} was observed, that is observation of a r.v. $Y_{ij} \sim N(0,1)$, where $j = 1, \ldots, n_i$, $i = 1,2,3$. Those r.v. Y_{ij} are independent.

a) Do those three doses give the same expected sleep time? Perform the appropriate test at level 0.01.
b) To investigate if the relationship between dose and sleep time is linear one want to examine

$$H_0: \mu_1 - \mu_2 = \mu_2 - \mu_3 \text{ against } H_1: \mu_1 - \mu_2 \neq \mu_2 - \mu_3$$

Perform the appropriate test at level 0.05.
c) Why H_0 is an interesting hypothesis when one wants to investigate the linearity?

Ex. 2.1.9 From an ore portion one has taken samples of four randomly selected places. Each sample is pulverized, mixed thoroughly and divided into three subsamples whose metal content determined. We let Y_{ij} denote metal content of observation j from place i. Results:

Place	Metal content			$\bar{y}_{i\cdot}$	s_i
1	50.1	49.6	51.2	50.30	0.8185
2	45.6	46.1	45.5	45.73	0.3215
3	47.0	46.0	46.4	46.47	0.5033
4	44.1	43.1	42.9	43.37	0.6429

Per and Stina not quite agree on how to analyze the data.
a) Per suggests model

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij},$$

where $\sum_{i=1}^{4} \tau_i = 0$ and $\epsilon_{ij} \sim N(0, \sigma)$, $i = 1, 2, 3, 4$, $j = 1, 2, 3$.
Construct a 95% confidence interval for μ.
b) Stina suggests model

$$Y_{ij} = m + \xi_i + \epsilon_{ij},$$

where $\xi_i \sim N(0, \sigma_\xi)$ and $\epsilon_{ij} \sim N(0, \sigma)$, $i = 1, 2, 3, 4$, $j = 1, 2, 3$, and ξ- and ϵ-variables are independent.
Construct a 95% confidence interval for m.
c) What is the difference between the parameters μ and m?

Ex. 2.1.10 In a study one wanted to investigate whether people with high average blood pressure har higher cholesterol levels than people with normal blood pressure (Rossi et al.). In the data below cholesterol values x_i for people with high blood pressure and y_j for people with normal blood pressure (unit: mg/l) are measured. The data is a subset of a larger data set.

```
MTB >  print c1
Data Display
x_i
   207   172   191   221   203   241   208   199   185   235
   214   134   226   221   223   181   217   208   202   218
   216   168   168   214   203   280   212   260   210   265
   206   198   210   211   274   223   175   203   168

MTB > print c2
Data Display
y_i
   286   226   187   204   203   206   196   168   229   184
   186   281   203   189   196   142   179   212   163   196
   189   142   168   121
```

Model: Random variables X_i are independent and $N(\mu_1, \sigma_1)$ distributed and the random variables Y_j are independent and $N(\mu_2, \sigma_2)$ distribured.
Two different analyzes were performed using Minitab:

```
ANALYS NR 1, SKILDA STANDARDAVVIKELSER (NOT EQUAL STDEV)
MTB > TwoSample 'x_i' 'y_i';
SUBC>   Confidence 90,0;
SUBC>   Test 0,0;
SUBC>   Alternative 0.

Two-sample T for x_i vs y_i
      N   Mean  StDev  SE Mean
x_i  39  209,5   29,6      4,7
y_i  24  194,0   37,6      7,7

Difference = mu (x_i) - mu (y_i)
Estimate for difference:  15,49
90% CI for difference:  (0,30; 30,68)
```

```
T-Test of difference = 0 (vs not =): T-Value = 1,72
P-Value = 0,094  DF = 40

---------------------------------------------------
ANALYS NR 2, SAMMA STANDARDAVVIKELSER (EQUAL STDEV)
MTB > TwoSample 'x_i' 'y_i';
SUBC>   Confidence 90,0;
SUBC>   Test 0,0;
SUBC>   Alternative 0;
SUBC>   Pooled.

Two-sample T for x_i vs y_i
      N   Mean  StDev  SE Mean
x_i  39  209,5   29,6      4,7
y_i  24  194,0   37,6      7,7

Difference = mu (x_i) - mu (y_i)
Estimate for difference:  15,49
90% CI for difference:  (1,26; 29,71)
T-Test of difference = 0 (vs not =): T-Value = 1,82
P-Value = 0,074  DF = 61
Both use Pooled StDev = 32,8318
```

a) Which of the two analyzes are most relevant? Justify your answer briefly.

b) Examine $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 > \mu_2$ at significance level 0.05 preferably by using Minitab analysis.

c) How does the test statistic look like in the first analysis and how to calculated degree of freedom?

d) Compare those two confidence intervals in the Minitab analysis. Which is more reliable?

Ex. 2.1.11 In a given project one wants to measure the discharge intensities of lightning in Florida, where thunderstorms are common. In a certain region three places (stations) have been randomly selected. At the stations proper equipment was installed and the measurements of the maximum intensity of five different lightning on each one were performed. Observed intensities z_{ij} :

Tracking Station	Intensities				
1	20	1050	3200	5600	50
2	4300	70	2560	3650	80
3	100	7700	8500	2960	3340

Since the measurement errors can hardly be constant over such a large range, the values wa transformed using ln function before analysis. We have model

$$Y_{ij} = ln(Z_{ij}) = \mu + \tau_i + \epsilon_{ij},$$

where $\tau_i \sim N(0, \sigma_\tau)$ and $\epsilon_{ij} \sim N(0, \sigma)$ and where τ- and ϵ-variables are independent.
Minitab analysis:

```
MTB > print c1-c3       (with log(intensities))
Data Display
Row      TS1      TS2      TS3
  1  2,99573  8,36637  4,60517
  2  6,95655  4,24850  8,94898
  3  8,07091  7,84776  9,04782
  4  8,63052  8,20248  7,99294
  5  3,91202  4,38203  8,11373

MTB > Describe 'TS1' - 'TS3';
SUBC>   Mean;
SUBC>   StDeviation;
SUBC>   Count.

Descriptive Statistics: TS1; TS2; TS3

          Total
Variable  Count   Mean  StDev
TS1           5   6,11   2,52
TS2           5  6,609  2,103
TS3           5  7,742  1,817
```

a) Construct 95% confidence interval for μ.

b) It is hoped through these first measurements to show that variation between stations was negligible. Examine on the significance level 10% hypothesis

$$H_0 : \sigma_\tau^2 = 0 \ \textit{against} \ H_1 : \sigma_\tau^2 \neq 0$$

Is it reasonable to continue to make measurements at the same locations in Florida?

Ex. 2.1.12 One wants to compare four different soil types for the presence of a particular bacterium. For each soil type one has taken seven soil samples and found the following number of bacteria.

Soil type	Observations							Mean $\bar{y}_{i\cdot}$	St.dev. s_i
1	92	94	89	78	91	99	76	88.43	8.42
2	76	72	65	68	59	80	67	69.57	7.04
3	50	48	63	55	54	42	43	50.71	7.34
4	72	75	83	81	77	64	70	74.57	6.55

Model: y_{ij} are observations of $Y_{ij} = \mu_i + \epsilon_{ij}$, where $\epsilon_{ij} \sim N(0, \sigma)$ for $i = 1, 2, 3, 4$ and $j = 1, \ldots, 7$. Random variables Y_{ij} are independent.

a) Is the significant difference between soil types regarding the presence of the relevant bacteria? Construct two-sided confidence interval for differences $\mu_i - \mu_j$ on simultaneous confidence level 95%.

b) For various reasons, one consider mixing soil type 1 and soil of type 2 in the ratio 2:1. Construct confidence interval for $(2\mu_1 + \mu_2)/3$ on confidence level 95%.

Ex. 2.1.13 The company produces steel at four different factories A, B, C and D. It has been long believed that the factory D has higher quality on his plate than other factories. In one study, samples were taken out of production for all the factories and the tensile strength was determined for each sample. Because of a misunderstanding the samples at different factories are of different size. Data from the various factories are in C1-C4 of the data output below.
Model: The data can be regarded as four samples from $N(\mu_i, \sigma)$, where $i = 1, 2, 3, 4$.

a) Does it seem reasonable to consider that A, B and C have approximately the same quality on their plates? Answer the question by constructing intervals for pairwise comparisons at simultaneous confidence level at least 94%.

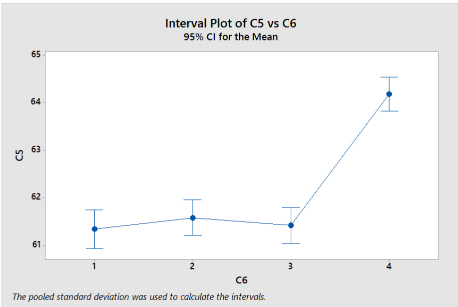
b) Examine if plates from factory D have at least 3% better strength than the other factories, i.e. examine

$$H_0 : \mu_4 = 1.03(\mu_1 + \mu_2 + \mu_3)/3 \ \ against \ \ H_1 : \mu_4 > 1.03(\mu_1 + \mu_2 + \mu_3)/3$$

at the level 0.05.

```
MTB > set c1
DATA>  61,2 62,0 60,9 62,1 61,8 61,3 62,4 62,1 60,1 59,8 61,0
DATA> end
MTB > set c2
DATA> 60,8 62,1 62,5 61,4 60,9 62,2 61,2 62,3 62,1 62,1 60,6
60,8 61,5
DATA> end
MTB > set c3
DATA> 60,8 61,5 61,9 61,5 61,7 62,0 61,2 60,2 60,5 61,3 61,4
62,3 62,1
DATA> end
MTB > set c4
DATA> 64,3 64,6 63,6 64,7 63,9 64,8 64,2 63,4 64,6 64,6 64,9
63,2 64,1 63,6
DATA> end
MTB > stack c1-c4 c5;
SUBC> subscript c6.
MTB > Name C7 "FITS1" C8 "RESI1".
MTB > OneWay;
SUBC> Response C5;
SUBC>   Categorical C6;
SUBC> TMethod;
SUBC>   TFactor;
SUBC>   TANOVA;
SUBC> TMeans.
```

```
One-way ANOVA: C5 versus C6
Factor Information
Factor  Levels  Values
C6           4  1; 2; 3; 4

Analysis of Variance
Source  DF  Adj SS
C6       3   76,06
Error   47   21,27
Total   50   97,33

Means
C6   N    Mean  StDev        95% CI
1   11  61,336  0,846  (60,928; 61,744)
2   13  61,577  0,670  (61,202; 61,952)
3   13  61,415  0,624  (61,040; 61,791)
4   14  64,179  0,558  (63,817; 64,540)
```



2.2 Two-way analysis of variance. Block design.

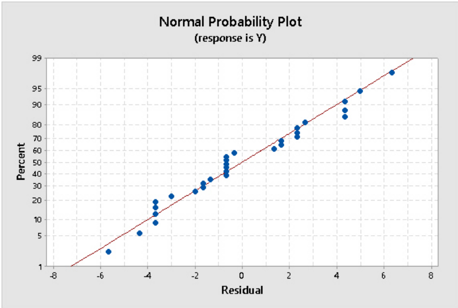
Ex. 2.2.1 To determine the optimum properties of a plating bath have been tried two different concentrations and five temperatures and measured reflectance of the treated metal. Results:

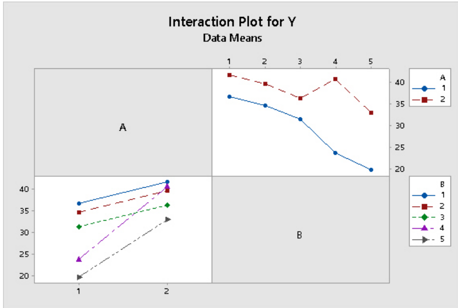
	Temperature ($^\circ F$)				
Conc (g/l)	75	100	125	150	175
5	35	31	30	28	19
	39	37	31	20	18
	36	36	33	23	22
10	38	36	39	35	30
	46	44	32	47	38
	41	39	38	40	31

The data were analyzed using Minitab, see below.
a) How does the model look like?
b) Is it reasonable to have an additive model or there is interaction between A and B? Perform the appropriate test at level 0.05.
c) According to which model should be the data analyzed?

d) Is it possible to find the best combination of concentration and temperature? Justify your answer using appropriate confidence intervals with simultaneous confidence level exactly 0.95.

```
MTB > ANOVA 'Y' = A | B;
SUBC> Means A|B.

ANOVA: Y versus A, B
Factor  Type  Levels          Values
A      fixed       2            1, 2
B      fixed       5   1, 2, 3, 4, 5

Analysis of Variance for Y
Source    DF       SS       MS
A          1   616.53   616.53
B          4   591.20   147.80
A*B        4   196.13    49.03
Error     20   280.00    14.00
Total     29  1683.87

S = 3.74166 R-Sq = 83.37% R-Sq(adj) = 75.89%

Means
A N Y
1 15 29.200
2 15 38.267

B N Y
1 6 39.167
2 6 37.167
3 6 33.833
4 6 32.167
5 6 26.333

A B N Y
1 1 3 36.667
1 2 3 34.667
1 3 3 31.333
1 4 3 23.667
1 5 3 19.667
2 1 3 41.667
2 2 3 39.667
2 3 3 36.333
2 4 3 40.667
2 5 3 33.000
```





Ex. 2.2.2 On a laboratory has measured the tensile strength of the five kinds of linen thread using four different instruments. Results:

	Instrument				
Thread	1	2	3	4	Mean $\bar{y}_{i\cdot}$
1	20.9	20.4	19.9	21.9	20.775
2	25.0	26.2	27.0	24.8	25.750
3	25.5	23.1	21.5	24.4	23.625
4	24.8	21.2	23.5	25.7	23.800
5	19.6	21.2	22.1	22.1	21.250
Mean $\bar{y}_{\cdot j}$	23.160	22.420	22.800	23.780	

a) Model 1: Thread No. i and instrument No. j give strength y_{ij}, where $Y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$ with $\epsilon_{ij} \sim N(0, \sigma)$ and $\sum_i \tau_i = 0$, $\sum_j \beta_j = 0$. Makes pairwise comparisons between thread types. The simultaneous confidence level should be at least 90%.

ANOVA table

	Sum of squares	df
Thread	66.3930	4
Instrument	5.02000	3
Error	24.9350	12

b) Model 2: $Y_{ij} = \mu_i + \tilde{\epsilon}_{ij}$, where $\tilde{\epsilon}_{ij} \sim N(0, \tilde{\sigma})$. Makes pairwise comparisons between thread types. The simultaneous confidence level should be exactly 90%.

c) Which of the two models work best? Justify your answer briefly.

