ANSWERS

TAMS38 - Experimental Design and Biostatistics, 4 hp / 6 hp Examination on 04 April 2018, kl 8-12.

1 **a)** Model: $Y_{ij} = \mu + \tau_i + \epsilon_{ij}$, where $\tau_1, ..., \tau_4$ are fixed, $\epsilon_{ij} \sim N(0, \sigma)$, and ϵ_{ij} are independent for $j = 1, ..., 7$, $i = 1, ...4$.

b) $H_0 : \tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$ and $H_1 :$ at least one $\tau_i \neq 0, i = 1, ..., 4$.

$\nu = \frac{SS_{TREAT}/(a-1)}{SSE/(N-a)} = \frac{384/3}{478/24} = 6.43$. The critical region is $C = (F_\alpha(3, 24), \quad \infty)$. If $\nu > (F_{0.05}(3, 24)$, then we reject H_0. That is, there exists significant difference in expected number of bacteria between the different soil types. We check the table for F distribution, we need the significance level $\alpha \geq 0.5\%$.

c) Tukey interval gives $1 - \alpha = 95\%$, then $\alpha = 0.05$.
Let $\mu_i = \mu + \tau_i, i = 1, \ldots, 4$. then

$$I_{\mu_i - \mu_j} = \bar{y}_{i.} - \bar{y}_{j.} \pm q_{0.05}(4, 24)\frac{s}{\sqrt{n}} = \bar{y}_{i.} - \bar{y}_{j.} \pm (3.9)\frac{\sqrt{478/24}}{\sqrt{7}} = \bar{y}_{i.} - \bar{y}_{j.} \pm 6.578.$$

Then $I_{\mu_1-\mu_2} = 79.71 - 73.71 \pm 6.578 = (-0.58, 12.58)$, $I_{\mu_1-\mu_3} = 79.71 - 69.29 \pm 6.578 = (3.84, 17.00)$
$I_{\mu_1-\mu_4} = 79.71 - 74.57 \pm 6.578 = (-1.44, 11.72)$, $I_{\mu_2-\mu_3} = 73.71 - 69.29 \pm 6.578 = (-2.16, 11.00)$,
$I_{\mu_2-\mu_4} = 73.71 - 74.57 \pm 6.578 = (-7.44, 5.72)$, $I_{\mu_3-\mu_4} = 69.29 - 74.57 \pm 6.578 = (-11.86, 7.88)$.

Thus, we get $\mu_1 > \mu_3$.

2 **a)** Additive two factor model, since there is no interaction term in linear model.

b) $I_{\beta_1} = \hat{\beta}_1 \pm t_{\alpha/2}(n-k-1) \times s\sqrt{h_{11}} = \hat{\beta}_1 \pm t_{0.025}(11) \times se(\hat{\beta}_1) = 57.50 \pm 2.20 \times 3.21 = (50.44, \quad 64.56)$. So $\beta_1 \neq 0$, there is significant difference between glass type that influences brightness.

c) $I_{\gamma_1} = \hat{\gamma}_1 \pm t_{\alpha/2}(n-k-1) \times s\sqrt{h_{22}} = \hat{\gamma}_1 \pm t_{0.025}(11) \times se(\hat{\gamma}_1) = -2.00 \pm 2.20 \times 3.85 = (-10.47, \quad 6.47)$. So it is possible $\gamma_1 = 0$, there is no significant difference between phosphor type 1 and phosphor type 3.

3 We apply case i on Kirkwoods list: $n \geq \frac{(u+v)^2 \cdot \sigma^2}{(\mu-\mu_0)^2}$, where $u = \lambda_{1-\text{ power}} = \lambda_{0.1} = 1.28$, $v = \lambda_\alpha = \lambda_{0.05} = 1.645$, $\mu = 0.3$, $\mu_0 = 0$ and $\sigma = 0.4$. So $n \geq 16$.

4 **a)** Model:

$$Y_{ijkl} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \gamma_k + \varepsilon_{ijkl},$$

where $\varepsilon_{ijkl} \sim N(0, \sigma)$, and ε_{ijkl} are independent for $i = 1, 2, 3$, $j = 1, 2$, $k = 1, 2$, and $l = 1, 2$.

b) $\nu_{\text{Temperature}} = \frac{SS_A/(a-1)}{SSE/(dfe)} = \frac{2151.58/2}{66.00/12} = 195.60 > F_{0.05}(2, 12) = 3.89$, then reject H_0, that is, temperature is significant.
$\nu_{\text{Pressure}} = \frac{SS_B/(b-1)}{SSE/(dfe)} = \frac{104.17/1}{66.00/12} = 18.94 > F_{0.05}(1, 12) = 4.75$, then reject H_0, that is, pressrue is significant.
$\nu_{\text{Day}} = \frac{SS_C/(c-1)}{SSE/(dfe)} = \frac{32.67/1}{66.00/12} = 5.94 > F_{0.05}(1, 12) = 4.75$, then reject H_0, that is, day is significant.
$\nu_{\text{Temperature*Pressure}} = \frac{116.58/2}{66.00/12} = 10.60 > F_{0.05}(2, 12) = 3.89$, then reject H_0, that is, interaction between temperature and pressrue is significant.
$\nu_{\text{Temperature*Day}} = \frac{3.08/2}{66.00/12} = 0.28 < F_{0.05}(2, 12) = 3.89$, then don't reject H_0, that is, interaction between temperature and day is NOT significant.
$\nu_{\text{Pressure*Day}} = \frac{0.17/1}{66.00/12} = 0.03 < F_{0.05}(1, 12) = 4.75$, then don't reject H_0, that is, interaction between pressure and day is NOT significant.
$\nu_{\text{Temperature*Pressure*Day}} = \frac{1.08/2}{66.00/12} = 0.10 < F_{0.05}(2, 12) = 3.89$, then don't reject H_0, that is, interaction between temperature, pressure and day is NOT significant.