Kurskod: TAMS38 - Provkod: TEN1

TAMS38 – Experimental Design and Biostatistics, 6 hp
August 20 2019, 14–18

The collection of the formulas in mathematical statistics prepared by Department of Mathematics LiU and calculator with empty memory are allowed on the exam. Dictionary English-other language are allowed. No extra notes in the formula collection is allowed.
Score limits: 7-9 points gives grade 3, 9.5-12 gives 4 and 12.5-15 gives 5.
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The result will *normally* be published via LADOK within 15 working days.
Clear answers and justifications are required for each problem.

1. Three different fertilizers, F_1, F_2, F_3, have been tested on an oat field and the harvest y_{ij} has been measured.

	y_{ij}							$\bar{y}_{i\cdot}$	s_i
F_1	12.7	13.8	19.8	15.2	13.9	19.1	16.2	15.81	2.73
F_2	20.7	18.5	20.0	19.2	20.3			19.74	0.88
F_3	24.5	23.4	22.8	20.2	23.2	24.3		23.07	1.55

Model: Fertilizer i have observations y_{ij} where $Y_{ij} \sim N(\mu_i, \sigma_i)$ and are independent.

(a) Give a single factor model and test using a F-test at level 5% if there are any difference between the fertilizers. Also give the hypothesis based on the given model. (1.5p)

(b) At level 5%, test the null hypotheses that the standard deviations are equal for the different fertilizers. It is enough if you only perform one test. Discuss your conclusions. (1.5p)

2. Test the hypothesis in 1(b) with a non-parametric test, i.e., do not assume normality for the data. Conclusion? (2p)

3. Suppose you wish to determine whether there are differences in average prices among four major supermarkets in a given city (identified as A, B, C, and D). From the regularly bought items (for example, milk, cereal, meat, cheese, and so on), four distinct items are randomly selected. In addition, four different brands of each item are available at each supermarket. The following sample data are the item prices (in dollars) and were recorded according to the indicated 4×4 Latin square design.

	Brands			
	1	2	3	4
Items				
1	D=3.09	A=2.84	B=2.90	C=2.80
2	A=0.51	B=0.56	C=0.53	D=0.65
3	B=1.07	C=1.09	D=1.20	A=1.05
4	C=3.70	D=3.99	A=3.69	B=3.65

(a) Provide a resonable graph of the sample data. What is your initial conclusion about price difference among the four supermarkets, on average? (1p)

(b) Give the model and use the ANOVA-table below to test the null hypothesis of no average price differences among the supermarkets at level 5%. (2p)

(c) Compare the supermarkets using confidence intervals with the exact simultaneous confidence level of 95%. Conclusion? (2p)

	DF	SS
Item	3	27.0282
Brand	3	0.0142
Market	3	0.1210
Error	6	0.0127
Total	15	27.1761

4. We want to conduct an experiment with the design 2^{5-1} given the factors A, B, C, D and E, where E is given as E=ABCD. The experiment gives the following observations and large values are better than small.

(1)	11.2	d	17.6
a	9.8	ad	16.8
b	13.5	bd	19.6
ab	11.5	abd	16.6
c	21.9	cd	24.7
ac	18.4	acd	21.5
bc	17.4	bcd	21.4
abc	21.0	abcd	24.3

and analysis for a complete four factor model with the observations y are in c17 in standard order with respect to the factors A, B, C and D.