Ex. 2.2.3 One has carried out an experiment to study the effects of the blow-through time and steam pressure when cleaning the filter. Amount of remaining particles:

Steam pressure	Blow-through time					
	1		2		3	
10	45.2	46.0	40.0	39.0	35.9	34.1
20	41.8	20.6	27.8	19.0	22.5	17.7
30	23.5	33.1	44.6	52.2	42.7	48.6

The data has partly been analyzed using Minitab, see below.

a) Should we use a two-factor additive model or a complete two-factor model? Answer the question with the help of a suitable test at significance level 0.05.

b) Which steam pressure should be chosen for three hour blowing time? The answer must be justified through appropriate confidence intervals with simultaneous confidence level at least 85%. Use the model you have chosen in a).

Data output:

```
ROW  PRESS  TIME    Y
  1      1     1  45.2
  2      1     1  46.0
  3      1     2  40.0
  4      1     2  39.0
  5      1     3  35.9
  6      1     3  34.1
  7      2     1  41.8
  8      2     1  20.6
  9      2     2  27.8
 10      2     2  19.0
 11      2     3  22.5
 12      2     3  17.7
 13      3     1  23.5
 14      3     1  33.1
 15      3     2  44.6
 16      3     2  52.2
 17      3     3  42.7
 18      3     3  48.6

MTB > ANOVA 'Y' = PRESS | TIME;
SUBC> Means PRESS|TIME.

ANOVA: Y versus PRESS, TIME
Factor  Type    Levels          Values
PRESS   fixed        3         1, 2, 3
TIME    fixed        3         1, 2, 3

Analysis of Variance for Y
Source     DF        SS        MS
PRESS       2    963.72    481.86
TIME        2     37.48     18.74
PRESS*TIME  4    680.76    170.19
Error       9    369.76     41.08
Total      17   2051.72

Means
PRESS  N       Y
    1  6  40.033
```

```
    2  6  24.900
    3  6  40.783

TIME  N      Y
  1   6   35.033
  2   6   37.100
  3   6   33.583

PRESS TIME N       Y
    1    1  2  45.600
    1    2  2  39.500
    1    3  2  35.000
    2    1  2  31.200
    2    2  2  23.400
    2    3  2  20.100
    3    1  2  28.300
    3    2  2  48.400
    3    3  2  45.650
```

Ex. 2.2.4 A chemist would like to study the combustion temperature on carbon monoxide content of the flue gases from a combustion process. He decides to use four different temperatures T_1, T_2, T_3 and T_4. The experiment will take one day, at his disposal, he has four experimental setups and he can carry four attempts on each of them. One can imagine following two ways to distribute the sixteen experiments in space and time:
I. A completely randomized design of four measurements at each temperature.
II. A block design with randomization within blocks (block = experimental setup).

a) Describe briefly how to make a design of type I.

b) Describe briefly how to make a design of type II.

c) Enter the appropriate models for the observed carbon dioxide levels from the two designs.

Ex. 2.2.5 An experiment has been conducted to see if the BOD test (BOD = biochemical oxygen demand) of water is affected by the presence of copper. One measures the amount of oxygen in the water at the beginning and end of a five day period, the difference between the measured values is attributed to microbial activity. The question is if dissolved copper inhibit bacterial activity and provides a low value of the difference in oxygen content. Three different water samples have been divided into five subsamples treated with different amounts of copper. BOD-values:

	Coppar concentration (ppm)					
Sample	0	0.1	0.3	0.5	0.75	Mean $\bar{y}_{i\cdot}$
1	210	195	150	148	140	168.60
2	194	183	135	125	130	158.40
3	138	98	89	90	85	100.00
Mean $\bar{y}_{\cdot j}$	180.67	158.67	124.67	121.00	118.33	

Model: Water sample No. i and copper concentration No. j gives BOD-value y_{ij}, such that y_{ij} are observations of $Y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$, where $\sum_i \tau_i = 0$, $\sum_j \beta_j = 0$ and ϵ-variables are independent and $N(0, \sigma)$ distributed.

ANOVA table

	Sum of squares	df
Sample	12980.9	2
Copper	9196.67	4
Residual (error)	913.733	8

a) Have copper concentration importance to the BOD value? Perform the appropriate test at significance level 0.01.

b) Is copper concentration 0.75 significant better than the concentration 0.3 according to this analysis? Answer the question with the help of a suitable test or confidence interval. Significance level 0.05.

Ex. 2.2.6 When soldering with two kinds of solders, L1 and L2, on three different materials, M1, M2 and M3, was obtained following strength data:

	M1	M2	M3
L1	102	86	78
	97	90	66
L2	94	118	40
	99	110	37

Minitab analysis:

```
MTB > print c1
Data Display
C1
   1   1   1   1   1   1   2   2   2   2   2   2
MTB > print c2
Data Display
C2
   1   2   3   1   2   3   1   2   3   1   2   3
MTB > print c3
Data Display
C3
   102    86    78    97    90    66    94   118    40    99
110    37

MTB > name c1 'L' c2 'M' c3 'Y'
MTB > ANOVA 'Y' = L| 'M';
SUBC>   Means L|M.
ANOVA: Y versus L; M
Factor  Type   Levels  Values
L       fixed       2  1; 2
M       fixed       3  1; 2; 3
```

```
Analysis of Variance for Y
Source  DF      SS      MS       F      P
L        1    36,8    36,8    1,56  0,258
M        2  5239,5  2619,8  111,08  0,000
L*M      2  1770,5   885,3   37,54  0,000
Error    6   141,5    23,6
Total   11  7188,3

Means
L  N       Y
1  6  86,500
2  6  83,000

M  N       Y
1  4   98,00
2  4  101,00
3  4   55,25

L  M  N       Y
1  1  2   99,50
1  2  2   88,00
1  3  2   72,00
2  1  2   96,50
2  2  2  114,00
2  3  2   38,50
```

a) Should you choose an additive two-factor model or complete model? Justify model selection using a suitable test at the significance level 5%.

b) Compare the two tin types properties by interval estimating suitable parameters, so that the simultaneous confidence level is at least 97%. (Hint: Which solder should be recommended for the material?)

2.3 ANOVA. Square design.

Ex. 2.3.1 When hardening steel, the steel is first heated 800–1200°C. Then it is cooled in a salt bath to a temperature of 300–600°C, and subjected to strong mechanical impact. Then again cooled rapidly to room temperature. A three-factor design was created with a factor A, heating temperature, at three levels (930°C, 985°C, 1040°C); factor B, salt bath temperature, at two levels (400°C, 550°C) and factor C, mechanical impact, on two levels (80%, 50%). Results (tensile property):

		B_1			B_2		
A_1	C_1	1209	1171	1250	1133	1065	1150
	C_2	1166	1081	1065	980	900	889
A_2	C_1	1098	1157	1099	1068	1115	1048
	C_2	1049	950	992	807	886	779
A_3	C_1	998	1015	1074	1088	1094	1010
	C_2	983	918	955	714	746	784

The analysis has been done with the help of Minitab according to two different models.

a) Which model is used in the analysis no. 2? Justify the choice of this model using analysis no. 1.

b) Can one on the basis of analysis no. 2 find the best combination of A, B and C? Motivate your answer using appropriate confidence intervals with simultaneous confidence level at least 90%.

```
MTB > MODEL No. 1
MTB > ANOVA 'Y' = A|B|C;

ANOVA: Y versus A, B, C
Factor  Type    Levels         Values
A      fixed        3        1, 2, 3
B      fixed        2           1, 2
C      fixed        2           1, 2
Analysis of Variance for Y
Source  DF       SS       MS       F
A        2   119225    59612   31.17
B        1   108241   108241   56.60
C        1   284089   284089  148.56
A*B      2     4246     2123    1.11
A*C      2     3706     1853    0.97
B*C      1    52441    52441   27.42
A*B*C    2     9145     4572    2.39
Error   24    45897     1912
Total   35   626987

MTB > MODEL No. 2
MTB > ANOVA 'Y' = A B|C;
SUBC> Means A B|C.

ANOVA: Y versus A, B, C
Factor  Type    Levels         Values
A      fixed        3        1, 2, 3
B      fixed        2           1, 2
C      fixed        2           1, 2

Analysis of Variance for Y
Source  DF       SS       MS       F       P
A        2   119225    59612   28.39   0.000
B        1   108241   108241   51.55   0.000
C        1   284089   284089  135.30   0.000
B*C      1    52441    52441   24.98   0.000
Error   30    62992     2100
Total   35   626987

Means
A   N       Y
```

```
1  12  1088.3
2  12  1004.0
3  12   938.3

B   N       Y
1  18  1068.3
2  18   958.7

C   N       Y
1  18  1102.3
2  18   924.7

B  C  N       Y
1  1  9  1119.0
1  2  9  1017.7
2  1  9  1085.7
2  2  9   831.7
```

Ex. 2.3.2 In a conservation area one wanted to examine how the treatment of grassland affected the occurrence of brinklosta (a kind of grass). A reasonably rectangular area, restricted on the west by a river and in the south by a highway, was divided into four rows and four columns which gave 16 experimental squares where four different treatments:
Treat. 1: The hay was cut and harvested;
Treat. 2: The hay was cut and left on the ground in windrows;
Treat. 3: The hay was cut with a scythe and left where it fell;
Treat. 4: The hay was not cut.
were applied according to a Latin square. In the following year n seedlings were chosen at random in each box and the share of brinklosta was registered. Results:

	0.32 (Treat.4)	0.81 (Treat.1)	0.64 (Treat.2)	0.57 (Treat.3)
River	0.84 (Treat.1)	0.27 (Treat.4)	0.58 (Treat.3)	0.62 (Treat.2)
	0.63 (Treat.3)	0.67 (Treat.2)	0.79 (Treat.1)	0.19 (Treat.4)
	0.72 (Treat.2)	0.65 (Treat.3)	0.24 (Treat.4)	0.70 (Treat.1)

Highway

Relative frequencies (X) are observations of r.v. with variance proportional to p(1-p), i.e. different variances. The transformation $Y = arcsin\sqrt{X}$ was applied to obtain r.v. with approximately the same variance, while the ranking of the observations preserved. Those Y-values have been analyzed using Minitab according to additive model. Results (row in the analysis represents a row in the data above):

```
MTB > let c5=asin(sqrt(c4))
MTB > name c1 'row' c2 'col' c3 'treat' c4 'x' c5 'y'
MTB > ancova y=row col treat;
SUBC> mean row col treat.
* NOTE * Unbalanced design. A cross tabulation of your factors will show
```

```
* where the unbalance exists.
* NOTE * Make sure your design is orthogonal.

ANCOVA: y versus row, col, treat
Factor  Levels       Values
row          4   1, 2, 3, 4
col          4   1, 2, 3, 4
treat        4   1, 2, 3, 4

Analysis of Variance for y
Source  DF         SS         MS       F
row      3   0.000923   0.000308    0.41
col      3   0.033162   0.011054   14.91
treat    3   0.693182   0.231061  311.63
Error    6   0.004449   0.000741
Total   15   0.731715

Means
row N       y
1   4  0.87599
2   4  0.86950
3   4  0.85539
4   4  0.86352

col N       y
1   4  0.92266
2   4  0.89069
3   4  0.84994
4   4  0.80110

treat N       y
1     4  1.0912
2     4  0.9515
3     4  0.8940
4     4  0.5277
```

a) Does the distance to the highway affect the incidence of brinklosta? Perform the appropriate test level 0.05.

b) Are any of the treatments superior to the other if you want to have a lot of brinklosta? Answer the question using appropriate confidence intervals with simultaneous confidence exactly 0.95.

Ex. 2.3.3 Production of a particular kind of alcohol is based on the fermentation of corn. In a survey temperature, type of yeast and maize have been varied. Then the yield of alcohol from the process was determined (in g alcohol per 250 grams of liquid). Results:

Temp	Yeasts	Maizes							
		MUS1		MUS2		MC1		MC2	
$21°C$	Y1	46.8	52.2	54.2	53.6	49.2	51.0	54.8	59.6
	Y2	54.4	53.8	55.0	60.8	54.8	55.4	65.2	61.4
	Y3	70.8	71.2	74.6	78.6	74.4	69.6	80.0	80.4
$26°C$	Y1	57.8	59.4	68.0	65.6	61.2	61.6	74.0	75.6
	Y2	69.4	71.6	77.8	83.0	66.6	71.8	84.8	86.0
	Y3	69.8	70.0	80.4	83.0	63.6	69.4	85.2	84.2
$31°C$	Y1	64.0	66.2	67.2	67.6	62.6	66.8	77.0	75.0
	Y2	66.6	62.2	69.2	71.6	65.4	64.6	75.8	75.0
	Y3	67.6	68.4	74.6	76.2	70.2	71.4	78.0	83.2

A Minitab analysis under a complete three factor model is available below.

a) Set up the model and specify the conditions that must be met.

b) Illustrates the potential interaction between temperature and yeast by making a so-called interaction plot.

c) Examine using a test of the level 0.05 if there is interaction between temperature and yeast.

d) Make paired comparisons of the various maize varieties from their main effect, which describes how they work *in average*. The simultaneous confidence level should be exactly 90%. Is some maize kind better than the other? You do not need to put all the intervals, but it should be clear how you draw your conclusions.

```
MTB > ANOVA 'y' = Temp| Yeast| Maize;
SUBC> Means Temp| Yeast| Majs.

ANOVA: y versus Temp, Yeast, Maize
Factor   Type  Levels     Values
Temp    fixed       3    1, 2, 3
Yeast   fixed       3    1, 2, 3
Maize   fixed       4 1, 2, 3, 4

Analysis of Variance for y
Source           DF        SS       MS
Temp              2   1545.51   772.76
Yeast             2   1934.70   967.35
Maize             3   1709.61   569.87
Temp*Yeast        4   1003.62   250.90
Temp*Maize        6    159.82    26.64
Yeast*Maize       6     29.63     4.94
Temp*Yeast*Maize 12     41.86     3.49
Error            36    170.76     4.74
Total            71   6595.52

Means
Temp N       y     Yeast  N       y    Maize  N       y
1    24  61.742        1 24  62.125        1 18  63.456
2    24  72.492        2 24  67.592        2 18  70.056
3    24  70.267        3 24  74.783        3 18  63.867
```

```
                                  4 18  75.289

Temp Yeast N      y     Temp Maize N      y     Yeast Maize N      y
   1     1 8 52.675        1     1 6 58.200        1     1 6 57.733
   1     2 8 57.600        1     2 6 62.800        1     2 6 62.700
   1     3 8 74.950        1     3 6 59.067        1     3 6 58.733
   2     1 8 65.400        1     4 6 66.900        1     4 6 69.333
   2     2 8 76.375        2     1 6 66.333        2     1 6 63.000
   2     3 8 75.700        2     2 6 76.300        2     2 6 69.567
   3     1 8 68.300        2     3 6 65.700        2     3 6 63.100
   3     2 8 68.800        2     4 6 81.633        2     4 6 74.700
   3     3 8 73.700        3     1 6 65.833        3     1 6 69.633
                           3     2 6 71.067        3     2 6 77.900
                           3     3 6 66.833        3     3 6 69.767
                           3     4 6 77.333        3     4 6 81.833
```

