TAMS38 (TAMS12)
Some extra formulas

Choice of Sample Size

Let u be related to the power of the test as follows: $Power = \Phi(u)$. Let v be related to the significance level of the test as $\Phi(v) = \begin{cases} 1-\alpha/2, & \text{if two-sided test,} \\ 1-\alpha, & \text{if one-sided test.} \end{cases}$

One sample, single mean: $H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$, and the variance σ^2 is known.

$$n \geq \frac{(u+v)^2\sigma^2}{(\mu-\mu_0)^2}$$

Two sample, comparison of two means: $H_0 : \mu_1 = \mu_2$ vs. $H_1 : \mu_1 \neq \mu_2$, and the variances σ_1^2 and σ_2^2 are known.

$$n \geq \frac{(u+v)^2(\sigma_1^2+\sigma_2^2)}{(\mu_1-\mu_2)^2}.$$

For the case when the variances are unknown but equal, i.e., $\sigma_1^2 = \sigma_2^2 = \sigma^2$, one should calculate $d = \frac{|\mu_1-\mu_2|}{2\sigma}$ and use an *operating characteristic curve* instead.

Balanced single-factor experiment: $H_0 : \mu_1 = \cdots = \mu_a$ vs. H_1 : not all μ_i equal. Calculate

$$\nu_1 = a-1, \quad \nu_2 = N-a, \quad \Phi^2 = n\sum_{i=1}^{a}\frac{\tau_i^2}{a\sigma^2}$$

and use an *operating characteristic curve.*

Binomial distribution, single probability: $H_0 : \pi = \pi_0$ vs. $H_1 : \pi \neq \pi_0$.

$$n \geq \frac{\left(u\sqrt{\pi(1-\pi)} + v\sqrt{\pi_0(1-\pi_0)}\right)^2}{(\pi-\pi_0)^2}$$

Two binomial distributions, comparison of two probabilities: $H_0 : \pi_1 = \pi_2$ vs. $H_1 : \pi_1 \neq \pi_2$.

$$n \geq \frac{\left(u\sqrt{\pi_1(1-\pi_1)+\pi_2(1-\pi_2)} + v\sqrt{2\bar{\pi}(1-\bar{\pi})}\right)^2}{(\pi_2-\pi_1)^2},$$

where $\bar{\pi} = \frac{\pi_1+\pi_2}{2}$.

Some nonparametric tests

The Wilcoxon signed rank test

Let r_i be the rank for the observations $|y_i| \neq 0, i = 1, \ldots, n$. Let $T_+ = \sum_{\{y_i>0\}} r_i$ and $T_- = \sum_{\{y_i<0\}} r_i$. When H_0 is true and $n > 15$ us that T_+ and $T_- \approx N\left(\frac{n(n+1)}{4}, \frac{n(n+1)(2n+1)}{24}\right)$. For $n \leq 15$ us table for Wilcoxons signed rank distribution.

For confidence interval us the $N = n(n+1)/2$ ordered pairwise means A_i and $P(A_{(k)} < \mu < A_{(N-k+1)}) = 1 - 2P(W_S \leq k-1)$, where W_S is Wilcoxons signed rank distributed.

The Wilcoxon-Mann-Whitney test

Let d_{ij} be the differences $d_{ij} = x_i - y_j$, $i = 1, \ldots, n_1$, $j = 1, \ldots, n_2$ and $d_{(k)}$, $k = 1, \ldots, n_1 n_2$, the ordered differences. The confidence interval for the difference in mean/median is given by

$$I = \left(d_{(c+1)}, d_{(n_1 n_2 - c)}\right),$$

where $c = T_l - \frac{n_1(n_1+1)}{2}$ and T_l is from the Wilcoxon table for the rank sum test.

The Kruskal-Wallis test

Assume a treatments. Let r_{ij} be the rank for the observation y_{ij}. Test statistic

$$T = \begin{cases} \dfrac{12 S_a}{N(N+1)} - 3(N+1), & \text{if no ties,} \\[2ex] \dfrac{(N-1)(S_a - C)}{S_r - C}, & \text{if ties,} \end{cases}$$

where $s_i = \sum_{j=1}^{n_i} r_{ij}$, $S_a = \sum_{i=1}^{a} \frac{s_i^2}{n_i}$, $S_r = \sum_{i=1}^{a} \sum_{j=1}^{n_i} r_{ij}^2$, $C = \frac{1}{4} N(N+1)^2$ and $N = \sum_{i=1}^{a} n_i$. For *small* values of $n_1, \ldots, n_a$ ($a \leq 3$ and $n_i \leq 5$) use table and for *large* values of $n_1, \ldots, n_a$ use that $T \approx \chi^2(a-1)$ when there is no treatmet effect.

The Friedman test

Assume t treatments and b blocks. Let r_{ij} be the rank of y_{ij} within each block, i.e., for each j, $r_{ij} = 1, \ldots, t$. Test statistic for the treatments is given by

$$T = \begin{cases} \dfrac{12 S_t}{t(t+1)} - 3b(t+1), & \text{if no ties,} \\[2ex] \dfrac{b(t-1)(S_t - C)}{S_r - C}, & \text{if ties,} \end{cases}$$

where $s_i = \sum_{j=1}^{b} r_{ij}$, $S_t = \frac{1}{b} \sum_{i=1}^{t} s_i^2$, $S_r = \sum_{i=1}^{t} \sum_{j=1}^{b} r_{ij}^2$ and $C = \frac{1}{4} bt(t+1)^2$. For *small* values of b and t ($t = 3$, $b \leq 15$ and $t = 4$, $b \leq 8$) use table and for *large* values of b and t use that $T \approx \chi^2(t-1)$ when there is no treatmet effect.