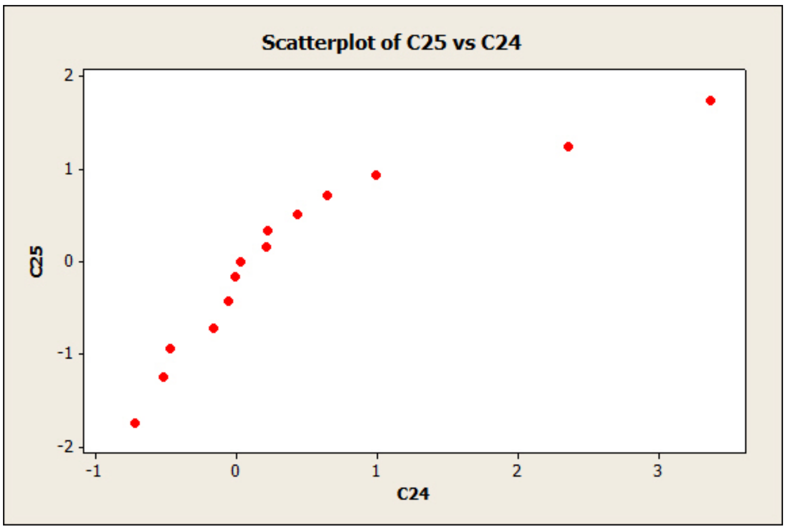
```
MTB > copy c1-c16 m1
MTB > copy c17 m2
MTB > trans m1 m3
MTB > mult m3 m2 m4
MTB > set c18
DATA> 1:16
DATA> end
```

```
MTB > copy m4 c19
MTB > let c20 = c19/16
MTB > Sort c20 c18 c21 c22;
SUBC>   By c20.
MTB > let c23 = 16*c21*c21
MTB > print c21-c23

Data Display

Row      C21  C22      C23
  1  -0,7125   13     8,12
  2  -0,5125    7     4,20
  3  -0,4625    2     3,42
  4  -0,1625   12     0,42
  5  -0,0500   10     0,04
  6  -0,0500   11     0,04
  7   0,0000   14     0,00
  8   0,0375   16     0,02
  9   0,2125    3     0,72
 10   0,2250   15     0,81
 11   0,4375    6     3,06
 12   0,6500    4     6,76
 13   1,0000    8    16,00
 14   2,3625    9    89,30
 15   3,3750    5   182,25
 16  17,9500    1  5155,24

MTB > copy c21 c24;
SUBC> omit 16.
MTB > nscores c24 c25
MTB > plot c25*c24
```



(a) Which effects seems to be most important in this experiment? Which parameter estimate includes the E-effect? (1p)

Given the result in a) we do another analysis.

```
MTB > ANOVA 'y' = C D;
SUBC>   Means C D.

ANOVA: y versus C; D

Analysis of Variance for y

Source  DF      SS      MS      F      P
C        1  182,25  182,25  54,31  0,000
D        1   89,30   89,30  26,61  0,000
Error   13   43,63    3,36
Total   15  315,18

S = 1,83193   R-Sq = 86,16%   R-Sq(adj) = 84,03%

Means

C   N       y
-1  8  14,575
 1  8  21,325

D   N       y
-1  8  15,587
 1  8  20,313
```

(b) Which model is used in the second analysis? Can you find the best combination of the used factors? Motivate your answer with some confidence intervals with the simultaneous confidence level at least 90%. (2p)

5. Consider the random effects model $Y_{ij} = \mu + \tau_i + \varepsilon_{ij}$, where $\tau_i \sim N(0, \sigma_\tau)$ and $\varepsilon_{ij} \sim N(0, \sigma)$, $i = 1, \ldots, a$, $j = 1, \ldots, n$.
Construct a 95% confidence interval for the ratio $\sigma_\tau^2/(\sigma_\tau^2 + \sigma^2)$. (2p)

Hint: $\dfrac{SS_E}{\sigma^2} \sim \chi^2(N-a)$ and $\dfrac{SS_{TREAT}}{n\sigma_\tau^2 + \sigma^2} \sim \chi^2(a-1)$.

The ratio $\sigma_\tau^2/(\sigma_\tau^2 + \sigma^2)$ is called the *intraclass correlation coefficient* and defines what proportion of the variance is from the random effects, i.e., the difference in treatment.