2.4 Factorial design 2^k. Fractional Factorial design 2^{k-p}

Ex. 2.4.1 In one experiment, one studied the gain of a semiconductor device depends on four factors

Factor	Low level	High level
A: Manufacture location	Laboratory	regular production
B: Pressure	10^{-15}	10^{-4}
C: Relative humidity	1%	30%
D: Time from production	72h	144h

Two replicates were made on two different occasions. Results:

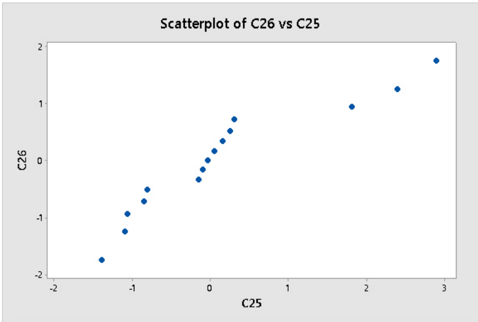
Level comb.	Rep. 1	Rep. 2	Level comb.	Rep. 1	Rep. 2
1	39.0	43.2	d	40.1	41.9
a	31.8	43.7	ad	42.0	40.5
b	47.0	51.4	bd	54.9	53.0
ab	40.9	40.3	abd	39.9	40.2
c	43.8	40.5	cd	43.1	40.2
ac	29.3	52.9	acd	30.1	39.9
bc	34.8	48.2	bcd	35.6	53.7
abc	45.6	58.2	abcd	41.4	49.5

a) First the mean values of the two replicates were analyzed using Minitabs matrix commands, see below. Which three effects appear to be most significant in this analysis? Justify your answer briefly.

b) Then, two analyzes using ANOVA command were performed, see below. According to which model the data were analyzed in the first ANOVA analysis? What three effects appear to be most significant in this analysis? Justify your answer briefly.

c) Can you find a best combination of B and C using the regular production? Answer the question by using appropriate confidence intervals with simultaneous confidence level at least 70%. Use ANOVA-analysis no. 2. They seek high gain.

```
MTB > Read c1-c16;
SUBC> File "C:...\design4.dat";
SUBC> Decimal ".".
Entering data from file: C:...\DESIGN4.DAT
16 rows read.
MTB > print c17

Data Display
C17
39.0 31.8 47.0 40.9 43.8 29.3 34.8 45.6 40.1 42.0 54.9 39.9 43.1
30.1 35.6 41.4

MTB > print c18

Data Display
C18
43.2 43.7 51.4 40.3 40.5 52.9 48.2 58.2 41.9 40.5 53.0 40.2 40.2
39.9 53.7 49.5

MTB > let c19=(c17+c18)/2
MTB > set c20
DATA> 1:16
DATA> end
MTB > copy c19 m2
MTB > copy c1-c16 m1
MTB > trans m1 m3
MTB > mult m3 m2 m4
MTB > copy m4 c21
MTB > let c22=c21/16
MTB > Sort C20 C22;
SUBC> By c22;
SUBC> After.
MTB > print c24 c23


Data Display
Sorted
Row     C22  C20
1  -1.3813  2
2  -1.0937  13
3  -1.0563  10
4  -0.8438  14
5  -0.8062  12
6  -0.1437  9
7  -0.0937  5
8  -0.0313  4
9   0.0563  7
10  0.1563  15
11  0.2562  11
12  0.3062  16
13  1.8188  6
14  2.3938  8
15  2.8938  3
16 43.0187  1
```




```
MTB > copy c24 c25;
SUBC> omit 16.
MTB > nscores c25 c26
MTB > Plot C26*C25

--------------------------
MTB > stack c17 c18 c31
MTB > print c31
```

```
Data Display
C31
39.0 31.8 47.0 40.9 43.8 29.3 34.8 45.6 40.1 42.0 54.9 39.9 43.1
30.1 35.6 41.4 43.2 43.7 51.4 40.3 40.5 52.9 48.2 58.2 41.9 40.5
53.0 40.2 40.2 39.9 53.7 49.5
MTB > Insert c1-c16;
SUBC> File "C:\...\design4.dat";
SUBC> Decimal ".".
Entering data from file: C:\...\DESIGN4.DAT
16 rows read.
MTB > name c2 'A' c3 'B' c5 'C' c9 'D' c31 'Y'
MTB > set c32
DATA> 16(1)
DATA> 16(2)
DATA> end
MTB > name c32 'R'
MTB > anova Y=A|B|C|D R

ANOVA: Y versus A, B, C, D, R

Analysis of Variance for Y
Source   DF      SS      MS     F      P
A         1   61.05   61.05  1.85  0.194
B         1  267.96  267.96  8.11  0.012
C         1    0.28    0.28  0.01  0.928
D         1    0.66    0.66  0.02  0.889
A*B       1    0.03    0.03  0.00  0.976
A*C       1  105.85  105.85  3.20  0.094
A*D       1   35.70   35.70  1.08  0.315
B*C       1    0.10    0.10  0.00  0.957
B*D       1    2.10    2.10  0.06  0.804
C*D       1   38.28   38.28  1.16  0.299
A*B*C     1  183.36  183.36  5.55  0.033
A*B*D     1   20.80   20.80  0.63  0.440
A*C*D     1   22.78   22.78  0.69  0.419
B*C*D     1    0.78    0.78  0.02  0.880
A*B*C*D   1    3.00    3.00  0.09  0.767
R         1  300.13  300.13  9.08  0.009
Error    15  495.87   33.06
Total    31 1538.75

----------------------------
MTB > anova Y=A|B|C R;
SUBC> means A|B|C R.
ANOVA: Y versus A, B, C, R

Analysis of Variance for Y
Source  DF      SS
A        1   61.05
B        1  267.96
C        1    0.28
A*B      1    0.03
A*C      1  105.85
B*C      1    0.10
A*B*C    1  183.36
R        1  300.13
Error   23  619.98
Total   31 1538.75

Means
A   N      Y     B  N      Y       C  N      Y
```

```
-1 16 44.400    -1 16 40.125      -1 16 43.112
 1 16 41.638     1 16 45.913       1 16 42.925

 A  B N      Y      A  C N      Y      B  C N      Y
-1 -1 8 41.475     -1 -1 8 46.313     -1 -1 8 40.275
-1  1 8 47.325     -1  1 8 42.487     -1  1 8 39.975
 1 -1 8 38.775      1 -1 8 39.913      1 -1 8 45.950
 1  1 8 44.500      1  1 8 43.362      1  1 8 45.875

 A  B  C N      Y           R  N      Y
-1 -1 -1 4 41.050           1 16 39.956
-1 -1  1 4 41.900           2 16 46.081
-1  1 -1 4 51.575
-1  1  1 4 43.075
 1 -1 -1 4 39.500
 1 -1  1 4 38.050
 1  1 -1 4 40.325
 1  1  1 4 48.675
```

Ex. 2.4.2 One has implemented a flame safety test for two different flame retardant treatments in the form of a 2^k design with factors of textile materials (A), flame-retardant treatment (B), laundry status (C) (low level = no laundry, high level = wash) and test method (D). They have used equal pieces and textiles taking as the response variable the number of inches of burned material. Results

(1)	42	d	40
a	31	ad	30
b	45	bd	50
ab	29	abd	25
c	39	cd	40
ac	28	acd	25
bc	46	bcd	50
abc	32	abcd	23

The data were analyzed using Minitab, see below.

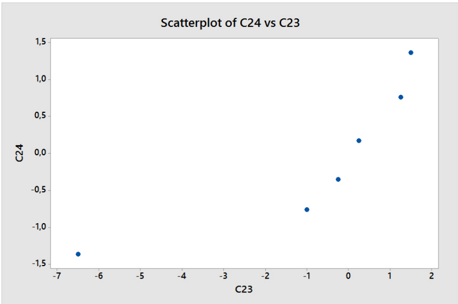
a) At first analyzes by 2^3-designs for each of the test methods were conducted. What effect seems to be most significant in each of analyzes? The answer must be justified.

b) This was followed by an analysis according to a complete three factor model with factors A, B and D. Can you recommend flame safety treatment for the different fabrics and test methods? Answer the question by using appropriate confidence intervals with simultaneous confidence level at least 80%.

Data output:

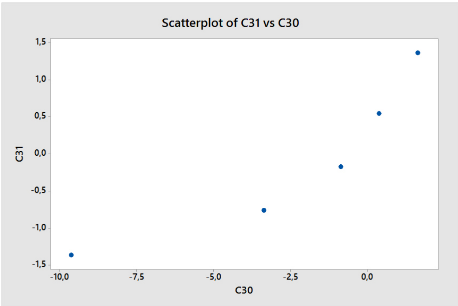
```
MTB > Read c1-c8;
SUBC>   File "C:\...\design3.dat";
SUBC>   Decimal ".".
Entering data from file: C:\...\DESIGN3.DAT
8 rows read.
MTB > set c17
DATA> 42 31 45 29 39 28 46 32
DATA> end
```

```
MTB > set c18
DATA> 1:8
DATA> end
MTB > copy c1-c8 m1
MTB > copy c17 m2
MTB > trans m1 m3
MTB > mult m3 m2 m4
MTB > copy m4 c19
MTB > let c20=c19/8
MTB > Sort C20 C18 c21 c22;
SUBC>   By c20.
MTB > print c21 c22

Data Display
Row     C21  C22
  1  -6,50    2
  2  -1,00    4
  3  -0,25    5
  4   0,25    6
  5   0,25    8
  6   1,25    7
  7   1,50    3
  8  36,50    1

MTB > copy c21 c23;
SUBC> omit 8.
MTB > nscores c23 c24
MTB > Plot C24*C23;
SUBC>   Symbol.
```




```
-------------------------------------
MTB > set c25
DATA> 40 30 50 25 40 25 50 23
DATA> end
MTB > copy c25 m2
MTB > trans m1 m3
MTB > mult m3 m2 m4
MTB > copy m4 c26
MTB > let c27=c26/8
MTB > Sort C27 C18 c28 c29;
SUBC>   By c27.
MTB > print c28 c29

Data Display
Row      C28  C29
  1  -9,625    2
  2  -3,375    4
  3  -0,875    5
  4  -0,875    6
```

```
  5   0,375    7
  6   0,375    8
  7   1,625    3
  8  35,375    1

MTB > copy c28 c30;
SUBC> omit 8.
MTB > nscores c30 c31
MTB > Plot C31*C30;
SUBC>   Symbol.
```




```
MTB > Read c1-c16;
SUBC>   File "C:\...\design4.dat";
SUBC>   Decimal ".".
Entering data from file: C:\...\DESIGN4.DAT
16 rows read.
MTB > stack c17 c25 c32
MTB > print c32

Data Display
C32
   42    31    45    29    39    28    46    32    40    30    50    25    40    25    50    23

MTB > name c32 'Y'
MTB > name c2 'A'
MTB > name c3 'B'
MTB > name c9 'D'
MTB > anova Y=A|B|D;
SUBC> means A|B|D.

ANOVA: Y versus A; B; D
Factor  Type   Levels  Values
A       fixed       2  -1;  1
B       fixed       2  -1;  1
D       fixed       2  -1;  1

Analysis of Variance for Y
Source  DF       SS       MS       F      P
A        1  1040,06  1040,06  291,95  0,000
B        1    39,06    39,06   10,96  0,011
D        1     5,06     5,06    1,42  0,267
A*B      1    76,56    76,56   21,49  0,002
A*D      1    39,06    39,06   10,96  0,011
B*D      1     0,06     0,06    0,02  0,898
A*B*D    1    22,56    22,56    6,33  0,036
Error    8    28,50     3,56
Total   15  1250,94

Means
```

A	N	Y
-1	8	44,000
1	8	27,875

B	N	Y
-1	8	34,375
1	8	37,500

D	N	Y
-1	8	36,500
1	8	35,375

A	B	N	Y
-1	-1	4	40,250
-1	1	4	47,750
1	-1	4	28,500
1	1	4	27,250

A	D	N	Y
-1	-1	4	43,000
-1	1	4	45,000
1	-1	4	30,000
1	1	4	25,750

B	D	N	Y
-1	-1	4	35,000
-1	1	4	33,750
1	-1	4	38,000
1	1	4	37,000

A	B	D	N	Y
-1	-1	-1	2	40,500
-1	-1	1	2	40,000
-1	1	-1	2	45,500
-1	1	1	2	50,000
1	-1	-1	2	29,500
1	-1	1	2	27,500
1	1	-1	2	30,500
1	1	1	2	24,000

Ex. 2.4.3 The quality of the fabric is judged on a scale from 0 to 10.0. A 2^4-factorial design was conducted to investigate the effects of

A: two machine operators
B: two machines
C: two different materials
D: two kinds of color.

Since the experiment interfere with the normal production, one could only carry eight attempts at a time of fairly large time intervals. The sixteen experiments were divided into two blocks according to the rule K=ABCD, where K is the block factor. The trials within each block was then performed in random order in the two periods. Results:

(1)	7.8	d	8.0
a	7.0	ad	8.3
b	7.3	bd	7.5
ab	8.4	abd	8.7
c	8.5	cd	9.0
ac	8.0	acd	8.0
bc	7.6	bcd	8.6
abc	9.0	abcd	9.5

a) How are the observations divided into two blocks?

b) Results from Minitab analysis are given below. The first analysis corresponds to a complete four factor model for A, B, C and D. In what effect is the effect of blocks overlaid? It that true that there is no difference between the blocks?

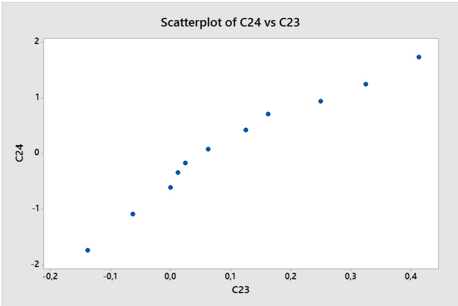
c) According to which model are data analyzed in the ANOVA analysis? What are the reasons to choose this particular model?

d) What can you say about the choice of materials and color? How to choose the machine for each of two operators? Justify your answer using appropriate confidence intervals with simultaneous confidence level at least 92% of all intervals together.

