ANSWERS

TAMS38 - Experimental Design and Biostatistics, 4 hp / 6 hp Examination on 12 January 2018, kl 14-18.

1 **a)** Model: $Y_{ij} = \mu + \tau_i + \epsilon_{ij}$, where $\tau_i \sim N(0, \sigma_\tau)$, $\epsilon_{ij} \sim N(0, \sigma)$, and ϵ_{ij} are independent for $j = 1, ..., 4$, $i = 1, ...4$.

b) $H_0 : \sigma_\tau^2 = 0$ and $H_1 : \sigma_\tau^2 \neq 0$.

2 **a)** Model: $Y_{ij} = \mu + \tau_i + \epsilon_{ij}$, where $\tau_1, ..., \tau_4$ are fixed, $\epsilon_{ij} \sim N(0, \sigma)$, and ϵ_{ij} are independent for $j = 1, ..., 4$, $i = 1, ...4$.

b) $H_0 : \tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$ and $H_1 :$ at least one $\tau_i \neq 0, i = 1, ..., 4$.

c) $\bar{y}_{..} = 137.9375$

$SS_{TREAT} = \sum_{i=1}^{4} 4(\bar{y}_{i.} - \bar{y}_{..})^2 \approx 844.69$, where $\bar{y}_{1.} = 145, \bar{y}_{2.} = 145.25, \bar{y}_{3.} = 132.25$ and $\bar{y}_{4.} = 129.25$.

$SS_E = \sum_{i=1}^{4} (4-1)s_i^2 \approx 236.25$, where $s_i^2 = \frac{1}{4-1} \sum_{j=1}^{4} (y_{ij} - \bar{y}_{i.})^2$, $N = 16$ and $a = 4$.

$\nu = \frac{SS_{TREAT}/(a-1)}{SSE/(N-a)} = 14.30 > F_{0.05}(3, 12) = 3.49$, then reject H_0. That is, there exists significant difference in conductivity due to coating type.

d) Model: $Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$, where

$$x_i = \begin{cases} 1, & \text{for type } i, \\ 0, & \text{otherwise,} \end{cases}$$

$i = 1, 2, 3$, and $\varepsilon \sim N(0, \sigma)$.

3 **a)** (i) 3 (ii) 3 (iii) 4.

b) $\nu_A = \frac{SS_A/(a-1)}{SSE/(dfe)} = \frac{1545.51/2}{170.76/36} = 162.91 > F_{0.05}(2, 36) = 3.28$, then reject H_0, that is, main effect A is significant.
$\nu_B = \frac{SS_B/(b-1)}{SSE/(dfe)} = \frac{1934.7/2}{170.76/36} = 203.94 > F_{0.05}(2, 36) = 3.28$, then reject H_0, that is, main effect B is significant.
$\nu_C = \frac{SS_C/(c-1)}{SSE/(dfe)} = \frac{1709.61/3}{170.76/36} = 120.14 > F_{0.05}(3, 36) = 2.88$, then reject H_0, that is, main effect C is significant.

c) $\nu_{AB} = 58.02 > F_{0.05}(4, 36) = 2.65$, then reject H_0, that is, interaction effect AB is significant.
$\nu_{AC} = 2.20 < F_{0.05}(6, 36) = 2.38$, then don't reject H_0, that is, interaction effect AB is NOT significant.
$\nu_{BC} = 1.04 < F_{0.05}(6, 36) = 2.38$, then don't reject H_0, that is, interaction effect BC is NOT significant.
$\nu_{ABC} = 0.74 < F_{0.05}(12, 36) = 2.05$, then don't reject H_0, that is, interaction effect ABC is NOT significant.

d) Model:

$$Y_{ijkl} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \gamma_k + \varepsilon_{ijkl},$$

where $\varepsilon_{ijkl} \sim N(0, \sigma)$, and ε_{ijkl} are independent for $i = 1, 2, 3$, $j = 1, 2, 3$, $k = 1, 2, 3, 4$, and $l = 1, 2$.

e)

Source	DF	SS
A	2	1545.51
B	2	1934.70
C	3	1709.61
A*B	4	1100.83
Error	60	304.86
Total	71	6595.51

f) The higher production the better, so we seek for higher means.

Tukey interval gives $1 - 2\alpha = 90\%$, then $\alpha = 0.05$.
Let $m_{ij} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij}$, then

$$I_{m_{ij}-m_{kl}} = \bar{y}_{ij..} - \bar{y}_{kl..} \pm q_{0.05}(9,60)\frac{s}{\sqrt{n}} = \bar{y}_{ij..} - \bar{y}_{kl..} \pm (4.55)\frac{\sqrt{304.86/60}}{\sqrt{8}} = \bar{y}_{ij..} - \bar{y}_{kl..} \pm 3.626.$$

Then we choose $A = 2, B = 2$ or $A = 2, B = 3$.

$$I_{\gamma_i-\gamma_j} = \bar{y}_{..i.} - \bar{y}_{..j.} \pm q_{0.05}(4,60)\frac{\sqrt{304.86/60}}{\sqrt{18}} = \bar{y}_{..i.} - \bar{y}_{..j.} \pm 1.99.$$

Then we choose $C = 4$. So the best combination is $A = 2, B = 2, C = 4$ or $A = 2, B = 3, C = 4$.

OR you can apply t- interval method which gives $\alpha = 0.002$.

4 **a)** (1) ad bd ab cd ac bc abcd.

b) nr. 2 : A+BCD effect; nr. 3: B+ACD effect; nr. 4: AB+CD effect.

c) nr. 8 : ABC+D effect; D effect seems to be negligible since the point estimate is -0.3687.