ANSWERS

TAMS38 - Experimental Design and Biostatistics, 4 hp / 6 hp Examination on 21 August 2018, 14:00-18:00.

1 **a)** Model: $Y_{ij} = \mu + \tau_i + \epsilon_{ij}$, where $\tau_1, ..., \tau_4$ are fixed, $\epsilon_{ij} \sim N(0, \sigma)$, and ϵ_{ij} are independent for $j = 1, ..., 6$, $i = 1, ...4$.

b) $\bar{y}_{..} = 19.133$

$SS_{TREAT} = \sum_{i=1}^{4} 6(\bar{y}_{i.} - \bar{y}_{..})^2 \approx 34.07$, and $SS_E = \sum_{i=1}^{4}(6-1)s_i^2 \approx 63.45$, $N = 24$ and $a = 4$.

Analysis of Variance

	Sum of squares	DF
Between groups	34.07	3
Within group	63.45	20

c)

$$\nu = \frac{SS_{TREAT}/(a-1)}{SS_E/(N-a)} = 3.58 > F_{0.05}(3, 20) = 3.10,$$

then reject H_0. That is, there exists significant difference in life between those four fluids.

d) t-interval method gives $1 - \binom{4}{2}\alpha = 94\%$, then $\alpha = 0.01$.
Let $\mu_i = \mu + \tau_i, i = 1, \ldots, 4.$ then

$$I_{\mu_i - \mu_j} = \bar{y}_{i.} - \bar{y}_{j.} \pm t_{0.01/2}(24-4)(s)\sqrt{\frac{1}{n_i} + \frac{1}{n_j}} = \bar{y}_{i.} - \bar{y}_{j.} \pm (2.85)\sqrt{\frac{SS_E}{N-a}}\sqrt{\frac{1}{6} + \frac{1}{6}} = \bar{y}_{i.} - \bar{y}_{j.} \pm 2.93.$$

Then $I_{\mu_1-\mu_2} = (-2.23, 3.63)$, $I_{\mu_1-\mu_3} = (-5.4, 0.46)$,$I_{\mu_1-\mu_4} = (-3.1, 2.76)$, $I_{\mu_2-\mu_3} = (-6.1, -0.24)$,

$I_{\mu_2-\mu_4} = (-3.8, 2.06)$, $I_{\mu_3-\mu_4} = (-0.63, 5.23)$.

Thus, we get $\mu_3 > \mu_2$, so I would select fluid type 3.

2 We choose non-parametric test: Kruskal - wallis test.

H_0 : There is no significant difference between 4 fluids.
H_1 : There is significant difference between 4 fluids.

Treatments 1	2	3	rank r_{ij} 4
17.6 r 8	16.9 r 3.5	21.4 r 21	19.3 r 13
18.9 r 12	15.3 r 1	23.6 r 24	21.1 r 20
16.3 r 2	18.6 r 11	20.4 r 18	16.9 r 3.5
17.4 r 6	17.1 r 5	18.5 r 10	17.5 r 7
20.1 r 16	19.5 r 14	20.5 r 19	18.3 r 9
21.6 r 22	20.3 r 17	22.3 r 23	19.8 r 15
$s_1 = 66$	$s_2 = 51.5$	$s_3 = 115$	$s_4 = 67.5$

$$T = T_{\text{ties}} = \frac{(N-1)(S_a - C)}{S_r - C} \approx 7.63,$$

where $S_a = \sum_{i=1}^{4} \frac{s_i^2}{n_i} \approx 4131.58$, $C = \frac{N(N+1)^2}{4} = 3750$ and $S_r = \sum_{i=1}^{4} \sum_{j=1}^{6} r_{ij}^2 = 4899.5$.

$T = 7.63 < \chi^2_{0.05}(3) = 7.82$, we don't reject H_0. That means, with 95% confidence we can say that there is NO significant difference in life between those four fluids.

3

$$\nu = \frac{SS_{Copper}/4}{SS_E/8} = 20.13 > F_{0.01}(4,8) = 7.01,$$

then reject H_0. That is, copper concentration seems to have importance to the BOD value.

4 According to the block $E = ABCD$, we have

$E = 1$, (1) ab ac bc ad bd cd abcd;
$E = -1$, a b c abc d abd acd bcd.

5 **a)** e a b abe c ace bce abc d ade bde abd cde acd bcd abcde. The generator for the design is $I = ABCDE$.
b) All alias structures are:
$I = ABCDE$, base effect + ABCDE effect ; $A = BCDE$, A effect + BCDE effect ;
$B = ACDE$, B effect + ACDE effect ; $AB = CDE$, AB effect + CDE effect ;
$C = ABDE$, C effect + ABDE effect ; $AC = BDE$, AC effect + BDE effect ;
$BC = ADE$, BC effect + ADE effect ; $ABC = DE$, ABC effect + DE effect ;
$D = ABCE$, D effect + ABCE effect ; $AD = BCE$, AD effect + BCE effect ;
$BD = ACE$, BD effect + ACE effect ; $ABD = CE$, ABD effect + CE effect ;
$CD = ABE$, CD effect + ABE effect ; $ACD = BE$, ACD effect + BE effect ;
$BCD = AE$, BCD effect + AE effect ; $ABCD = E$, ABCD effect + E effect .