No. 1:

```
MTB > Read c1-c16;
SUBC>   File "C:\....\design4.dat";
SUBC>   Decimal ".".
Entering data from file: C:\....\DESIGN4.DAT
16 rows read.
MTB > set c17
DATA> 7,8 7,0 7,3 8,4 8,5 8,0 7,6 9,0 8,0 8,3 7,5 8,7 9,0 8,0 8,6 9,5
DATA> end
MTB > set c18
DATA> 1:16
DATA> end
MTB > copy c1-c16 m1
MTB > copy c17 m2
MTB > trans m1 m3
MTB > mult m3 m2 m4
MTB > copy m4 c19
MTB > let c20=c19/16
MTB > Sort C20 C18 c21 c22;
SUBC>   By c20.
MTB > print c21 c22

Data Display
Row      C21  C22
  1  -0,1375   14
  2  -0,0625   12
  3  -0,0625    6
  4   0,0000   11
  5   0,0000   13
  6   0,0125   10
  7   0,0250    7
  8   0,0625    8
  9   0,0625   16
 10   0,1250    3
 11   0,1250   15
```

```
 12   0,1625    2
 13   0,2500    9
 14   0,3250    5
 15   0,4125    4
 16   8,2000    1

MTB > copy c21 c23;
SUBC> omit 16.
MTB > nscores c23 c24
MTB > plot c24*c23
```



No. 2:

```
MTB > name c2 'A'
MTB > name c3 'B'
MTB > name c5 'C'
MTB > name c9 'D'
MTB > name c17 'Y'
MTB > anova Y=A|B C D;
SUBC> means A|B C D.

ANOVA: Y versus A; B; C; D
Factor  Type   Levels  Values
A       fixed       2  -1;  1
B       fixed       2  -1;  1
C       fixed       2  -1;  1
D       fixed       2  -1;  1

Analysis of Variance for Y
Source  DF      SS      MS      F      P
A        1  0,4225  0,4225   5,18  0,046
B        1  0,2500  0,2500   3,07  0,110
A*B      1  2,7225  2,7225  33,40  0,000
C        1  1,6900  1,6900  20,74  0,001
D        1  1,0000  1,0000  12,27  0,006
Error   10  0,8150  0,0815
Total   15  6,9000

Means
A   N       Y
-1  8  8,0375
 1  8  8,3625

B   N       Y
-1  8  8,0750
 1  8  8,3250

A   B   N       Y
-1  -1  4  8,3250
```

```
-1   1  4  7,7500
 1  -1  4  7,8250
 1   1  4  8,9000

C   N       Y
-1  8  7,8750
 1  8  8,5250

D   N       Y
-1  8  7,9500
 1  8  8,4500
```

Ex. 2.4.4 In the 2^{5-1}-fractional factorial design with factors A, B, C, D and E the factor E was applied according to the rule ABCD=E. Results:

	y		y
e	14.8	d	16.0
a	14.5	ade	15.1
b	18.1	bde	18.9
abe	19.4	abd	22.0
c	18.4	cde	19.8
ace	15.7	acd	18.9
bce	27.3	bcd	29.9
abc	28.2	abcde	27.4

Analysis performed in Minitab is available below.

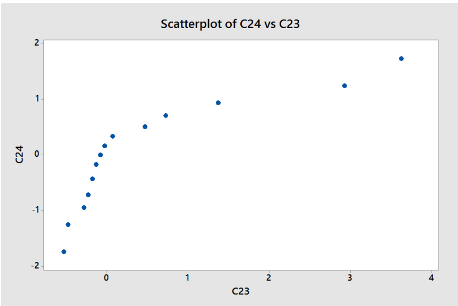
a) At first the data was analyzed according to a model with sixteen possible parameters. Which effects appear to be of greatest importance and which interaction effects of higher order were overlayed on them? The answer must be justified. In which parameter estimation the main effect of E-factor is included?

b) The data were also analyzed using a reduced model, where we take into account only factors B and C. Write up this model. What level combination would you recommend for B and C, if one seeks high y values. Justify your answer by constructing appropriate confidence intervals with simultaneous confidence level at least 95%.

c) Which of the factors that we disregarded in b) would primarily like to be explored further? Justify your answer briefly.

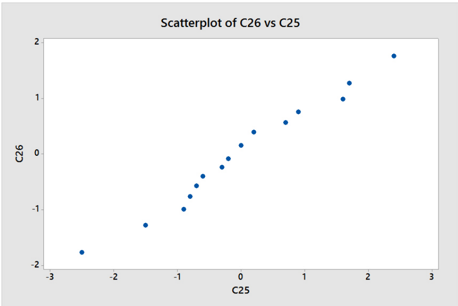
```
MTB > Read c1-c16;
SUBC>   File "C:\....\design4.dat";
SUBC>   Decimal ".".
Entering data from file: C:\....\DESIGN4.DAT
16 rows read.
MTB > set c17
DATA> 14,8 14,5 18,1 19,4 18,4 15,7 27,3 28,2 16,0 15,1 18,9 22,0 19,8 18,9
29,9 27,4
DATA> end
MTB > set c18
DATA> 1:16
DATA> end
MTB > copy c1-c16 m1
MTB > copy c17 m2
```

```
MTB > trans m1 m3
MTB > mult m3 m2 m4
MTB > copy m4 c19
MTB > let c20=c19/16
MTB > Sort C20 C18 c21 c22;
SUBC>   By c20.
MTB > print c21 c22

Data Display
Row      C21  C22
  1  -0,525    6
  2  -0,475   16
  3  -0,275   15
  4  -0,225    8
  5  -0,175   14
  6  -0,175   12
  7  -0,125    2
  8  -0,075   11
  9  -0,025   10
 10   0,075   13
 11   0,475    4
 12   0,725    9
 13   1,375    7
 14   2,925    5
 15   3,625    3
 16  20,275    1

MTB > copy c21 c23;
SUBC> omit 16.
MTB > nscores c23 c24
MTB > plot c24*c23
```




```
MTB >name c3 'B'
MTB > name c5 'C'
MTB > name c17 'Y'
MTB > anova Y=B| C;
SUBC> residuals c25;
SUBC> means B|C.
 ANOVA: Y versus B; C
Factor  Type   Levels  Values
B       fixed       2  -1;  1
C       fixed       2  -1;  1

Analysis of Variance for Y
Source  DF      SS      MS       F      P
B        1  210,25  210,25  107,45  0,000
C        1  136,89  136,89   69,96  0,000
B*C      1   30,25   30,25   15,46  0,002
Error   12   23,48    1,96
```

```
Total    15  400,87

Means
B   N       Y
-1  8  16,650
 1  8  23,900

C   N       Y
-1  8  17,350
 1  8  23,200

B   C   N       Y
-1  -1  4  15,100
-1   1  4  18,200
 1  -1  4  19,600
 1   1  4  28,200

MTB > nscores c25 c26
MTB > plot c26*c25
```



Ex. 2.4.5 In a study of capillary zone electrophoresis (CZE) of heterocyclic amino acids (MCA) one has varied five factors in order to optimize CZE separation, Journal of Chromatographic Science (1996).

Factors	Low level	High level
A: pH	2.5	3.5
B: methanol	0%	3.5%
C: NaCl	0mM	30mM
D: temp.	35°C	25°C
E: voltage	20 kV	15 kV

A 2^{5-1}-fractional factorial design with E=BCD has been conducted and the values indicated in the table below are *electrophoresis response values minus 47*. High values are good. Data sorted as if it were a 2^4-factorial design with factors A, B, C and D.

(1)	5.08	de	9.33
a	4.97	ade	4.04
be	7.66	bd	12.25
abe	3.58	abd	0.46
ce	7.78	cd	8.02
ace	3.37	acd	1.36
bc	12.21	bcde	11.34
abc	6.90	abcde	2.10

The data have been analyzed partly using the complete model and partly using the reduced model, see below.

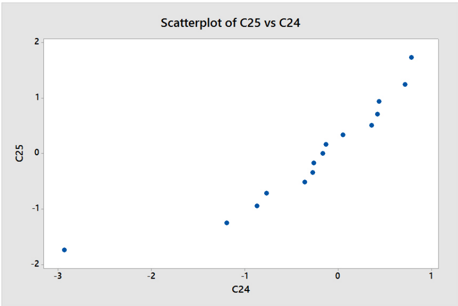
a) Consider analysis no. 1. Which three effects seem to have the greatest impact? Motivate briefly. Both the effects, possible overlays and the corresponding parameter estimates shall be stated. Which parameter estimate includes E-effects?

b) According to which model, the data was analyzed in the analysis no. 2? From the first analysis it seems clear which A-level should be selected. Consider this choice as obvious. Can one from the second analysis recommend levels B and D? Construct appropriate confidence intervals with simultaneous confidence level at least 70%-80% and report your findings. Normal distribution may be assumed.

Computer output:

```
MTB > Read c1-c16;
SUBC>   File "C:\....\design4.dat";
SUBC>   Decimal ".".
Entering data from file: C:\....\DESIGN4.DAT
16 rows read.
MTB > set c17
DATA> 5,08 4,97 7,66 3,58 7,78 3,37 12,21 6,9 9,33 4,04 12,25 0,46 8,02
1,36 11,34 2,1
DATA> end
MTB > set c18
DATA> 1:16
DATA> end
MTB > copy c1-c16 m1
MTB > copy c17 m2
MTB > trans m1 m3
MTB > mult m3 m2 m4
MTB > copy m4 c19
MTB > let c20=c19/16
MTB > Sort C20 C18 c21 c22;
SUBC>   By c20.
MTB > let c23=16*c21**2
MTB > print c21-c23

Data Display
Row       C21  C22      C23
  1  -2,93063    2  137,417
  2  -1,19187   10   22,729
  3  -0,87187    4   12,163
  4  -0,76438   13    9,348
  5  -0,35937   11    2,066
  6  -0,27187    6    1,183
  7  -0,26313   12    1,108
  8  -0,16562    9    0,439
  9  -0,12813   15    0,263
 10   0,05313   16    0,045
 11   0,35688    5    2,038
 12   0,41937   14    2,814
 13   0,43687    8    3,054
 14   0,71813    7    8,251
 15   0,78438    3    9,844
 16   6,27812    1  630,638


MTB > copy c21 c24;
```

```
SUBC> omit 16.
MTB > nscores c24 c25
MTB > plot c25*c24
```




```
MTB >  anova Y=A|B|D;
SUBC> means A|B|D.

ANOVA: Y versus A; B; D
Factor  Type   Levels  Values
A       fixed       2  -1;  1
B       fixed       2  -1;  1
D       fixed       2  -1;  1

Analysis of Variance for Y
Source  DF       SS       MS      F      P
A        1  137,417  137,417  40,72  0,000
B        1    9,844    9,844   2,92  0,126
D        1    0,439    0,439   0,13  0,728
A*B      1   12,163   12,163   3,60  0,094
A*D      1   22,729   22,729   6,74  0,032
B*D      1    2,066    2,066   0,61  0,456
A*B*D    1    1,108    1,108   0,33  0,582
Error    8   26,996    3,374
Total   15  212,761

Means
A    N       Y
-1   8  9,2088
 1   8  3,3475

B    N       Y
-1   8  5,4938
 1   8  7,0625

D    N       Y
-1   8  6,4438
 1   8  6,1125

A    B   N       Y
-1  -1   4   7,553
-1   1   4  10,865
 1  -1   4   3,435
 1   1   4   3,260

A    D   N       Y
-1  -1   4   8,183
-1   1   4  10,235
 1  -1   4   4,705
 1   1   4   1,990
```

```
B   D   N      Y
-1  -1  4  5,3000
-1   1  4  5,6875
 1  -1  4  7,5875
 1   1  4  6,5375

A   B   D   N      Y
-1  -1  -1  2   6,430
-1  -1   1  2   8,675
-1   1  -1  2   9,935
-1   1   1  2  11,795
 1  -1  -1  2   4,170
 1  -1   1  2   2,700
 1   1  -1  2   5,240
 1   1   1  2   1,280
```

Ex. 2.4.6 In a particular construction includes steel elements joined together with rubber gaskets glued to steel. The structure will be used in water. One has conducted a 2^{5-1}-fractional factorial design where each one of factors

A: The concentration of sea water
B: Temperature
C: pH-value
D: Voltage
E: Loading

had a low and high level. As a generator for the study plan one used I = ABCDE. The following values, y, of the total number of cracks in the rubber joints have been measured:

462 746 714 1070 474 832 764 1087

522 854 773 1068 572 831 819 1104

where observations, if you read line by line, are sorted as if one had in complete 2^4-factorial design with A, B, C and D.

a) Which observations have been taken regarding level of factors A, B, C, D and E? Describe each observed y-values above following the designations (1), a, b, ab, ..., but it is not certain that these are included.

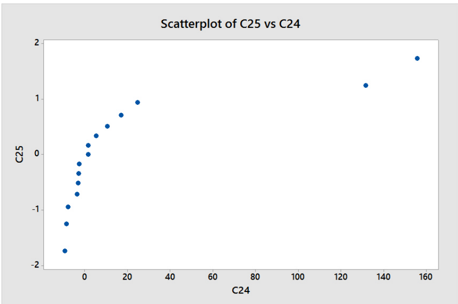
b) The data was at first analyzed under a complete model, where the effects overlaid on each other. Which are the two most important parameters according to this analysis and what effects they contain? In which parameter estimate e_1 is included, i.e., the parameter that describes the main effect of E?

c) The ANCOVA analysis a reduced normal distribution model is utilized. Construct on the basis of this analysis, a 95% confidence interval for E(Y) for the level combination that seems to work the worst.