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Some extra formulas

Some nonparametric tests

Tukey-Duckworth's quick test

If $4 \leq n_1 \leq n_2 \leq 30,$ and $n_2 \leq \frac{4n_1}{3} + 3$ then we can test the null hypothesis H_0 with the test:

1. Find the smallest and largest observation in each sample, respectively.
2. For the sample with the largest observation, count how many observation that is larger in that sample than in the other sample.
3. For the other sample, count how many observations that is smaller then the smallest observation in the first sample.
4. Let C be the sum of the number of observations in 2. and 3. For $\alpha = 0.05$, 0.01 or 0.001, reject H_0 if $C \geq 7$, 10, or 13, respectively.

The Wilcoxon signed rank test

Let r_i be the rank for the observations $|y_i| \neq 0, i = 1, \ldots, n$. Let $T_+ = \sum_{\{y_i>0\}} r_i$ and $T_- = \sum_{\{y_i<0\}} r_i$. When H_0 is true and $n > 15$ us that T_+ and $T_- \approx N\left(\frac{n(n+1)}{4}, \sqrt{\frac{n(n+1)(2n+1)}{24}}\right)$. For $n \leq 15$ us table for Wilcoxons signed rank distribution.

For confidence interval us the $N = n(n+1)/2$ ordered pairwise means A_i and $P(A_{(k)} < \mu < A_{(N-k+1)}) = 1 - 2P(W_S \leq k-1)$, where W_S is Wilcoxons signed rank distributed.

The Wilcoxon-Mann-Whitney test

Let d_{ij} be the differences $d_{ij} = x_i - y_j$, $i = 1, \ldots, n_1$, $j = 1, \ldots, n_2$ and $d_{(k)}$, $k = 1, \ldots, n_1 n_2$, the ordered differences. The confidence interval for the difference in mean/median is given by

$$I = \left(d_{(c+1)}, d_{(n_1 n_2 - c)}\right),$$

where $c = T_l - \frac{n_1(n_1+1)}{2}$ and T_l is from the Wilcoxon table for the rank sum test.

The Kruskal-Wallis test

Assume a treatments. Let r_{ij} be the rank for the observation y_{ij}. Test statistic

$$T = \begin{cases} \dfrac{12S_a}{N(N+1)} - 3(N+1), & \text{if no ties,} \\ \\ \dfrac{(N-1)(S_a - C)}{S_r - C}, & \text{if ties,} \end{cases}$$

where $s_i = \sum_{j=1}^{n_i} r_{ij}$, $S_a = \sum_{i=1}^{a} \frac{s_i^2}{n_i}$, $S_r = \sum_{i=1}^{a} \sum_{j=1}^{n_i} r_{ij}^2$, $C = \frac{1}{4}N(N+1)^2$ and $N = \sum_{i=1}^{a} n_i$. For *small* values of $n_1, \ldots, n_a$ ($a \leq 3$ and $n_i \leq 5$) use table and for *large* values of $n_1, \ldots, n_a$ use that $T \approx \chi^2(a-1)$ when there is no treatment effect.

The Friedman test

Assume t treatments and b blocks. Let r_{ij} be the rank of y_{ij} within each block, i.e., for each j, $r_{ij} = 1, \ldots, t$. Test statistic for the treatments is given by

$$T = \begin{cases} \dfrac{12S_t}{t(t+1)} - 3b(t+1), & \text{if no ties,} \\ \\ \dfrac{b(t-1)(S_t - C)}{S_r - C}, & \text{if ties,} \end{cases}$$

where $s_i = \sum_{j=1}^{b} r_{ij}$, $S_t = \frac{1}{b} \sum_{i=1}^{t} s_i^2$, $S_r = \sum_{i=1}^{t} \sum_{j=1}^{b} r_{ij}^2$ and $C = \frac{1}{4}bt(t+1)^2$. For *small* values of b and t ($t = 3$, $b \leq 15$ and $t = 4$, $b \leq 8$) use table and for *large* values of b and t use that $T \approx \chi^2(t-1)$ when there is no treatment effect.