Computer output:

```
MTB > Read c1-c16;
SUBC>   File "C:\....\design4.dat";
SUBC>   Decimal ".".
Entering data from file: C:\....\DESIGN4.DAT
16 rows read.
```

```
MTB > set c17
DATA> 462 746 714 1070 474 832 764 1087
DATA> 522 854 773 1068 572 831 819 1104
DATA> end
MTB > set c18
DATA> 1:16
DATA> end
MTB > copy c1-c16 m1
MTB > copy c17 m2
MTB > trans m1 m3
MTB > mult m3 m2 m4
MTB > copy m4 c19
MTB > let c20=c19/16
MTB > Sort C20 C18 c21 c22;
SUBC>   By c20.
MTB > let c23=16*c21**2
MTB > print c21-c23

Data Display
Row       C21  C22       C23
  1    -9,375   10      1406
  2    -8,500   11      1156
  3    -7,750   14       961
  4    -3,500   13       196
  5    -3,000   12       144
  6    -2,750    8       121
  7    -2,625    6       110
  8     1,500    7        36
  9     1,625    4        42
 10     5,375   15       462
 11    10,625   16      1806
 12    17,125    5      4692
 13    24,625    9      9702
 14   131,625    3    277202
 15   155,750    2    388129
 16   793,250    1  10067929

MTB > copy c21 c24;
SUBC> omit 16.
MTB > nscores c24 c25
MTB > plot c25*c24
```




```
MTB > ancova Y=A B C D E
ancova Y=A B C D E
* NOTE * Unbalanced design. A cross tabulation of your factors will show
       * where the unbalance exists.
* NOTE * Make sure your design is orthogonal.

ANCOVA: Y versus A; B; C; D; E
```

```
Analysis of Variance for Y
Source  DF      SS      MS
A        1  388129  388129
B        1  277202  277202
C        1    4692    4692
D        1    9702    9702
E        1    1806    1806
Error   10    4635     464
Total   15  686167
```

2.5 Non-parametric methods

Ex. 2.5.1 For each of the ten different blood samples, the number of white blood cells was determined by two laboratory assistants. Results:

Lab-ass	1	2	3	4	5	6	7	8	9	10
1	243	275	270	280	271	230	251	225	293	294
2	259	255	274	391	309	251	254	244	300	290

It has been suspected that laboratory assistant 2 obtains rather too high values, that is, he gets i average higher number of cells than laboratory assistant 1 for the same blood sample.
Examine if the suspicion is justified

a) using a sign test at a maximum level of 0.10.

b) using Wilcoxons sign rank test at a maximum level of 0.10.

c) Construct two-sided 80% confidence interval for the systematic difference μ_D between measurements of both laboratory assistants based on the sign rank test.

The following computer printout from Minitab facilitates calculations. Data from lab-ass 1 are place in column c1 and data for lab-ass 2 in column c2.

```
MTB> let c3=c2-c1
```

We have differences $z_i = y_i - x_i$ in column c3.

```
MTB> print c3
Data Display
z_i
    16 -20  4  111  38  21  3  19  7  -4
```

Via Stat/Nonparametrics/Pairwise Averages we create pairwise mean values for z_i

```
MTB> Walsh C3 C4.
```

We have pairwise averages of z_i in c4 and we sort them in increasing order.

```
MTB> sort c4 c5
MTB> print c5
Data Display
A_j
-20.0   -12.0   -8.5   -8.0   -6.5   -4.0   -2.0   -0.5   -0.5
  0.0     0.5    1.5    3.0    3.5    4.0    5.0    5.5    6.0
  7.0     7.5    8.5    9.0    9.5   10.0   11.0   11.5   11.5
 12.0    12.5   13.0   14.0   16.0   17.0   17.5   18.5   19.0
 20.0    20.5   21.0   21.0   22.5   27.0   28.5   29.5   38.0
 45.5    53.5   57.0   57.5   59.0   63.5   65.0   66.0   74.5
111.0
```

Ex. 2.5.2 In a study one wanted to examine if cyclazocine has a beneficial effect on the heroin addict's psychological dependence on heroin. A group of fourteen chronic addicts were treated and then they were asked to answer a questionnaire about their psychological dependence. For each person a so called Q-score was determined from the responses. The minimum possible value of Q-score is 11 and maximum possible value is 55, where a high value indicates **low** psychological dependence. Results:

51 53 43 36 55 55 39 43 45 27 21 26 22 43

The questionnaire was designed so that the results for heroin addicts who do not receive treatment have a distribution that is symmetrical around $\mu = 28$.
Examine the treated group using Wilcoxon sign rank test

$$H_0 : \mu = 28 \text{ against } H_1 : \mu > 28$$

at level 0.01,
a) using the table for sign ranked test,
b) using normal approximation.

Ex. 2.5.3 Replace the test task in Ex. 2.1.1a) with an appropriate non-parametric test on the level of approximately 0.01 (notice that test statistic should be adjusted for ties)

Ex. 2.5.4 For a certain type of components there are two brands A and B. One wanted to investigate whether there was any difference between those two brands and therefore chose randomly fifteen components of every make, put all in operation and registered life for these components, until all of one kind is broken. One sample is **not complete**. Results:

Type A:	13.1	16.4	18.5	21.3	24.0	26.3	30.1	38.4
	41.2	49.5	57.0	63.4	86.7	94.2	99.0	
Type B:	17.3	21.5	26.5	34.3	46.5	58.3	69.0	77.9
	86.4	97.8						

Examine H_0: The same life distribution for the two brands
against H_1: Different life distribution for the two brands
using
a) Wilcoxons rank sum test on the level 0.05.
b) Tukey-Duckworths test on the level 0.05.

Ex. 2.5.5 Investigate using Friedman's test at the level of approximately 0.05 if the threads resorts in Ex. 2.2.2 are the same good. What are the blocks in this case? Note that you would use test statistic corrected for ties.

Ex. 2.5.6 Two groups with pigs of equal size were injected with sedatives and for each pig time in minutes between injection and onset of sleep was measured. The pigs in the two groups were given 0.5 mg and 1.5 mg of the product, respectively. Results:

Dose		$\bar{y}_i$	s_i
0.5 mg:	21 23 19 24	21.75	2.22
1.5 mg:	15 10 13 14 11 15	13.00	2.10

Model: The r.v. $X_i = \mu_1 + \epsilon_i$ and r.v. $Y_j = \mu_2 + \tilde{\epsilon}_j$, where ϵ-variables have expectation 0.
Have dose any significance for falling sleep time? Answer the question by:

a) assuming that all ϵ-variables are normal distributed with the same variance and by construction of a suitable 95% confidence interval.

b) using a 95% confidence interval that is constructed according to Wilcoxon-Mann-Whitneys method.

Ex. 2.5.7 Fourteen cars of brand A were considered and number of mil that corresponds to their lifetimes were recorded. Results:

7980 12644 21013 2014 13007 11084 11011

4711 15013 11043 13142 12112 8910 13014

while seven cars of brand B has been scrapped after the following number of mil

3014 12142 7890 8810 9450 6100 9088

We assume that the cars were selected randomly among all cars in Sweden of the current brands manufactured in a given period.

Examine on the level 5%
H_0: Lifetime distribution of the two brands is the same.
mot
H_1: Lifetime distribution of the two brands is NOT the same.
The comparison is of course valid only for cars used in Sweden.

Ex. 2.5.8 One wanted to investigate the influence of treatment on reparation ability of nerves. Three groups were examined, where the nerves had been under treatment by 1, 3 and 7 days, respectively. The growth you had for two days after repair procedure was measured. Results in mm: Results in mm:

Group								
1	1.79	2.11	1.20	0.64	1.60	2.13	1.06	1.19
2	2.47	2.17	2.82	2.16	2.30	2.86	1.95	2.09
3	2.40	1.50	1.97	1.54	1.04	1.93	1.48	1.67
	1.39	1.68	1.40	1.13				

Has the time under treatment importance for growth after repair? Perform an appropriate non-parametric test at the level of approximately 0.05.

Ex. 2.5.9 a) Examine with a non-parametric test, at the level of approximately 0.05, if the copper ion concentration appears to be significant for the BOD value in the Ex. 2.2.5

b) Examine using Wilcoxon sign rank test at the level 0.05 if copper ion concentration no. 5 is better than no. 3.

2.6 Response surface

Ex. 2.6.1 The following data sets have studied with two factors A, temperature, and B, the amount of *acetic anhydride*, in connection with measurement of phenol in the soil samples.

		Factors		Coded factors		Total phenol
	Run	A ($°C$)	B(μL)	X_1	X_2	recovery (%), Y
Factorial	7	90	80	-1	-1	71.23
	3	110	80	+1	-1	88.70
	5	90	130	-1	+1	82.24
	8	110	130	+1	+1	90.09
Centre	1	100	105	0	0	81.57
	10	100	105	0	0	84.31

Initial analysis has been carried out:

```
Full Factorial Design
Factors:  2   Base Design:         2; 4
Runs:     6   Replicates:             1
Blocks:   1   Center pts (total):     2

Factorial Regression: C7 versus A; B; CenterPt

Analysis of Variance
Source               DF   Adj SS   Adj MS  F-Value  P-Value
Model                 4  221,873   55,468    14,78    0,192
  Linear              2  198,716   99,358    26,47    0,136
    A                 1  160,276  160,276    42,70    0,097
    B                 1   38,440   38,440    10,24    0,193
  2-Way Interactions  1   23,136   23,136     6,16    0,244
    A*B               1   23,136   23,136     6,16    0,244
  Curvature           1    0,021    0,021     0,01    0,953
Error                 1    3,754    3,754
Total                 5  225,626

Coded Coefficients
Term      Effect    Coef  SE Coef  T-Value  P-Value
Constant          83,065    0,969    85,75    0,007
A         12,660   6,330    0,969     6,53    0,097
B          6,200   3,100    0,969     3,20    0,193
A*B       -4,810  -2,405    0,969    -2,48    0,244
Ct Pt              -0,13     1,68    -0,07    0,953
```

a) Examine using a suitable test on the level 0.10 if there is tendency of curvature of the underlying functional surface. See also how test statistic is calculated from the observed Y-value.

b) How should one continue the investigation?
(i) Make additional measurements so that you can adapt a second degree polynomial and seek an optimum point?
(ii) Making new measurements involving a step up in x_1x_2 plane from the old zero point to a new better center point? In which direction should we in such a case step?
The answer must be justified.

Ex. 2.6.2 In the production of a certain kind of mechanical devices one are committed to obtain an end product that in service should have as little vibration as possible. One have done experiment with production by varying two factors and measure the strength of the vibrations. After an initial 2^2-factorial design one have found an interesting area and in this area implemented a new 2^2-factorial design. Results

Orginal factor settings		Coded factor settings		Response
X_1	X_2	X_1	X_2	Y
2.25	2.5	0	0	0.248
2.25	2.5	0	0	0.251
2.25	2.5	0	0	0.252
2.40	2.7	1	1	0.290
2.40	2.3	1	-1	0.270
2.10	2.7	-1	1	0.263
2.10	2.3	-1	-1	0.251

Analysis using Minitab gave

```
Factorial Regression: C7 versus x1; x2; CenterPt

Analysis of Variance
Source              DF    Adj SS    Adj MS
Model                4  0,001367  0,000342
  Linear             2  0,000785  0,000393
    x1               1  0,000529  0,000529
    x2               1  0,000256  0,000256
  2-Way Interactions 1  0,000016  0,000016
    x1*x2            1  0,000016  0,000016
  Curvature          1  0,000566  0,000566
Error                2  0,000009  0,000004
Total                6  0,001375

Coded Coefficients
Term       Effect      Coef  SE Coef
Constant            0,26850  0,00104
x1        0,02300   0,01150  0,00104
x2        0,01600   0,00800  0,00104
x1*x2     0,00400   0,00200  0,00104
Ct Pt              -0,01817  0,00159
```

a) Examine using a suitable test or confidence interval if the response surface in the area is curved. Level 0.05.

b) Suggest four new measuring points with which you may find an optimal point. State also according to which model the data should be analyzed when additional measurements are made. Should one make additional measurements in the zero point?

Ex. 2.6.3 One has carried out a 2^2-factorial design with measurements in the zero point when the levels of the two factors temperature and pressure were encoded -1 and +1 in the usual manner. At the subsequent analysis one has not been shown any tendency to curvature. A regression analysis has given the estimated regression relationship

$$y = 30.5 + 2.1x_1 - 3.5x_2$$

where y represents the response variable and x_1 and x_2 is the coded values of temperature and pressure. One aims high y-values and wants, via new experiments, to find best combinations of analyzed two factors. Propose new combinations of x_1 and x_2 that should be analyzed (in coded values).

2.7 Choice of sample size

Ex. 2.7.1 Two sicknesses A and B are related to the increase of the increased blood calcium level. One wants to investigate if any of those two sicknesses is significantly more influenced by the calcium level. For n A-patients and n B-patients one is going to measure calcium level and count how many people have abnormally high values. Let π_A and π_B be probabilities that A-, B-patient are abnormal Ca-value, respectively.

a) One wants to examine

$$H_0 : \pi_A = \pi_B \ \ against \ \ H_1 : \pi_A \neq \pi_B$$

at significance level 0.05 and one decided to design test so that the conclusion $H_0 rejected$ is given with probability 0.95 if $|\pi_A - \pi_B| = 0.20$. One guesses that $\pi_A = 0.15$ and $\pi_B = 0.35$ or vice versa. Determine n. Take advantage of the Kirkwood's formula sheet.

b) Determine also n' for $H_1 : \pi_B > \pi_A$.

Observe that one when you perform the measurements the results can be presented by means of an appropriate confidence interval for $\pi_A - \pi_B$ and reject H_0 if 0 is not included in the obtained interval.

Ex. 2.7.2 The laboratory in a pharmaceutical factory will compare two pupil astringent eye drops, where A is a previously used type and B is a new type. Let p be probability that type B is more effective than type A when using of a randomly chosen subject. It is believed that B is better than A and therefore one wants to examine

$$H_0 : p = 0.5 \ \ against \ \ H_1 : p > 0.5.$$

They do this by using the A and B each in his own eye of n randomly selected volunteers. The test variable is the number of volunteers that found B as more effective than A. One wants to have significance level approx 0.05 and reject H_0 with probability approx. 0.8 if $p = 0.7$. Determine n. Normal approximation shall be used.

Ex. 2.7.3 In a sociological study of the body's adaptation to the cold one should measure the change in body temperature where the subjects spent 90 minutes in the cold water. We assume that the change, i.e. the temperature before the minus temperature after cooling, is normally distributed with mean μ and standard deviation $\sigma = 0.4$ (unit: $°C$). How many test subjects (volunteers) are needed if you want to examine

$$H_0 : \mu = 0 \ \ against \ \ H_1 : \mu > 0$$

at level 0.05, so that the power of test for $\mu = 0.3$ is at least 0.90.

Ex. 2.7.4 A new process for the production of silicon panels is assumed to reduce the error rate to well below 10%. To investigate this, it is planned to take 250 panels at random and examine them. Let X denote the number of incorrect among the surveyed plates and let p denote the true probability that a panel is faulty. One should examine

$$H_0 : p = 0.10 \ \ against \ \ H_1 : p < 0.10.$$

One decides to reject H_0 if $x \leq 18$.

a) Calculate approximate significance level for the test.

b) Calculate the approximate strength of test for $p = 0.04$.

In both a) and b) one can assume that silicon panels become faulty independently of each other.

2.8 Linear models. Regression.

Ex. 2.8.1 In this exercise we should analyze date from Ex. 2.2.6 using regression model with dummy variables. Usually two factor model is more convenient in case of balanced design as in this example, but if we do not have the same amount of observations for different level combinations method with dummy variables can be usefull. Let us define three dummy variables (amount of needed dummy variables is given by (a-1)+(b-1)):

$$u_1 = \begin{cases} 1 & \text{for material 1,} \\ 0 & \text{otherwise,} \end{cases}$$

$$u_2 = \begin{cases} 1 & \text{for material 2,} \\ 0 & \text{otherwise,} \end{cases}$$

$$v_1 = \begin{cases} 1 & \text{for solder 1,} \\ 0 & \text{otherwise.} \end{cases}$$

In order to take into account the interaction effects one forms products of dummy variables belonging to different factors, see data output.

The data were then analyzed according to the model

$$Y = \beta_0 + \beta_1 u_1 + \beta_2 u_2 + \gamma_1 v_1 + \delta_1 u_1 v_1 + \delta_1 u_2 v_1 + \epsilon,$$

where $\epsilon \sim N(0, \sigma)$.

a) What parameters take care of the interaction effects? Are they significantly different from 0 at significance level $\alpha = 0.01$?

b) Estimate $E(Y)$ for level combinations L1M1, L2M2 and L2M3 and compare with corresponding estimators from Ex. 2.2.6.

c) Compare SS_E for regressions model with SS_E in Ex. 2.2.6.

```
MTB > print c1-c6

Data Display
Row    Y  u1  u2  v1  u1v1  u2v1
  1  102   1   0   1     1     0
  2   97   1   0   1     1     0
  3   86   0   1   1     0     1
  4   90   0   1   1     0     1
  5   78   0   0   1     0     0
  6   66   0   0   1     0     0
  7   94   1   0   0     0     0
  8   99   1   0   0     0     0
  9  118   0   1   0     0     0
 10  110   0   1   0     0     0
 11   40   0   0   0     0     0
 12   37   0   0   0     0     0

MTB > Regress;
SUBC>   Response 'Y';
SUBC>   Nodefault;
SUBC>   Continuous 'u1' - 'u2v1';
SUBC>   Terms u1 u2 v1 u1v1 u2v1;
SUBC>   Constant;
SUBC>   Tanova;
SUBC>   Tsummary;
SUBC>   Tcoefficients;
SUBC>   Tequation.

Regression Analysis: Y versus u1; u2; v1; u1v1; u2v1

Analysis of Variance
Source      DF  Adj SS   Adj MS  F-Value  P-Value
Regression   5  7046,7  1409,35    59,76    0,000
  u1         1  3364,0  3364,00   142,64    0,000
  u2         1  5700,3  5700,25   241,71    0,000
  v1         1  1122,3  1122,25    47,59    0,000
  u1v1       1   465,1   465,12    19,72    0,004
  u2v1       1  1770,1  1770,13    75,06    0,000
Error        6   141,5    23,58
Total       11  7188,2

Model Summary
      S    R-sq  R-sq(adj)  R-sq(pred)
```

```
4,85627  98,03%     96,39%      92,13%

Coefficients
Term        Coef  SE Coef  T-Value  P-Value
Constant   38,50     3,43    11,21    0,000
u1         58,00     4,86    11,94    0,000
u2         75,50     4,86    15,55    0,000
v1         33,50     4,86     6,90    0,000
u1v1      -30,50     6,87    -4,44    0,004
u2v1      -59,50     6,87    -8,66    0,000

Regression Equation
Y = 38,50 + 58,00 u1 + 75,50 u2 + 33,50 v1 - 30,50 u1v1 - 59,50 u2v1
```

Ex. 2.8.2 Mortality in flour beetles

The idea with this experiment was to study the effect of gaseous carbon disulfide (CS_2) on a sort of flour beetles, Tribolium confusum. In experiments vials, two tissue cage with about 30 flour beetles in each, were placed. Various amounts of liquid CS_2 was placed in the bottles. After five hours, the actual concentration of gaseous CS_2 was measured and the number of dead beetles were counted. Mortality in the table below.

Concentration	Cage 1		Cage 2	
of CS_2	y	n	y	n
49.06	2	29	4	30
52.99	7	30	6	30
56.91	9	28	9	34
60.84	14	27	14	29
64.76	23	30	29	33
68.69	29	31	24	28
72.61	29	30	32	32
76.54	29	29	31	31

Table 1: Number of dead beetles, y, out of n placed in the cage for different concentrations of CS_2.

Data have been analyzed using the model

$$logit\ p = \beta_0 + \beta_1 x + \beta_2 x^2$$

see Minitab output below (a preliminary analysis showed that the replicates (cages) were equal).

a) Write out the estimated model for $logit\ \hat{p}$.

b) Construct confidence intervals for β_1 and β_2 each with confidence level 95%.

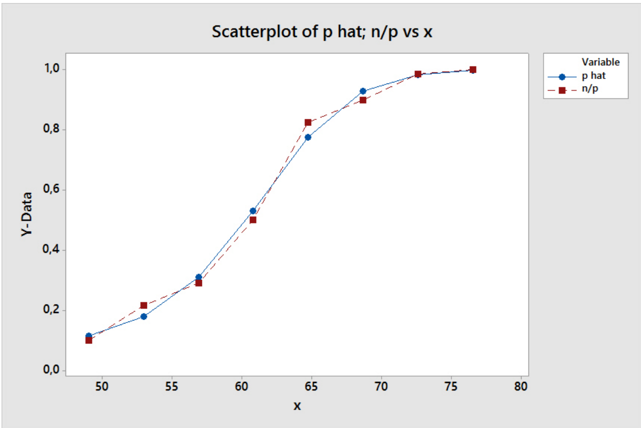
c) Compare $(SECoef)^2$ with diagonal elements of estimated covariance matrix.

d) Does the current model give a good adaptation to the observed data? Perform the appropriate test at level 5%.

e) Estimate the concentration for which 90% of the beetles dies.

```
Binary Logistic Regression: y versus x

Method
Link function  Logit
Rows used      8

Response Information
                              Event
Variable  Value      Count  Name
y         Event        291  Event
          Non-event    190
n         Total        481

Coefficients
Term          Coef  SE Coef         95% CI         Z-Value  P-Value
Constant       8,0     11,0  (  -13,7;     29,6)     0,72    0,470
x           -0,517    0,374  ( -1,249;   0,216)    -1,38    0,167
x*x        0,00637  0,00314  (0,00021; 0,01253)     2,03    0,043

Regression Equation
P(Event) = exp(Y')/(1+exp(Y'))
Y' = 8,0- 0,517x + 0,00637 x*x

Goodness-of-Fit Tests
Test             DF  Chi-Square  P-Value
Deviance          5        2,99    0,702
Pearson           5        2,84    0,724
Hosmer-Lemeshow   6        2,84    0,828

Observed and Expected Frequencies for Hosmer-Lemeshow Test
            Event
         Probability          Event              Non-event
Group       Range       Observed  Expected  Observed  Expected
    1  (0,000; 0,115)        6       6,8        53      52,2
    2  (0,115; 0,180)       13      10,8        47      49,2
    3  (0,180; 0,311)       18      19,3        44      42,7
    4  (0,311; 0,531)       28      29,8        28      26,2
    5  (0,531; 0,775)       52      48,8        11      14,2
    6  (0,775; 0,928)       53      54,7         6       4,3
    7  (0,928; 0,983)       61      60,9         1       1,1
    8  (0,983; 0,997)       60      59,8         0       0,2

MTB > Print 'XPWX1'.
Data Display

Matrix XPWX1
```

```
121,801  -4,11585   0,0344355
 -4,116   0,13960  -0,0011722
  0,034  -0,00117   0,0000099

MTB > print c11
Data Display
FITS1 (p hat)
   0,115200   0,180447   0,311482   0,531293
   0,775149   0,927528   0,982939   0,996855

MTB > let c12=y/n
MTB > plot c11*x C12*x;
SUBC> Symbol;
SUBC> Connect;
SUBC> Overlay.
```



Ex. 2.8.3 Survival of roots from the coffee plant

On a test station for vegetative reproduction of coffee plants, pieces of the roots of old plants were cut. Half of the pieces were planted as soon as possible, while the others were embedded into sand under cover and planted in the spring. Two lengths of root pieces, 6 cm and 12 cm, were used. For each of the four combinations of length and planting time, 240 pieces was used in the experiment.

Root pieces	Planting time	Number of survived out of 240	Proportion survived
Short	Direct	107	0.45
	Spring	31	0.13
Long	Direct	156	0.65
	Spring	84	0.35

Four analysis have been done using Minitab.

Analysis 1 is a logit-analysis with two additive factors A(length) and B(planting time).

Analysis 2 is a logit analysis with dummy-variables in model

$$logit\ p = \beta_0 + \beta_1 x_1 + \beta_2 x_2,$$

where

$$x_1 = \begin{cases} 1 & \text{for long roots,} \\ 0 & \text{otherwise,} \end{cases}$$
$$x_2 = \begin{cases} 1 & \text{for planting in spring,} \\ 0 & \text{otherwise.} \end{cases}$$

Analysis 3 is so called probit- (normit-)analysis, where link function is based on normal distribution

$$p = \Phi(\beta_0 + \beta_1 x_1 + \beta_2 x_2).$$

Analysis 4 is so called gompit-analysis, where link function is based on Gompertz extreme value distribution

$$p = 1 - \exp[-\exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2)].$$

a) See that first two analyzes are equivalent.

b) Put up the formula for $\hat{p}$ in analysis 2, 3 and 4.

c) Which of the analysis describes the data best. Is fitting sufficient? Perform an appropriate test at level 5%.

d) For the best model, do the length of the root and the planting time effect the proportion of survival? Construct confidence intervals with the simultaneous confidence level at least 90%.

```
ANALYS 1 ----------------------------------------
Binary Logistic Regression: y versus A; B

Method
Link function  Logit
Rows used      4

Response Information
                               Event
Variable  Value      Count  Name
y         Event        378  Event
          Non-event    582
n         Total        960

Coefficients
Term        Coef  SE Coef        95% CI        Z-Value  P-Value   VIF
Constant   0,106    0,291  (-0,464;  0,675)     0,36    0,715
A          1,018    0,145  ( 0,733;  1,303)     7,00    0,000  1,03
B         -1,428    0,146  (-1,715; -1,140)    -9,75    0,000  1,03
```

```
Odds Ratios for Continuous Predictors
   Odds Ratio       95% CI
A      2,7668  (2,0804; 3,6797)
B      0,2399  (0,1800; 0,3197)

Goodness-of-Fit Tests
Test             DF  Chi-Square  P-Value
Deviance          1        2,29    0,130
Pearson           1        2,27    0,132
Hosmer-Lemeshow   2        2,27    0,321

Observed and Expected Frequencies for Hosmer-Lemeshow Test
            Event
         Probability          Event            Non-event
Group       Range       Observed  Expected  Observed  Expected
    1  (0,000; 0,150)         31      36,1       209     203,9
    2  (0,150; 0,329)         84      78,9       156     161,1
    3  (0,329; 0,425)        107     101,9       133     138,1
    4  (0,425; 0,671)        156     161,1        84      78,9

ANALYS 2 -----------------------------------------
Binary Logistic Regression: y versus x1; x2

Method
Link function                 Logit
Categorical predictor coding  (1; 0)
Rows used                     4

Response Information
                            Event
Variable  Value      Count  Name
y         Event        378  Event
          Non-event    582
n         Total        960

Coefficients
Term         Coef  SE Coef       95% CI        Z-Value  P-Value   VIF
Constant  -0,304    0,117  (-0,534; -0,074)    -2,59    0,009
x1
  1        1,018    0,145  ( 0,733;  1,303)     7,00    0,000  1,03
x2
  1       -1,428    0,146  (-1,715; -1,140)    -9,75    0,000  1,03

Odds Ratios for Categorical Predictors
Level x1    Level x2   Odds Ratio       95% CI
x1     1      0            2,7668  (2,0804; 3,6797)
x2     1      0            0,2399  (0,1800; 0,3197)

Goodness-of-Fit Tests
```

Test	DF	Chi-Square	P-Value
Deviance	1	2,29	0,130
Pearson	1	2,27	0,132
Hosmer-Lemeshow	2	2,27	0,321

Observed and Expected Frequencies for Hosmer-Lemeshow Test

	Event Probability	Event		Non-event	
Group	Range	Observed	Expected	Observed	Expected
1	(0,000; 0,150)	31	36,1	209	203,9
2	(0,150; 0,329)	84	78,9	156	161,1
3	(0,329; 0,425)	107	101,9	133	138,1
4	(0,425; 0,671)	156	161,1	84	78,9

ANALYS 3 ---

Binary Logistic Regression: y versus x1; x2

Method

Link function	Normit
Categorical predictor coding	(1; 0)
Rows used	4

Response Information

Variable	Value	Count	Event Name
y	Event	378	Event
	Non-event	582	
n	Total	960	

Coefficients

Term	Coef	SE Coef	95% CI	Z-Value	P-Value	VIF
Constant	-0,1841	0,0720	(-0,3252; -0,0429)	-2,56	0,011	
x1						
1	0,6197	0,0870	(0,4493; 0,7902)	7,13	0,000	1,01
x2						
1	-0,8713	0,0872	(-1,0422; -0,7005)	-9,99	0,000	1,01

Goodness-of-Fit Tests

Test	DF	Chi-Square	P-Value
Deviance	1	1,62	0,203
Pearson	1	1,61	0,205
Hosmer-Lemeshow	2	1,61	0,447

Observed and Expected Frequencies for Hosmer-Lemeshow Test

	Event Probability	Event		Non-event	
Group	Range	Observed	Expected	Observed	Expected
1	(0,000; 0,146)	31	34,9	209	205,1
2	(0,146; 0,332)	84	79,6	156	160,4
3	(0,332; 0,427)	107	102,5	133	137,5

4	(0,427; 0,668)	156	160,4	84	79,6

ANALYS 4 --
Binary Logistic Regression: y versus x1; x2

Method

Link function	Gompit
Categorical predictor coding	(1; 0)
Rows used	4

Response Information

Variable	Value	Count	Event Name
y	Event	378	Event
	Non-event	582	
n	Total	960	

Coefficients

Term	Coef	SE Coef	95% CI	Z-Value	P-Value	VIF
Constant	-0,6247	0,0924	(-0,8058; -0,4435)	-6,76	0,000	
x1						
1	0,739	0,109	(0,526; 0,953)	6,79	0,000	1,00
x2						
1	-1,081	0,113	(-1,304; -0,859)	-9,53	0,000	1,00

Goodness-of-Fit Tests

Test	DF	Chi-Square	P-Value
Deviance	1	5,34	0,021
Pearson	1	5,22	0,022
Hosmer-Lemeshow	2	5,22	0,074

Observed and Expected Frequencies for Hosmer-Lemeshow Test

	Event Probability	Event		Non-event	
Group	Range	Observed	Expected	Observed	Expected
1	(0,000; 0,166)	31	39,9	209	200,1
2	(0,166; 0,316)	84	75,9	156	164,1
3	(0,316; 0,415)	107	99,5	133	140,5
4	(0,415; 0,674)	156	161,8	84	78,2

Data Display

Row	y	n	f	x1	x2	A	B	FITS1	FITS2	FITS3	FITS4
1	107	240	0,445833	0	0	1	1	0,424599	0,424599	0,426980	0,414587
2	31	240	0,129167	0	1	1	2	0,150401	0,150401	0,145618	0,166069
3	156	240	0,650000	1	0	2	1	0,671234	0,671234	0,668462	0,674259
4	84	240	0,350000	1	1	2	2	0,328766	0,328766	0,331538	0,316434

Ex. 2.8.4 In an experiment one want to test the lifetime (y) for a transistor which

has been kept under different storage conditions. Another factor that can effect the lifetime is the leakage current (x), which was also measured. Result:

Storage condition 1		Storage condition 2		Storage condition 3	
x	y	x	y	x	y
4.8	9912	6.4	9952	8.8	9596
7.2	9383	8.7	9482	6.2	9697
5.5	9734	7.1	9435	7.5	9700
6.0	9551	5.3	9915	4.9	9610
8.3	8959	4.6	9492	5.4	10145
7.6	9474	6.0	9565	5.8	10191
5.9	9179	7.2	9704	7.3	9855
8.0	9359	8.8	9636	8.6	9682
4.3	9580	5.4	9608	8.8	10160
5.1	9245	7.8	9548	6.0	9982

The data has been analyzed using the following model.

$$Y = \beta_0 + \beta_1 x + \beta_2 z_2 + \beta_3 z_3 + \epsilon,$$

where $\epsilon \sim N(0, \sigma)$ and

$$z_2 = \begin{cases} 1 & \text{for storage condition 2,} \\ 0 & \text{otherwise,} \end{cases}$$

$$z_3 = \begin{cases} 1 & \text{for storage condition 3,} \\ 0 & \text{otherwise.} \end{cases}$$

```
MTB > set c7
DATA> (0 1 0)10
DATA> end
MTB > set c8
DATA> (0 0 1)10
DATA> end
MTB > Name M1 "XMAT1".

Now run regression with continuous variable 'x' and categorical 'z2' 'z3';

Regression Analysis: y versus x; z2; z3

Analysis of Variance
Source         DF   Adj SS   Adj MS
Regression      3  1083491   361164
Error          26  1299902    49996
Total          29  2383393

Model Summary
      S    R-sq  R-sq(adj)  R-sq(pred)
223,598  45,46%     39,17%      25,83%
```

```
Coefficients
Term         Coef  SE Coef
Constant     9795      200
x           -57,0     29,9
z2            222      101
z3            462      102

MTB > set c5
DATA> (1)30
DATA> end
MTB > copy c5-c8 m1
MTB > print m1

Data Display
Matrix XMAT1
1  4,8  0  0
1  7,2  0  0
1  5,5  0  0
1  6,0  0  0
1  8,3  0  0
1  7,6  0  0
1  5,9  0  0
1  8,0  0  0
1  4,3  0  0
1  5,1  0  0
1  6,4  1  0
1  8,7  1  0
1  7,1  1  0
1  5,3  1  0
1  4,6  1  0
1  6,0  1  0
1  7,2  1  0
1  8,8  1  0
1  5,4  1  0
1  7,8  1  0
1  8,8  0  1
1  6,2  0  1
1  7,5  0  1
1  4,9  0  1
1  5,4  0  1
1  5,8  0  1
1  7,3  0  1
1  8,6  0  1
1  8,8  0  1
1  6,0  0  1

MTB > trans m1 m2
MTB > mult m2 m1 m3
MTB > invert m3 m4
```

```
MTB > print m4

Data Display
Matrix M4
 0,801978  -0,111958  -0,048499  -0,026108
-0,111958   0,017856  -0,008214  -0,011785
-0,048499  -0,008214   0,203778   0,105421
-0,026108  -0,011785   0,105421   0,207778
```

a) Does the leakage current effect the lifetime for a transistor? In what way? Motivate your answer with a confidence interval or test at level 0.10.

b) Is there any difference between storage condition 1 and 2? Motivate your answer with a confidence interval with confidence level 95%.

c) Estimate the parameter (linear combination) that is the differences between storage condition 2 and 3.

3 Answers

Ex. A a) 0.16 b) 0.02

Ex. B We have pairwise measurements. We construct differences $d_i = x_i - y_i$. Model: The r.v. $D_i \sim N(\mu_D, \sigma)$, where μ_D describes the systematic difference between the models. $I_{\mu_D} = (\bar{d} \pm 2.26 \cdot s_d/\sqrt{10}) = (-0.11; 5.11)$, where s_d is sample standard deviation fpr d_i-values. Since $0 \in I_{\mu_D}$, we cannot conclude that there is a systematic difference between the methods.

Ex. C a) Test statistics $\frac{\bar{x}-5}{0.87/\sqrt{24}} = -2.20 > -2.33$ H_0 cannot be rejected.
b) Power $h(4.5) = \Phi(0.486) \approx 0.69$.
c) For those who will drink the water's test H_0 against H_1 is better.

Ex. D a) $I_{\mu_1-\mu_2} = (\bar{u} - \bar{v} - 1.72s\sqrt{\frac{22}{117}}, \infty) = (0.864, \infty)$
Allergic people on average have higher values than non-allergic.

Ex. E a) $\mu > 2.165$
b) $I_\mu = (2.29, \infty)$; condition in a) is with high probability satisfied.
c) Help variable $15S^2/\sigma^2 \sim \chi^2(15)$ and it gives $I_\sigma = (0.392, 0.882)$, so $\sigma = 0.5$ seems to be reasonable assumption for our model.

Ex. F $I_\sigma = (0, s\sqrt{21/11.59}) = (0, 0.317)$

Ex. G Difference $\mu_1 - \mu_2$ describes the systematic difference between the indicators

$$I_{\mu_1-\mu_2} = (\bar{x} - \bar{y} \pm 2.02 \cdot s \cdot \sqrt{\frac{1}{16} + \frac{1}{26}}) = (-0.00024; 0.00114).$$

We see that $0 \in I_{\mu_1-\mu_2}$ and that the interval is short. The systematic difference seems negligible.

Ex. H a) Test statistic $\frac{\bar{x}-2.5}{0.32/\sqrt{15}} = -0.97 > -1.645$; H_0 can not be rejected.
b) $1 - \Phi(0.44) \approx 0.33$, i.e. poor power for $\mu = 2.40$.

Ex. I a) The observed points follow the curved curve much better. The straight line in the first plot seems to be systematically wrong in relation to the observed values.
b) $I_{\beta_2} = (\hat{\beta}_2 \pm t \cdot s \cdot \sqrt{h_{22}}) = (-7.11; -4.10)$. We see that $0 \notin I_{\beta_2}$. Hence, x^2 is useful as an explanatory variable.
c) For the estimated regression relationship $x = 10.22$ is the value that gives highest reduction of the phosphate. This is only an estimate of the optimum x-value.
d) $\hat{m}_{10} - \hat{m}_{11} = -\hat{\beta}_1 - 21\hat{\beta}_2 = 3.188$. Hence, pH=10 seems to be better than pH=11.

Ex. 2.1.1 a) F-testet ger $v = 7.75 > 5.95$; signifikant skillnad.
b) $I_{\mu_i - \mu_j} = (\bar{y}_{i\cdot} - \bar{y}_{j\cdot} \pm 5.50 \cdot \frac{s}{\sqrt{4}}) = (\bar{y}_{i\cdot} - \bar{y}_{j\cdot} \pm 0.52)$; μ_2 signifikant större än μ_4.

Ex. 2.1.2 a) $v_{14} = \frac{s_4^2}{s_1^2} = 3.14$. As $\frac{1}{7.15} < 3.14 < 7.15$ we cannot claim that $\sigma_4 \neq \sigma_1$, etc. The other comparisons also do not point out any significant difference between standard deviations. It seems reasonable to assume that $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma$. The simultaneous confidence level $\leq 6 \cdot 0.05 = 0.30$.
b) Let $\theta = \mu_1 - 3\mu_3 + 2\mu_4$. We obtain $I_\theta = (-5.76, 0.63)$. Hence, it is possible that $\mu_1 - \mu_3 = 2(\mu_3 - \mu_4)$.

Ex. 2.1.3 a) $v = 51.33 > 6.93$, where value 6.93 is given by F(2,12)-table. With high probability there is difference between batches with respect to their tensile strengths.
b) $I_\mu = (\bar{y}_{\cdot\cdot} \pm t \cdot \frac{\tilde{s}}{\sqrt{3}}) = (7839, 8570)$, where $t = 4.30$ and $\tilde{s}$ is calculated using $y_{i\cdot}$.

Ex. 2.1.4 a) Signifikant on level 5% but not on level 1%.
b) t-interval $I_{\mu_i - \mu_j} = (\bar{y}_{i\cdot} - \bar{y}_{j\cdot} \pm 3.17)$;
Scheffe-interval $I_{\mu_i - \mu_j} = (\bar{y}_{i\cdot} - \bar{y}_{j\cdot} \pm 3.30)$;
Tukey-interval $I_{\mu_i - \mu_j} = (\bar{y}_{i\cdot} - \bar{y}_{j\cdot} \pm 2.91)$;
c) Tukey-intervals are always the shortest ones.
d) The decision about the choice of analysis should be made **before** one see measurements. To be able to find the biggest difference one has to do all pairwise comparisons and then the simulatanous significance level for test is at most 5%.

Ex. 2.1.5 a) $\hat{\mu} = \bar{y}_{\cdot\cdot} = 7004.6$; $\hat{\sigma}^2 = \frac{SS_E}{df_E} = 11.60$; $\hat{\sigma}_\tau^2 = 224.8$.
b) $v = 59.13$. F(4,10)-table gives critical region 5.99<59.13. With high probability there is variation between sensors.

Ex. 2.1.6 a) Construct t-interval with confidence level 98%; use the pooled variance estimator for variance.
$I_{\mu_A} = (51.8 \pm 23.3)$, $I_{\mu_B} = (82.2 \pm 18.0)$, $I_{\mu_C} = (84.6 \pm 25.5)$.
b) We obtain test statistic $v = 4.20$. A r.v. $V \sim F(2, 18)$ is expected values are equal. As 3.55< 4.20< 6.01, we can show the significant difference between expected values on level 5% but not on level 1%.
c) $I_{\mu_A - \mu_B} = (-30.4 \pm 29.4)$; $I_{\mu_A - \mu_C} = (-32.8 \pm 34.5)$; $I_{\mu_B - \mu_C} = (-2.4 \pm 31.2)$; simultaneous confidence level is at least 94%.
d) $I_{\mu_A - \mu_B} = (-30.4 \pm 30.7)$; $I_{\mu_A - \mu_C} = (-32.8 \pm 36.0)$; $I_{\mu_B - \mu_C} = (-2.4 \pm 32.6)$; simultaneous confidence level is at least 95%.

Ex. 2.1.8 a) $H_0' : \mu_1 = \mu_2 = \mu_3$ against H_1' : not all μ_j are the same
is examined with F-test. Test statistics $v = 31.9 > 6.7$. H_0' is rejected.
b) $I_{\mu_1 - 2\mu_2 + \mu_3} = (-9.51, -0.99)$; H_0 is rejected.

Ex. 2.1.9 a) $I_\mu = (46.07, 46.87)$.
b) $I_m = (41.9, 51.0)$.
c) μ= true average metal content of the four samples; m= true average metal content of an ore portion. We see that we have more precise information about μ than about m (that is what one can expect).

Ex. 2.1.10 a) As we have quite large samples and the clear difference between standard deviation can be observed we choose the first analysis as more relevant. It is especially important to choose this analysis as the samples are of different sizes.
b) Test statistic $t = 1.72 > 1.68$ where the critical region comes from $t(40)$-table. H_0 is rejected. On average, those with high blood pressure have higher cholesterol levels, but there are large individual variations.
c) $t = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_1^2}{39}+\frac{s_2^2}{24}}}$.
d) The interval from the first analysis has approximately the right level of confidence. The second interval is shorter and therefore have a lower level of confidence, because the conditions for the method are not fully satisfied.

Ex. 2.1.11 a) $I_\mu = (4.75, 8.89)$
b) Test statistic $v = \frac{SS_{TREAT}/2}{SS_E/12} = 0.74 < 2.81$. Variation mellan tracking stations seems to be negligible. It may be sufficient to measure in one place, but it is wiser to measure at several.

Ex. 2.1.12 a) Tukeys method gives $I_{\mu_1-\mu_2} = (8.0, 29.7)$, $I_{\mu_1-\mu_3} = (26.9, 48.6)$, $\ldots$, $I_{\mu_3-\mu_4} = (-34.7, -13.0)$.
b) $\theta = (2\mu_1 + \mu_2)/3$; $I_\theta = (77.9, 86.4)$.

Ex. 2.1.13 a) $I_{\mu_1-\mu_2} = (-0.9, 0.4)$, $I_{\mu_1-\mu_3} = (-0.7, 0.6)$, $I_{\mu_2-\mu_3} = (-0.5, 0.8)$; $s = 0.6727$; $df = 47$; $t = 2.41$. There seems to be no major differences in quality between A, B and C.
b) Let $\theta = \mu_4 - 1.03(\mu_1 + \mu_2 + \mu_3)/3$; $I_\theta = (0.5, \infty)$. H_0 is rejected in favor of H_1; D gives, in average, at least 3% better tensile strength that the alternatives.

Ex. 2.2.1 b) $v = 3.50 > 2.87$; with high probability we have interaction.
c) $Y_{ijk} = \mu_{ij} + \epsilon_{ijk}$, i.e. complete two factor model.
d) $A = 2, B = 1$ are significantly better than $A = 1, B = 4$ and $A = 1, B = 5$. $I_{\mu_{21}-\mu_{14}} = (41.667 - 23.667 \pm 10.82) = (7.18, 28.82)$ etc.
One does not find no clear choice of level combination.

Ex. 2.2.2 a) $I_{\tau_i-\tau_q} = (\bar{y}_{i\cdot} - \bar{y}_{q\cdot} \pm 3.108)$.
b) $I_{\mu_i-\mu_q} = (\bar{y}_{i\cdot} - \bar{y}_{q\cdot} \pm 2.706)$.
c) Two factor model gives $s = 1.441$ and one factor model $\tilde{s} = 1.413$, i.e. s and $\tilde{s}$ are approximately of the same size. Hence, we choose easier model, i.e. model no. 2.

Ex. 2.2.3 a) Interaction effect is examined with $v = 4.14 > 3.63$; we conclude that with high probability there is interaction. The data should be analyzed as a complete two-factor model, i.e. $Y_{ijk} = \mu_{ij} + \epsilon_{ijk}$.
b) $I_{\mu_{13}-\mu_{23}} = (0.4, 29.4)$, $I_{\mu_{13}-\mu_{33}} = (-25.1, 3.8)$, $I_{\mu_{33}-\mu_{23}} = (11.1, 40.0)$; steam pressure 20 gives clearer filter than the other steam pressures.

Ex. 2.2.5 a) Use F-test, $v = 20.13 > 7.01$; Coppar concentration seems to have impact on the results.
b) $I_{\beta_3-\beta_5} = (-9.89, \infty)$; $0 \in I_{\beta_3-\beta_5}$; concentration 0.75 is not significantly better than concentration 0.3 according to analysis.

Ex. 2.2.6 a) $v = 37.54 > 5.14$. Significant interaction effect.
b) $I_{\mu_{11}-\mu_{21}} = (3.0 \pm 18.0)$,
$I_{\mu_{12}-\mu_{22}} = (-26.0 \pm 18.0)$; choose L2 for M2,
$I_{\mu_{13}-\mu_{23}} = (33.5 \pm 18.0)$; choose L1 for M3.

Ex. 2.3.1 a) For the complete model we have no significant interaction effects with A, so A is assume to be additive. New model:

$$Y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + (\beta\gamma)_{jk} + \epsilon_{ijkl}$$

b) $I_{\tau_i-\tau_q} = (\bar{y}_{i\cdots} - \bar{y}_{q\cdots} \pm 46.16)$; A_1 is best (Tukeys method; $1-\alpha_{sim \approx 95\%}$).
Let $m_{jk} = \mu+\beta_j+\gamma_k+(\beta\gamma)_{jk}$, so we have $I_{\mu_{jk}-\mu_{rs}} = (\bar{y}_{\cdot jk\cdot} - \bar{y}_{\cdot rs\cdot} \pm 58.80)$.
Choose C_1 and B_1 or B_2 (Tukeys method; $1-\alpha_{sim \approx 95\%}$).
A total simultaneous confidence level is at least 90%.
Alternatively one can do 9 t-intervals, each on confidence level 99%.

Ex. 2.3.2 a) $v_{RAD} = 0.41 << 4.76$. We cannot conclude that the distance from highway has impact on our measurements.
b) $I_{\gamma_i-\gamma_q} = (\bar{y}_{\cdot\cdot i} - \bar{y}_{\cdot\cdot q} \pm 0.0667)$. Treatment no. 1 is significantly better that remaining treatments.

Ex. 2.3.3 b) In interaction plot we have $\bar{y}_{ij\cdot\cdot}$ ploted against temperature levels and keeps track of yeast resort. We see that yeast type no. 3 is not as temperature sensitive as the no. 1 and 2.
c) The interaction between temperature and type of yeast is confirmed by $v_{AB} = 52.90 > 2.64$.
d) Tukey interval $I_{\gamma_i-\gamma_q} = (\bar{y}_{\cdot\cdot i\cdot} - \bar{y}_{\cdot\cdot q\cdot} \pm 1.73)$. Type no. 4 is better than no. 2 that is better than no. 1 and 3.

Ex. 2.4.1 a) B-, ABC- och AC-effects.
b) B-, ABC- och R-effects.
c) $I_{\mu_{1jk}-\mu_{1rs}} = (\bar{y}_{1jk\cdot\cdot} - \bar{y}_{1rs\cdot\cdot} \pm 7.60)$. Levels B=C=1 are the best for regular production (A=1).

Ex. 2.4.2 a) For D=-1 is a no. 2 i.e. A-effect that seems to be more significant and for D=1 is no. 2 and no. 4 i.e. A-effect and AB-effect.
b) We work with three factor complete model with A, B,D.

$$I_{\mu_{i1k}-\mu_{i-1k}} = (\bar{y}_{i1k} - \bar{y}_{i-1k} \pm 4.36) \qquad (t = 2.31)$$

We choose B=-1 for A=-1 and D=-1 and also for A=-1 and D=1, i.e. for A=-1. No clear choice for A=1.

Ex. 2.4.3 a) Block 1: (1), ab, ac, bc, ad, bd, cd, abcd; Block 2: a, b, c, abc, d, abd, acd, bcd.
b) Block effect overlaid with interaction ABCD; $\widehat{(\tau\beta\gamma\delta)}_{1111} = 0.0625$, and this effect appears to be negligible.
c) $Y_{ijkl} = m_{ij} + \gamma_k + \delta_l + \epsilon_{ijkl}$. Model is motivated by significance of AB, C and D.
d) $I_{m_{-1,1}-m_{-1,-1}} = (-1.13, -0.02)$; choose machine B=-1 for A=-1.
$I_{m_{1,1}-m_{1,-1}} = (0.52, 1.63)$; choose machine B=1 for A=1.
$I_{\gamma_1-\gamma_{-1}} = (0.26, 1.04)$; choose C=1.
$I_{\delta_1-\delta_{-1}} = (0.11, 0.89)$; choose D=1.

Ex. 2.4.4 a) No. 3: B+ACDE, No. 5: C+ABDE and No. 7: BC+ADE according to normal probability plot. Effect no. 16 is E+ABCD. Estimated value is -0.475.
b) $Y_{ijk} = \mu_{ij} + \epsilon_{ijk}$ for B-level i and C-level j. $I_{\mu_{ij}-\mu_{uv}} = (\bar{}_{ij\cdot} - \bar{}_{uv\cdot} \pm 2.94)$.
High level for both B and C is better than the other combinations.
c) D-factor has the biggest impact among those effects that we neglected.

Ex. 2.4.5 a) The most important effects are A+ABCDE, AD+ABCE and AB+ACDE, that we interpret as A, AD and AB. Main effect of E is included in the parameter estimate together with parameter estimate for interaction BCD, i.e. no. 15: -0.128, which seems quite negligible.
b) A complete three factor model with A, B and D. A should be on low level according to result in a). $I_{\mu_{-1jk}-m_{-1pq}} = (\bar{y}_{-1jk} - \bar{y}_{-1pq} \pm 4.24)$, where we use $t = 2.31$, that gives simultaneous confidence level at least 70%.
$I_{\mu_{-1,1,1}-m_{-1,1,-1}} = (\bar{y}_{-1,1,1} - \bar{y}_{-1,1,-1} \pm 4.24) = (-2.38, 6.10)$. Choice of B- and D- levels is not clear.

Ex. 2.4.6 a) Observations are e a b abe c ace bce abc d ade bde abd cde acd bcd abcde.
b) The most important effects are no. 2, i.e. effect A+BCDE, and no. 3, i.e. B+ACDE, which we interpret as the main effects of A and B. $\hat{e}$ is included in no. 16 and estimated with 10.625.
c) Model: $Y_{ijkl(v)} = \mu + \tau_i + \beta_j + \gamma_k + \delta_l + e_v + \epsilon_{ijkl(v)}$ where $\epsilon_{ijkl(v)} \sim N(0, \sigma)$ independent + usual constains. The worst result i.e. The worst result that most cracks one gets when A, B, C, D and E are on high level, that gives estimated value $\hat{\theta} = \hat{\mu} + \hat{\tau}_1 + \hat{\beta}_1 + \hat{\gamma}_1 + \hat{\delta}_1 + \hat{e}_1 = 1133.00$. By using the fact that r.v. $\hat{\mu}, \ldots, \hat{e}$ are independent and normally distributed with variance $\sigma^2/16$ we obtain t-interval: $I_\theta = (1133.0 \pm 29.40) = (1103.6, 1162.4)$, where t(10)-distributed help variable was used.

Ex. 2.5.1 Constuct difference between results for lab-ass 2 and lab-ass 1.
a) One sided test: v=number of positive observations; $v = 8 \geq 8$; the hypothesis of equal value measurement is rejected; $\alpha = 0.0547$. Lab-ass 2 tends to get higher value.
b) $T_- = 9.5 < 14$. the same conclusion as in a).
c) $I_{\mu_D} = (A_{15}, A_{41}) = (4.0, 22.5)$.

Ex. 2.5.2 a) $W_- = 10 < 16$; H_0 is rejected.
b) $z = \frac{10-52.5}{\sqrt{253.75}} = -2.67 < -2.33$; H_0 is rejected.
The results suggest that drug users psychological dependence on heroin decreased.

Ex. 2.5.3 Kruskal-Wallis test with χ^2-approximation: $T = 10.29 < 11.32$. We can not claim that there is a difference between splicing methods.

Ex. 2.5.4 a) $T_1 = 181 < 184$; hypothesis about the same distributions is rejected at the level 0.05.
b) Number of observations that *stand out* is $2 + 5 = 7 \geq 7$; the hypothesis of equal distribution is rejected at the level 0.05.
B-components seems to last longer.

Ex. 2.5.5 We can block design with block=instrument. Test statistic $T = 10.89 > 9.49$. With high probability there is significant difference between threads.

Ex. 2.5.6 a) $I_{\mu_1-\mu_2} = (5.5, 12.0)$
b) $I_{\mu_1-\mu_2} = (d_{(3)}, d_{(22)}) = (5, 13)$
I both cases the conclusion is that the higher dose gives shorter time to fall asleep.

Ex. 2.5.7 Wilcoxons rank sum test.
H_0: The same lifetime distribution for A and B
H_1: Different lifetime distribution for A and B
H_0 can not be rejected as $50 < T_{OBS} < 104$.

Ex. 2.5.8 Kruskal-Wallis test with χ^2-approximation: $T = 12.75 > 5.99$. With high probability time has impact on results.

Ex. 2.5.9 a) Friedmans test. $T = 10.67 > 9.49$. Coppar concentration seems to have impact on results.
b) Make differences $y_{i3} - y_{i5}$. We obtain $T_- = 0$ with $P = 0.125 > 0.05$. We can not state that the concentration no. 5 is better than no. 3. We do one sided test as we already in advance could argue that if there was a difference, then it should be that the higher the concentration ab copper inhibits bacteria growth more effectively.

Ex. 2.6.1 a) Curvature is examined with $v_{PQ} = 0.0056 << 39.86$. No tendency to curvature.
$SS_{PQ} = \frac{(\bar{y}_F - \bar{y}_C)^2}{\frac{1}{4}+\frac{1}{2}} = 0.02083$
$SS_E = (2-1) \cdot s_C^2 = 3.7538$, where s_C^2 =sample standard deviation for measurements from centrum point.
b) Since we did not find any tendency to curvature, it is not likely that (i) will be succesfull. We follow (ii) and move from $x_1 = 0$, $x_2 = 0$ in direction $(6.33, 3.10)$.

Ex. 2.6.2 a) $v_{PQ} = 130.51 > 18.51$. There is, with high probability, curvature of the response surface, which means that there is an optimum point in that particular area.
b) New measurements should be taken in $(-\sqrt{2}, 0)$, $(\sqrt{2}, 0)$, $(0, -\sqrt{2})$, $(0, \sqrt{2})$. In addition, you should make additional measurements in the centrum point to get safer σ^2-estimator.

Ex. 2.6.3 Starting from zero (0,0) one should more in direction (2.1,-3.5), for example make new measurements of y-value in points (0.6,-1), (1.2, -2), (1.8, -3), ... and continue so long the value y is increasing and both x_1 and x_2 remains within the acceptable range.

Ex. 2.7.1 a) $n \approx 120$
b) $n' \approx 100$

Ex. 2.7.2 $0.05 = 1 - \Phi\left(\frac{K - \frac{n}{2}}{\sqrt{n}/2}\right)$

$0.8 = 1 - \Phi\left(\frac{K - 0.7n}{\sqrt{0.21n}}\right)$

gives $n = 37$. Calculation using binomial distribution without approximation provide $n = 37$: $K = 24$, $\alpha = 0.049$, $1 - \beta = 0.807$.

Ex. 2.7.3 at least 16 people.

Ex. 2.7.4 a) Significe level $\alpha \approx 0.07$ (using normal approximation)
b) Power ≈ 0.993 (using Poisson approximation)

Ex. 2.8.1 a) In regression model δ_1 and δ_2 are the interaction parameters. Both of them have p-value<0.01 so they differ from 0 significantly on given level. The simultaneous significance level is <0.02.
b)$\widehat{E(Y)} = 99.5$ for L1M1 that is consistent with $\bar{y_{11}}..$
$\widehat{E(Y)} = 114.0$ for L2M2 that is consistent with $\bar{y_{22}}..$
$\widehat{E(Y)} = 38.5$ for L2M3 that is consistent with $\bar{y_{23}}..$
c) Regression have $SS_E = 141.5$ that is the same value as in complete two factor model in Ex. 2.2.6., which is due to the fact that two models are equivalent.

Ex. 2.8.2 a) $\hat{\beta_0} = 7.968$, $\hat{\beta_1} = -0.517$, $\hat{\beta_2} = 0.00637$.
b) $I^{0.95}_{\beta_1} = (-1.249, 0.215)$ and $I^{0.95}_{\beta_2} = (0.000212, 0.0125)$
c) $D = 2.99$ and p-value$= 0.702 > 0.05$. Our *small* model (logistic regression model) seems to be ok.
d) $x_{opt} = 67.69$.

Ex. 2.8.3 c) Analysis no. 3. $D = 1.62$ och $P = 0.203 > 0.05 \Rightarrow$ Model in analysis no. 3 seems to be ok.
d) $I^{0.95}_{\beta_1} = (0.449, 0.790)$ and $I^{0.95}_{\beta_2} = (-1.042, -0.701)$

Ex. 2.8.4 a) $I_{\beta_1} = (-108.1, -5.9)$. The leakage current appears to be important because $0 I_{\beta_1}$. $\beta_1 < 0$ indicate that the service life decreases as the leakage current increases.
b) Difference between method 2 and method 1 is described by β_2. We have $I_{\beta_2} = (14.4, 430.2) > 0$ Method 2 seems to be better than method 1.
c) Difference between method 3 and method 2 is described by $\beta_3 - \beta_2$. We have $\hat{\beta_3} - \hat{\beta_2} = 239.5$.